

The Home Math Environment: More Than Numeracy

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Abstract

The goal of the current study is to develop a more complete understanding of the early *home math environment*, encompassing both numeracy and non-numeracy aspects of that environment. Parents of preschoolers ($n = 63$) were surveyed about their support of three components of early mathematics knowledge (i.e., numeracy, spatial, and pattern) as well as parents' math-related beliefs about themselves and their children. Children were administered a broad math knowledge assessment which included a numeracy subscale, and individual measures of spatial and patterning skills in the fall (concurrently). Broad math knowledge was measured again in the spring of the preschool year. Parents indicated providing some support of early math development through numeracy, spatial, and patterning activities, with a stronger emphasis on numeracy than pattern and space. Parents' child-specific ability beliefs were related to their numeracy, pattern, and broad math support, while their parent-specific ability beliefs were related to their spatial support. Parent support was rarely linked to child skills, except that numeracy support related to concurrent numeracy knowledge. Findings suggest that although parents do support a broad range of early math skills at home, parents tend to prioritize supporting early numeracy. Parents' beliefs, especially about their child's academic abilities, may influence components of the early home math environment, but future research is needed to better understand the relations between parent's academic beliefs and the home math environment they create.

The Home Math Environment: More Than Numeracy

Children's knowledge of mathematics upon school entry is critical to their futures. Math knowledge is strongly predictive of later math and reading skills, which are imperative for academic achievement (Watts, Duncan, Siegler, & Davis-Kean, 2014). In turn, individuals with better math knowledge enter into more prestigious careers, attain higher earnings, and make better healthcare decisions (Lipkus & Peters, 2009; Ritchie & Bates, 2013; Shapka, Domene, & Keating, 2006). Math knowledge varies substantially before school entry (Starkey, Klein, & Wakeley, 2004). This timing suggests that much can be learned about mathematics outside of school, such as during interactions with parents. Most research on parents' contribution to early math development, like that on math development itself, has focused on numeracy, which is only one component of mathematics. Too little is known about how parents support children's mathematics development, broadly defined.

The current study had three aims. The first aim was to develop a better understanding of the home math environment. We did this by directly assessing and comparing the frequency with which parents of preschoolers reported providing home-based support of three components of early math knowledge: numeracy, pattern, and space. The second aim was to explore relations between parents' academic beliefs about themselves and their children and the broad math support parents provide. The third aim was to explore whether parental support of each component of math was correlated with children's corresponding skills and math knowledge at two time points (beginning- and end-of-the-pre-kindergarten school year). Related to these aims, we review what is currently known about the home math environment that parents of young children provide, the relations between parental beliefs and the early home math environment, and how the early home math environment relates to children's knowledge and skills.

Early Home Math Environment

Parents support their children's academic readiness and growth outside of school. Considerable research has demonstrated that the *home literacy environment* parents provide, such as shared book reading, is related to their children's reading achievement (Sénéchal & LeFevre, 2002). Based largely off of this framework, research has also shown that parents provide support for numeracy skills at home, such as counting objects together, sometimes referred to as the *home numeracy environment* (LeFevre et al., 2009).

Our goal is to develop a more complete understanding of the early *home math environment*, encompassing both numeracy and non-numeracy aspects of that environment. Math knowledge is more than numeracy knowledge, even in early childhood (LeFevre, Fast, et al., 2010; National Research Council, 2009; Sarama & Clements, 2004). The National Research Council (2009) report on mathematics learning in early childhood notes: "Mathematics provides a powerful means for understanding and analyzing the world. Mathematical ways of describing and representing quantities, shapes, space, and patterns help to organize people's insights and ideas about the world in systematic ways" (p. 21). Such a description highlights the importance of patterns, space and shapes in math. Working with repeating patterns (e.g., blue-blue-green-blue-blue-green) provides children with early opportunities to think about relationships, rules and regularities fundamental to algebraic thinking (National Research Council, 2009). In addition, mathematical thinking often involves spatial thinking, such as representing magnitudes along a mental number line and identifying and manipulating geometric shapes (Mix & Cheng, 2012). When asked to identify their top three math activities, preschool teachers most often selected patterning and spatial relations among non-numeracy math activities (Sarama, 2002). Further, there is strong evidence that repeating patterning and spatial knowledge in preschool is related to

children's later math performance. In contrast, there is little direct evidence for such a link beginning in preschool for other topics, such as shape and measurement (Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). Thus, in addition to understanding the numeracy experiences that parents provide, we need to understand how parents support other important early math skills, specifically repeating pattern and spatial thinking. Below we summarize the research on parents' support of children's numeracy, pattern, and spatial thinking, also referred to as the home numeracy, spatial, and pattern environments.

Numeracy support. Numeracy activities focus on numbers, including count words, numeral names, and combining and comparing magnitudes (LeFevre et al., 2009; Ramani & Siegler, 2008; Skwarchuk, Sowinski, & LeFevre, 2014). Some home experiences are formal, with explicit direct practice with numeracy skills (e.g., naming numerals), and other experiences are informal, with numeracy practice embedded in everyday activities, such as reading number-related storybooks and playing card and board games (Hart, Ganley, & Purpura, 2016; Huntsinger, Jose, & Luo, 2016; Skwarchuk et al., 2014). Parents provide formal numeracy activities frequently at home. For example, they report frequent (i.e., daily to weekly) engagement in counting and using number words (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009; Thompson, Napoli, & Purpura, 2017). However, parents of preschoolers report less frequent (e.g., monthly) support of some formal number skills, such as comparing numerical magnitudes and doing arithmetic (Thompson et al., 2017; Zippert & Ramani, 2017) and of many informal numeracy activities (Thompson et al., 2017).

Observational studies also indicate that parents frequently support numeracy knowledge. Naturalistic home observations suggested that parents produce up to 1,700 number words (e.g., numbers 1-10, *count*) a week to their children between the ages of 14- and 30-months (Levine,

Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Additionally, number talk (e.g., counting, labeling cardinal values, naming numerals) accounted for over a third of parents' total talk to their 3- to 5-year-olds during play (Ramani, Rowe, Eason, & Leech, 2015).

Although home numeracy activities may be frequent for young children, they occur less often than non-numeracy activities. Parents reported supporting their kindergarteners' literacy skills (e.g., helping their child read, print words) several days a week, but numeracy activities (e.g., learning simple sums, asking about quantities) on a weekly or monthly basis (Skwarchuk et al., 2014). Similarly, much of preschoolers' time at home is spent on non-numeracy focused activities (e.g., arts and crafts, play with action figures; Ramani et al., 2015)

Spatial support. Spatial thinking involves visual imagery and mental manipulation of spatial information, which is often needed when solving math problems (Mix & Cheng, 2012). Although less studied than the home numeracy environment, research has identified a range of home spatial activities in which parents engage their children. For example, parents build with blocks, play with puzzles, and draw maps and plans with their preschoolers and elementary schoolers on a monthly basis (Dearing et al., 2012; Hart et al., 2016). Parents also engage in spatial talk with their young children. For example, most parents report using spatial relation words (e.g., “between”, “below”) to describe locations and positions of objects and people with their preschoolers (Verdine et al., 2014). Similarly, parents of 3- to 6-year-old children reported using spatial location words (e.g., “over”, “on”) as often as 3- to 5-times a week (Missall, Hojnoski, Caskie, & Repasky, 2015). Observations of parent-child talk during a play session also suggests that parents use spatial words including but not limited to spatial relation and location words, with spatial words accounting for 6% of total words spoken when children were between 14 and 46 months old (Ho, Lee, Wood, Kassies, & Heinbuck, 2017).

Patterning support. Patterning activities encompass identifying and using predictable sequences, such as arrays of shapes or sounds (Burgoyne, Witteveen, Tolan, Malone, & Hulme, 2017). In early childhood, the focus is on repeating patterns, or linear sequences that have repeating units, such as ABBABB (National Association for the Education of Young Children, 2014). Only two studies have examined the variety of patterning activities done at home, and they suggest that parents frequently do patterning activities with their 3- to 5-year-old children (Missall et al., 2015; Rittle-Johnson, Fyfe, Loehr, & Miller, 2015). Specifically, a small but diverse set of parents reported that some activities (i.e., reading books and watching television shows involving patterns) occurred multiple times a week, and other patterning activities (i.e., making and copying patterns, figuring out what comes next in a pattern) occurred weekly (Rittle-Johnson et al., 2015). The latter activities align with early childhood guidelines for patterning, which emphasize children making, duplicating, and extending repeating patterns (National Association for the Education of Young Children, 2014).

Broad support. Although no study has comprehensively examined all three of these components of the home math environment, a few studies have simultaneously considered two components of early math knowledge or sampled a few example activities of each type (Dearing et al., 2012; Huntsinger et al., 2016; Missall et al., 2015; Rittle-Johnson et al., 2015). The most comprehensive explored the numeracy and spatial activities parents reported providing for preschool and elementary school age children (Hart et al., 2016). Although the authors did not compare the frequency of the different types of math activities, parents reported engaging in 18 numeracy activities about once a week and engaging in 5 spatial activities a few times a month. This suggests that parents may provide broad math experiences but tend to engage in more numeracy than spatial activities, at least with their children ages 3 to 8. Research is needed that

substantially and simultaneously assesses the home number, spatial, and patterning environments and directly compares their frequencies before children begin formal schooling.

Relations Between Parents' Academic Beliefs and the Early Home Math Environment

Past work suggests that the home academic environment includes not only the experiences parents provide, but the beliefs and cognitions that motivate their efforts (Eccles et al., 1983; Jacobs, Davis-Kean, Bleeker, Eccles, & Malanchuk, 2005; Pellegrini & Stanic, 1993). A theory of parent socialization, developed by Eccles and colleagues, suggest two types of parental beliefs that may influence parents' provision of home-based academic support: (a) general beliefs, henceforth referred to as parent-specific academic beliefs (e.g., personal academic attitudes), and (b) child-specific academic beliefs, such as ratings of their child's interest and abilities in math and the importance of math (Eccles, 1993). Parents who are not confident in their own math skills, devalue the importance of math for themselves, and find the subject uninteresting are posited to provide infrequent support of their child's math learning at home compared to parents with more positive academic beliefs about themselves. Similarly, parents who believe their children have low interest and ability in math are thought to provide infrequent math experiences compared to parents who believe their children are more interested in and able to engage in math activities. The theory further suggests parents' beliefs and practices will impact children's math achievement, with some empirical evidence in support of the theory for math achievement with preschool- and kindergarten-aged children (Huntsinger, Jose, & Fong-Ruey, 1997; Huntsinger, Jose, Larson, Balsink Krieg, & Shaligram, 2000; Huntsinger et al., 2016). Ultimately, theorists posit that parents' positive academic beliefs about their child and themselves lead them to create more frequent opportunities for their child to learn academic content in the home (e.g., one-on-one academic support). We review studies that have

examined parental beliefs about themselves and their children as predictors of the home math environment for preschool- and elementary-school-aged children.

Parent beliefs and the home numeracy environment. Past research has focused on how parents' beliefs about their own math abilities and interests related to the home numeracy environment, and evidence is mixed. Some studies report that more positive parental beliefs about their own interest and ability in math helps predict the home numeracy environment they reported providing for their preschoolers (Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Skwarchuk, 2009). Observational research corroborates these results. Specifically, mothers' positive beliefs about their own numeracy knowledge positively predicted their observed numeracy support during an informal free play interaction with their 5- to 6-year-old child (Elliott, Braham, & Libertus, 2017). However, some studies on the home numeracy environment have found near-zero correlations between parents' math beliefs about themselves and the frequency of the numeracy support they reported providing in a sample of kindergarten-aged children (Skwarchuk et al., 2014) and a sample of preschoolers through fourth graders (Sonnenschein et al., 2012). In another study, parents' reported math anxiety was not related to their reported frequency of home numeracy activities for children ages 3 to 8 (Hart et al., 2016), but in another there was a significant negative correlation for 5-year-old children (del Río, Susperreguy, Strasser, & Salinas, 2017). Although the reasons for these mixed findings have not been identified, potential explanations include differences in the age of the studies' samples, variation in survey questions (e.g., asking about math vs. numeracy) and presentation format (e.g., via a one-on-one interview versus a paper survey completed privately), and the demographic nature of the participants (e.g., convenience versus representative sample).

A few studies suggest that parents' beliefs about their children's abilities in and the importance of numeracy may also be related to the early home numeracy environment. Parents who believed their children ages 3 to 8 had high math and spatial ability (measured as a single variable) also reported frequent numeracy activities in the home (Hart et al 2016). In addition, parents' beliefs about the importance they place on their children ages 5 to 6 attaining specific numeracy skills was correlated with the early home numeracy environment (Musun-Miller & Blevins-Knabe, 1998; Skwarchuk et al., 2014), though evidence of null findings also exists (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010). Overall, some parent-specific and child-specific academic beliefs are sometimes related to the early home numeracy environment, but evidence is limited and mixed.

Parent beliefs and the home pattern and spatial environment. We only identified one study that considered the relation between parents' academic beliefs and the early home spatial environment, and no study did so for the home pattern environment. Parents who reported high math anxiety reported providing frequent spatial activities for their 3- to 8-year-old children, after controlling for general home math environment (Hart et al., 2016). This finding was unexpected, difficult to interpret because of the control for the general home math environment, and a potential explanation for it was not offered. In addition, parents who believed their children had high math and spatial ability (measured as a single variable) also reported frequent spatial activities in the home (Hart et al., 2016). This study suggests that parent-specific and child-specific academic beliefs may be related to the early home spatial environment, but little is known about relations with beliefs specific to spatial tasks.

Parent beliefs and the broad home math environment. A few studies suggest that parents' beliefs about their children's abilities and interests in math are related to the early home

math environment. Parents' ratings of their preschoolers' math interests and abilities predicted parental reports of formally teaching general math concepts at home to their elementary school-aged children (Huntsinger et al., 1997). Further, parents who believed their children had high math and spatial ability (measured as a single factor) also reported frequent math activities in the home (Hart et al 2016). In addition, we expected findings on the home numeracy environment would generalize to the home math environment.

Summary. According to Eccles' theory of parent socialization, parents' academic beliefs about themselves and their child influence the home environment they create (Eccles et al., 1983; Jacobs et al., 2005). Evidence related to the early home math environment in particular is fairly limited, with mixed evidence. More positive parental beliefs about their own interest and ability in math predicted the home numeracy environment they provided in preschool in some studies (Blevins-Knabe et al., 2000; Skwarchuk, 2009), but not in others (Sonnenschein et al., 2012). Further, parents' beliefs about their children's abilities and interests in math were related to the early home math environment they created for their elementary school students in a few studies (Musun-Miller & Blevins-Knabe, 1998; Skwarchuk et al., 2014). Overall, we know too little about relations between parents' academic beliefs about themselves and their child and the early home math environment they create, with almost no evidence on relations with beliefs about spatial and patterning tasks.

Relations Between the Early Home Math Environment and Children's Knowledge and Skills

Interest in the home academic environment is often based on the claim that it supports children's academic performance. Although this is generally true (Melhuish et al., 2008), evidence for a relation between the early home math environment and children's math knowledge measured more broadly is limited, with mixed findings.

Numeracy. First consider research on the home *numeracy* environment and its relation to children's numeracy knowledge. The frequency of parent numeracy support was predictive of children's numeracy knowledge both concurrently (Blevins-Knabe & Musun-Miller, 1996; Ramani et al., 2015; Skwarchuk, 2009; Zippert & Ramani, 2017) and longitudinally (Gunderson & Levine, 2011; LeFevre, Clarke, & Stringer, 2002; LeFevre et al., 2009; LeFevre, Polyzoi, et al., 2010; Niklas & Schneider, 2014; Skwarchuk et al., 2014), with few studies failing to find a relation between the two (e.g., Blevins-Knabe et al., 2000). Further, recent evidence suggests specific links between type of numeracy support and type of numeracy outcome. Formal numeracy support at home predicted children's symbolic numeracy knowledge, (e.g., comparing the magnitudes of symbolic numbers and counting), whereas informal numeracy support did not (Hart et al., 2016; LeFevre, Polyzoi, et al., 2010; Ramani et al., 2015; Skwarchuk et al., 2014). In contrast, informal numeracy support at home predicted kindergarteners' non-symbolic numeracy knowledge (e.g., quickly deciding which set of objects has more without counting), whereas formal numeracy support did not (Skwarchuk et al., 2014). Whether numeracy support of either type is related to broad math knowledge, patterning, or spatial skills is unknown. Because of evidence for specific links between type of numeracy support and type of numeracy outcome, home numeracy support is likely not related to non-numeracy skills like patterning and spatial.

Spatial. Some past research supports a positive relation between home spatial support and children's spatial skills, although evidence is limited and inconsistent. First, frequency of observed parent spatial talk at 14 and 46 months correlated with their children's spatial skills in preschool (Pruden & Levine, 2017; Pruden, Levine, & Huttenlocher, 2011). Also, 4- through 7-year-old children whose parents reported frequent puzzle, block, and board game play at home had higher spatial skills than children who engaged in these activities sometimes or rarely in a

nationally-representative, large study of almost 850 children (Jirout & Newcombe, 2015).

However, the size of this effect was small. In a study of 127 diverse families, parental reports of home spatial activities (i.e., building with construction toys and playing with puzzles) was not related to first graders' concurrent spatial skills (Dearing et al., 2012).

Limited evidence also supports a positive relation between one component of the home spatial environment – spatial talk - and children's numeracy skills. Frequency of parents' spatial talk with their children at age 36 months correlated with children's later numeracy knowledge in preschool and first grade (Lombardi, Casey, Thomson, Nguyen, & Dearing, 2017). In contrast, parental reports of home spatial activities was not associated with first-graders' concurrent arithmetic skills (Dearing et al., 2012).

Thus, the frequency of parents' spatial talk in infancy and toddlerhood is related to future spatial and numeracy skills. In contrast, the frequency of parent-reported spatial experiences at home seems to be weakly related to children's spatial skills given that a small effect was only detected in a study with a large sample (Jirout & Newcombe, 2015).

Patterning. In the only relevant study on patterning, parent-reported pattern support marginally related to their child's patterning skills at the end of preschool (Rittle-Johnson et al., 2015). Given their small sample ($n = 20$), the results require replication with a larger sample.

Broad math. Research on the relations between the broad home math environment and math knowledge is limited with mixed findings. Parents' reports of engaging in general math, numeracy and spatial activities correlated positively with their preschool child's broad math knowledge (DeFlorio & Beliakoff, 2014). Similar results were found for parent-reported broad math support and numeracy knowledge with elementary schoolers (Huntsinger et al., 2016). In contrast, parent-reported broad math support (i.e., numeracy, geometry and a few patterning

items) with preschoolers was not related to early numeracy knowledge (Missall et al., 2015), perhaps due to a misalignment of the survey and outcome. Ultimately, synthesizing findings of prior work is challenging due to inconsistencies in surveys items and child outcomes across studies.

Summary. There is some evidence that components of the early home math environment are related to components of early math knowledge. Most of the evidence shows a relation within the same component (e.g., numeracy support and numeracy knowledge), and a few studies have found no relations. There is only evidence for one cross-component relation: frequency of spatial talk was related to numeracy knowledge in one study.

Current Study

To help address some of the limitations in past research, we focused on three research questions. First, do parents emphasize some math topics more than others? We hypothesized that parents would emphasize numeracy topics more than patterning or spatial topics, based on comparing different activity frequencies across studies.

Our second research question is: Do parents' beliefs about math, spatial, and patterning abilities and interests relate to the home math environment they create? Our tentative hypothesis, drawing on Eccles' theory of parent socialization (Eccles, 1993), is that a mix of parents' beliefs about their own and their child's academic abilities and interests would relate to the early home math environment parents provide. Limited past research with mixed findings made it difficult to predict specific relations, so we aimed to provide exploratory evidence to inform future research.

Our third research question is: Is the early home math environment related to children's math knowledge at the beginning and end of the final year of preschool? We consider math broadly (both in the home environment and as a child outcome) and as individual components

(numeracy, pattern and space). We hypothesize that the broad early math home environment will positively correlate with broad math knowledge, and that individual components of the home math environment will correlate with their respective skills. We also explored cross-component relations (e.g., early numeracy environment to spatial skills), although we did not expect them to be substantial or significant. In order to focus on relations between the early home math environment and math knowledge specifically, we controlled for child verbal ability, a common non-math correlate of early math knowledge (LeFevre, Fast, et al., 2010; Watts et al., 2014).

Method

Participants

Sixty-three primary caregivers of preschoolers enrolled in a larger study on the contributions of patterning and spatial skills to mathematics development participated in the study (see Table S1 for demographic details). The researchers recruited families from six preschool programs (three public, one Head Start center, and two private) in a Southeastern U.S. state. The primary caregivers (henceforth referred to as parents) were mostly mothers (86%), and slightly over half of all the primary caregivers (55%) reported being racial and ethnic minorities (see Table 1). The majority of children's parents (91% of mothers and 73% of fathers) had at least an associate's degree or some college education.

Children (52% girls) were 4.5 years old on average ($SD = .29$) when first assessed. Parents reported that 57% of the children were racial and ethnic minorities, and that 8% of children were bilingual. Almost half of families received some amount of financial assistance for their child's preschool attendance fees, and 10% of children received special education services.

Procedure

Parents (82% of the larger study's sample) completed a 10-minute survey on paper or on line, with most filling out the survey privately, and received a \$10 gift card. Children's math, patterning, spatial, and language skills were assessed at their schools during the fall of the school year (Time 1), and math skills were re-assessed during the spring of the school year (Time 2), 6.8 months ($SD = 7.6$ days) after Time 1.

Measures

Parent survey.

Numeracy, patterning, and spatial support. Parents reported the frequency with which they engage their children in various numeracy, pattern, and spatial activities at home using 25 items on a 6-point Likert-type scale, where 0 = never, 1 = once a month or less, 2 = 2- to 3-times a month, 3 = 1- to 2-times a week, 4 = 3- to 4-times a week, 5 = daily (see Table 1 for a list of all items and their reliabilities for each component of math). Nine items were numeracy-related, seven were related to spatial activities, and nine were pattern related. We adapted the majority of items from previous reliable instruments (Dearing et al., 2012; LeFevre et al., 2009; Rittle-Johnson et al., 2015). We created composite scores for each activity type by averaging across items of a particular type, as well as a broad math support composite, calculated by averaging across all activity items. Internal consistency was good for composites of numeracy and pattern activity items, and moderate for spatial activities.

Parents' academic beliefs. Parents responded to questions adapted from previous surveys (Eccles et al., 1983; LeFevre et al., 2009; Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015) regarding their math-related beliefs (see Table 2 for list of item types and scales, and Table S2 for full survey). The first section focused on parent-specific academic beliefs. Parents reported their math and spatial related beliefs, attitudes and expectations for themselves

by responding to 30 items on 7-point Likert-type scales. Ten of these items asked parents to report their math- and spatial-related beliefs about themselves. In particular, parents rated their math and spatial abilities currently and when in school and reported how important their abilities were to them. They also rated how much they liked math and spatial tasks, and their related anxiety and nervousness surrounding math and spatial tasks. Questions on patterning were not included in this section because it was difficult to generate items that would be relevant for parents. The second section (30 items) asked parents to report their academic beliefs about their children. Twenty-one of these items were related to parents' child-specific beliefs about numeracy (i.e., counting and naming numbers), spatial (i.e., building with blocks and doing puzzles), and patterning (i.e., noticing and making patterns) topics. Specifically, parents rated their children's current and future abilities in these topics, how important abilities in each of these areas were, and how useful parents believed related activities would be to their child in the future. Additionally, parents reported how much their children liked each type of activity. Distractor items such as reporting parents' abilities in writing and life sciences, and children's language abilities and interests were included to mask the study's focus on math.

Child measures.

Math, including numeracy, knowledge. The Research-Based Early Mathematics Assessment Short-Form (Weiland et al., 2012), was used to assess preschooler's math knowledge. The first section assessed children's numeracy knowledge (13 items such as matching numerals 1-5 to corresponding set sizes, object counting, and number comparison) while the second section assessed their geometric knowledge (6 items such as shape identification and shape creation) and patterning (1 item). See (Authors, in press) for details on item scoring and the IRT estimation procedure.

Spatial skills. Three spatial skills assessments were administered and averaged to create a composite measure. We measured children's form perception using The Position in Space subtest of the *Developmental Test of Visual Perception—Second Edition* (Hammill, Pearson, & Voress, 1993) as used by Lachance and Mazzocco (2006). The assessment required children to identify an image matching a target image from a set of four or more figures. We used the Block Design subtest of the Wechsler Preschool and Primary Scale of Intelligence—Fourth Edition (Wechsler, 2012) to measure spatial visualization. The assessment required children to recreate pictures or models of block structures using red and white colored blocks. We used the Corsi Block Tapping Task to measure visual-spatial WM. Children used a version of the Corsi Block that is adapted for use with young children on an iPad (available at <https://hume.ca/ix/pathspan.html>) and used in previous research (LeFevre, Fast, et al., 2010; Xu & LeFevre, 2016). Children needed to copy a jumping frog's path by touching lily pads in the same or backward order the frog jumped on them. Children's scores on the Corsi task were calculated as the number of trials correct on both the forward and backward order of the task.

Patterning skills. The first patterning assessment was a well-established measure developed using items from past research (Papic & Mulligan, 2011; Sarama & Clements, 2004; Starkey et al., 2004). It consisted of nine items measuring preschoolers' ability to duplicate, extend, abstract, and identify units of repeating visual patterns. We also developed a new patterning assessment, using tasks commonly used by teachers in preschool and kindergarten classrooms, including easier tasks than those included on the existing measure (e.g., completing patterns and AB patterns). Thus, this new measure captured earlier developing patterning skills and was more aligned with classroom activities than the first assessment. See (Authors, 2018) for

additional information. Children's ability estimates on the two measures were moderately correlated, $r(61) = .61, p < .01$, and scores were averaged to create a patterning composite.

Verbal ability. We used The Picture Vocabulary Test from version 1.6 of the NIH Toolbox app as a measure of children's verbal ability (Weintraub et al., 2013).

Missing data imputation. Fifteen percent (10 subjects) had missing data on at least one parent survey item. The majority of participants were missing only 2- to 3 percent of data and the total overall missing values were minimal (1.44%). Values for missing parent survey items were generated using the multiple imputations approach in SPSS, through which 30 datasets were generated and aggregated. Paternal education values were missing for 3 parents, so means were imputed. Two children with parent survey data did not complete the math assessment at Time 2, so they were dropped from analyses involving child assessment data.

Analysis Plan

We first explored which demographic factors were associated with home math support and children's broad math knowledge using zero-order Pearson correlations for continuous or polytomous variables (i.e., child age, language, parent education, and financial assistance), and an independent samples t-test for child gender. For our first research question, 3 paired samples t-tests compared parents' number, spatial, and pattern support.

Next, in line with our second research question, which was exploratory, we conducted zero-order correlations between parents' 4 types of support (i.e., numeracy, spatial, pattern, and broad math support) with the 8 parent-specific math beliefs (i.e., ratings of (a) importance of, (b) ability, (c) anxiety, and (d) liking for two academic task types (i.e., math and spatial) and 9 child-specific math beliefs (i.e., ratings of (a) importance of, (b) ability, and (c) interest for three

academic activities and tasks: numeracy, spatial, and pattern). Given the number of analyses conducted for this research question, we flagged findings as significant at the $p < .01$ level.

To address our third research question, we conducted partial correlations, controlling for some child and parent characteristics, between the 4 types of parents' math support and 6 child outcomes (i.e., broad math and numeracy knowledge at both time points, spatial and patterning skills at Time 1). Note that we conducted many correlations when addressing our second and third research questions, so all findings are preliminary and require replication in future research.

Results

Preliminary Analyses

Differences by parent and child demographic factors were tested via correlations (i.e., child age, language, parent education and financial assistance) and t-tests (i.e., child gender) for child assessment data and parent math support variables. Most tests were not significant. For parent math support, only level of financial assistance related to parents' pattern support ($r = .273, p = .033$), with families receiving some or full financial assistance reporting more pattern support at home than families receiving no financial assistance. For child outcomes, child age and language ability related to all child outcome variables (r 's = .348 to .538, p 's < .01), and fathers' education was correlated with children's broad math and numeracy knowledge at both time points, and Time 1 spatial skills (r 's = .314 to .396, p 's < .014). Maternal education only correlated with Time 1 numeracy knowledge ($r = .316, p = .013$). In our analyses with child ability, we controlled for child age at test time, child language ability, and fathers' education level because they were most consistently and strongly related to our variables of interest.

Early Home Math Environment: Parent Emphasis on Different Math Topics

Parents reported engaging in some numeracy activities multiple times a week and most spatial and pattern activities 1 to 3 times a month (see Table 1 for means for items and composites). There was substantial variability in the reported frequency of most items, with the full range of response options used on most items. In line with Hypothesis 1, paired samples *t*-tests confirmed that parents provided numeracy support more frequently than spatial support, $t(62) = 12.16, p < .01, d = 1.52$ and pattern support $t(62) = 9.94, p < .01, d = 1.29$. Spatial and patterning support did not differ significantly in frequency, $t(62) = 1.42, p < .16, d = .19$. Further, all three activity types were significantly and positively correlated—*r*'s ranged from .556 to .677, *p*'s < .01. In line with our hypothesis, parents emphasized math and numeracy over space and patterning at home.

Relations Between Parent Academic Beliefs and The Early Home Math Environment

See supplemental materials for comparisons of parents' academic beliefs (Table S3) and validity evidence for the child-specific belief measures (Table S4). We acknowledge limited evidence for the reliability and validity of our parent belief measures.

In accordance with our second research question, we explored which parental beliefs were associated with parents' numeracy, spatial, and pattern, and broad math support. As shown in Table 2, child-specific beliefs about ability were the primary correlates of parent support, with one exception. Numeracy, pattern, and broad math support were only correlated with parents' child-specific beliefs about ability. Broad math support correlated with child-specific beliefs about ability in numeracy and in patterning. Numeracy and pattern support correlated only with child-specific beliefs about ability in numeracy and patterning, respectively. Child-specific beliefs about interest and importance were not related to parent support, and parent-specific beliefs were also not related to numeracy, patterning, or broad math support. In contrast, spatial

support related to parent-specific, but not child-specific ability beliefs. In particular, spatial support correlated with parents' beliefs about their own spatial abilities.

Overall, child-specific ability beliefs were related to numeracy, pattern, and broad math support, and parent-specific ability belief was related to spatial support. Further, the ability belief was specific to the type of support (e.g., belief about child patterning ability was related to patterning support). These findings are tentative given the exploratory nature of these analyses.

Relations Between the Early Home Math Environment and Children's Knowledge & Skills

Descriptive information on child knowledge and skills and partial correlations with activity composites and child skills are presented at the bottom of Table 2. Overall, parent-reported support was rarely related to children's knowledge, with one exception. Specifically, in partial alignment with our hypothesis, parent numeracy support was significantly positively correlated with children's Time 1 numeracy knowledge. Given past evidence of a link between formal numeracy support and concurrent and future early numeracy knowledge (Hart et al., 2016; Skwarchuk et al., 2014), we created a formal numeracy support subscale (averaging item frequencies of the two counting items, talking about written numbers, adding simple sums, and comparing quantities; $M = 3.10$, $SD = .74$; $\alpha = .80$). Formal numeracy support correlated with numeracy knowledge at Time 1 $r(56) = .305$, $p = .020$, but not at Time 2 $r(56) = .094$, $p = .481$.

Discussion

The current study continues efforts to broaden how we define the early home math environment, recognizing that parents support a broader range of math skills than just numeracy. Further, we explored relations between parent support, parent beliefs, and child skills. We found that parents provided their preschoolers with numeracy, pattern, and spatial support at home, that numeracy activities were the most frequent, and parents' beliefs about ability helped to explain

some of their math support. The home math environment rarely related to child knowledge and skills, although numeracy support related to concurrent numeracy knowledge.

In this section, we discuss children's broad experiences with math at home, how this support is related to parental beliefs, and how parents' efforts could relate to children's broad math knowledge and corresponding skills (e.g., numeracy, patterning, and spatial). We discuss suggestions for future research and end with practical recommendations for parental support of preschoolers' broad math development. Our findings should be interpreted with caution given our fairly small sample size and limited reliability and validity evidence for our parent survey.

Early Home Math Environment: Parent Emphasis on Different Math Topics

An important contribution of this study was the adaptation and compilation of pre-existing surveys to describe and compare preschool children's broad home math experiences. The current study was the first to comprehensively measure and directly compare how often parents reported providing numeracy, pattern, and spatial support at home to their preschoolers.

First, parents are providing a range of math-related activities at home, including patterning and spatial activities. The most common were counting, using spatial words, talking about written numerals, and building with construction toys, which parents reported doing 3- to 4- times a week on average. Activities such as making and copying patterns, playing with puzzles, and adding simple sums also occurred, but were less common, with parents reporting engaging in them multiple times a month on average. Some individual patterning and spatial activities, such as doing mazes and playing games involving patterns, occurred only monthly. There was also large variability in the frequency of home support, with some parents reporting never or rarely engaging in some activities, and others reported almost daily engagement. In line with past work, parents reported engaging in many numeracy activities multiple times a week

(Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009; Thompson et al., 2017) and spatial activities on a monthly basis (Dearing et al., 2012; Hart et al., 2016). It may be that only a narrow range of spatial activities (i.e., construction toy play and spatial talk) are well-integrated into the home lives of young children. In contrast with our findings, two previous studies found that parents of preschoolers reported engaging in patterning activities a few times a week (Missall et al., 2015; Rittle-Johnson et al., 2015). Past research may have overestimated the frequency of patterning activities due to the saliency of the studies' focus on patterning (e.g., the majority of questions on one past survey measured home patterning support, which may have biased parents to inflate their reports of home patterning activities). Alternatively, large variability may exist in home patterning support, leading to differences across small studies.

Second, in line with our hypothesis, though this support was broad, parents in our study emphasized numeracy significantly more than patterning or space. These results suggest that parents may prioritize numeracy for their child over other math topics. This finding aligns with our informal comparison of the frequency of different activities across different studies as well as the greater emphasis in the media on numeracy over other early math activities. Parents may also emphasize numeracy over other math skills with their preschooler because they may not recognize non-numeracy skills as part of math knowledge. When parents of preschoolers were asked to spontaneously recall examples of math activities they typically engaged in with their children, parents overwhelmingly cited examples of numeracy activities, such as talking about numerical magnitudes, counting, and simple arithmetic, but not spatial or patterning activities (Cannon & Ginsburg, 2008). This suggests that parents may not recognize a broader range of math knowledge as relevant for young children. For example, it may surprise parents that repeating patterning skill is an important early math skill, or that spatial skills support math

knowledge. Future work should study parents' broader conceptions of early math for preschoolers and how to aid them in recognizing activities supporting non-numeracy math skills.

Parent Academic Beliefs and The Early Home Math Environment

Parents' beliefs about math-related ability were related to the math-related support they provided, but their beliefs about importance and interest were not. Further, which ability belief was related to parent math support varied with the type of support. Our findings are exploratory given the large number of correlations we conducted.

Numeracy support. Parents' ability beliefs about their child, but not themselves, were associated with the numeracy support parents provided. Specifically, parents who believed their children had high ability in numeracy tasks engaged in frequent home numeracy support. This finding expands past evidence that child-specific beliefs about ability is associated with the home numeracy environment (Hart et al., 2016). Hart et al. (2016) used a single variable for child-specific math and spatial ability. Our findings that child-specific beliefs about numeracy ability were related to home numeracy support further suggest that parents' home numeracy support is influenced by their beliefs about their child's domain specific abilities related to math.

We did not replicate past findings that parent-specific ability, anxiety or interest beliefs are related to home numeracy support (Blevins-Knabe et al., 2000; del Río et al., 2017; Elliott et al., 2017; LeFevre, Polyzoi, et al., 2010). Others have also failed to find such relations (Hart et al., 2016; Skwarchuk et al., 2014; Sonnenschein et al., 2012). Similarly, we did not replicate a past finding that child-specific beliefs about importance are related to home numeracy support (Musun-Miller & Blevins-Knabe, 1998; Skwarchuk et al., 2014), though others have also failed to find a relation (LeFevre, Polyzoi, et al., 2010). One substantial limitation in identifying the reasons for mixed findings is large variations in belief survey items used across studies. For

example, the number of items used for each construct varied widely (e.g., studies like ours rely on 1-3 items per construct, whereas others include many items per construct). There is a natural trade-off between depth (e.g., many items on a single construct) and breadth (e.g., a few items on a variety of constructs). Another explanation may be the age of the child, as parent-specific beliefs may be more important for school-age children learning more advanced math topics.

Patterning. Similar to numeracy support, parents' ability beliefs about their child, but not themselves, were associated with the patterning support parents provided. In particular, child-specific beliefs about patterning ability were positively related to home patterning support. This suggests that parents are sensitive to their child's patterning ability and that patterning support may be primarily influenced by this belief. Previous research has not examined relations between parents' beliefs and patterning support, and our work suggests specific links between child-specific ability beliefs and the type of math-related support that is provided.

Broad support. Similar to numeracy and patterning support, parents' ability beliefs about their child, but not themselves, were associated with the broad math support parents provided. In particular, child-specific beliefs about both numeracy and patterning ability were positively related to broad math support. This finding aligns with evidence that child-specific beliefs about ability are associated with the home math environment (Hart et al., 2016; Huntsinger et al., 1997). The current findings suggest that future research on parents' academic beliefs should include their beliefs about child numeracy *and* patterning abilities.

Past research with elementary-school children suggested that child-specific beliefs about interest were also related to the home math environment (Huntsinger et al., 1997). The math measures used in that study differed substantially from our own, and future research is needed to explore relations with child-specific beliefs about interest and the early home math environment.

We predicted that findings on relations between parent-specific academic beliefs and the home numeracy environment would generalize to the home math environment. We failed to replicate those findings for numeracy support or extend those findings to broad math support. The multiple reasons discussed for numeracy support above may also explain the lack of relations in this study.

Spatial support. In contrast to numeracy, patterning and broad math support, parents' spatial support was positively related to ratings of their own ability, but not their child's ability. Further, this was specific to beliefs about parents' own spatial abilities. Spatial reasoning is theorized to be important in mathematical cognition, and negative feelings about spatial skills are associated with poor performance on spatial tasks, especially for women (Lawton, 1994; Mix & Cheng, 2012; Ramirez, Gunderson, Levine, & Beilock, 2012). Our data show evidence that this effect may extend to parents' (mostly mothers') support of preschoolers' spatial skills at home.

Only one previous study has considered the relations between parents' academic beliefs and the early home spatial environment (Hart et al., 2016). In that study, child-specific beliefs about ability in math and spatial tasks (as a single variable) and parent-specific beliefs about math anxiety were each positively associated with home spatial support. Hart et al. (2016) included children from a larger age range and controlled for the general home math environment in their analyses. Future research is needed to consistently identify parent-specific and child-specific beliefs that are related to the home spatial environment for preschoolers.

Summary. Our work was important in exploring which parent-specific and child-specific beliefs were related to a range of specific types of math support. Parents' beliefs about their school-aged child's math abilities are predictive of children's math outcomes (e.g., self-ratings of their math ability), over and above children's past performance in math in school

(Eccles Parsons, Adler, & Kaczala, 1982). Our work suggests that parents' child-specific ability beliefs are related to the math support they provide early in development. Although we suggest that parents' beliefs motivate their home math support, providing home math support likely also impacts parents' beliefs about children's math abilities. Finally, our findings suggest that parent-specific beliefs about spatial ability are related to the early home spatial environment, a finding that highlights the need for future research on the topic.

Linking the Early Home Math Environment to Children's Knowledge and Skills

Finally, we explored links between parents' broad math support and children's broad math skills, and corresponding skills for each type of support. However, we rarely found links between parent support and children's skills.

We did confirm our hypothesis that the frequency of numeracy support would be correlated with preschoolers' concurrent numeracy knowledge, in line with past research (Blevins-Knabe & Musun-Miller, 1996; Ramani et al., 2015; Skwarchuk, 2009; Zippert & Ramani, 2017). However, this support was not correlated with children's broader math knowledge (which included the numeracy subscale). Past research has used the term "math knowledge" as the outcome, but used measures such as the TEMA that assess numeracy skills exclusively (del Río et al., 2017; Ginsburg & Baroody, 2003; Missall et al., 2015), though others are more specific in their sole focus on numeracy development (Skwarchuk et al., 2014). Thus, the current findings suggest a more specific link from home numeracy support to numeracy knowledge. Further, in line with past research, frequency of formal numeracy support in particular was related to concurrent numeracy knowledge (Ramani et al., 2015; Skwarchuk et al., 2014; Thompson et al., 2017; Zippert & Ramani, 2017). Our numeracy subscale almost exclusively measured symbolic numeracy knowledge, and others have found that formal

numeracy support, such as practicing simple sums, is related to symbolic numeracy knowledge (Mutaf Yıldız, Sasanguie, De Smedt, & Reynvoet, 2018; Skwarchuk et al., 2014). In contrast, informal numeracy support, such as parent reports of knowledge of children's math games, was related to non-symbolic numeracy knowledge in past research (Skwarchuk et al., 2014). Thus, an emerging picture reveals fairly specific links between the types of numeracy activities parents provide and the knowledge and skills directly related to those activities (formal symbolic numeracy activities promote symbolic numeracy knowledge, but not other components of math knowledge).

Contrary to our hypothesis and some past research, frequency of numeracy input was not correlated with later numeracy knowledge (Gunderson & Levine, 2011; LeFevre et al., 2002, 2009; LeFevre, Polyzoi, et al., 2010; Niklas & Schneider, 2014; Skwarchuk et al., 2014). This may be because most of the longitudinal evidence linking numeracy support to later numeracy knowledge is with school-aged children, and parent input may make the most impact for more advanced numeracy skills (e.g., advanced calculation ability). Further, parents of school-aged children are sometimes offered structured ways to support their children's math development via homework assignments that parents and children can complete together. If the formal numeracy support parents report for school-aged children occurs within the context of homework help, this might explain why formal numeracy support is more reliably predictive of children's numeracy outcomes (Skwarchuk et al., 2014). In addition, because our sample all attended preschool programs, their experiences in the classroom may be more strongly linked to their end-of-preschool numeracy knowledge than home support (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). More work is needed comparing the effects of parent support of preschool- to school-aged children on numeracy development at different time points.

Contrary to our hypothesis, frequency of spatial support was not related to spatial skills, and our cross-component exploratory analyses did not reveal a relation to numeracy or broad math knowledge. Past research finding relations between the home spatial environment and children's spatial skills has relied on observations of spatial talk or used surveys with large samples spanning a larger age range (Jirout & Newcombe, 2015; Pruden et al., 2011). Our spatial talk item focused on spatial location words (e.g., "below" and "under") and did not cohere well with the other items. The nature of the home spatial environment may need to be better distinguished. First, consider spatial talk. Parents' observed spatial language to their infants and toddlers that involved "what words" (describing features of objects) predicted their spatial skills at age 3, even after controlling for other types of spatial input ("where words" describing locations and directions; Pruden & Levine, 2017). Second, parental reports of their use of specific "where words" (e.g., spatial location words such as "between" and "below") to their 3-year-olds was related to children's concurrent numeracy knowledge (Verdine et al., 2014). Research could determine the usefulness of parents' reports of their use of "what words" and "where words" throughout the preschool years to explain numeracy and spatial development.

Second, there is very little research linking reports of spatial activities (e.g., block play) to young children's spatial skills. Specifically, one study with parents of first-graders found no concurrent relations between home spatial activities and spatial skills (Dearing et al., 2012). In another large survey study with parents of 4- to 7-year-olds, only a small advantage in spatial skills existed for children who played with blocks and puzzles at home "often" versus "sometimes" or "rarely" (Jirout & Newcombe, 2015). The nature of home spatial activities may need to be further specified. For example, semi-structured spatial play was more beneficial than free play in improving early spatial skills (Casey et al., 2008; Schmitt, Korucu, Napoli, Bryant,

& Purpura, 2018). Thus, if home spatial activities with parents typically take the form of free play, these experiences may not effectively support spatial skills.

Finally, frequency of patterning support was not related to children's patterning skills. This is contrary to a previous study of a demographically comparable but much smaller sample ($n = 20$ vs. 63) of parents using similar survey items and child pattern measures (Rittle-Johnson et al., 2015). The current home patterning activity survey was reliable and suggested that parents were engaging in some patterning activities; however, parents in this study reported engaging in patterning activities less often than in the previous study, which mostly focused on patterning. Further, parents in the current study reported engaging in more sophisticated patterning activities, such as copying a pattern by making the same kind of pattern but with different materials, infrequently. Whether parents understand and accurately report this activity is questionable given that many preschool teachers do not engage in it (Rittle-Johnson et al., 2015). More generally, the concern that preschool teachers often engage children in only lower-level patterning activities, neglecting attention to the rules and regularities in patterns (Economopoulos, 1998; Warren & Cooper, 2006), likely extends to parents. Future research needs to (a) validate parent surveys through observations and interviews and (b) identify if some types of patterning activities relate to patterning skills, as is true for numeracy knowledge.

The lack of a correlation between the broad home math environment and children's broad math knowledge suggests that more effort is needed to understand appropriate ways to uncover the link between early math knowledge and how it is supported, broadly defined. Researchers could consider a more comprehensive measure of early broad math knowledge. Although correlated with the full measure, the abbreviated broad math assessment we used (REMA-brief) included a limited number of geometry items and only one patterning item. Thus, using the full

REMA, or other broad math measures (Klein & Starkey, 2002) may be more effective in revealing this link. Researchers should also consider a more comprehensive measure of the early home math environment, including items on early geometry activities.

Limitations and Considerations

A number of limitations should be considered. First, we could not assess directionality and causality of the relations found in our study due to its correlational design. Additionally, our data on the early home math environment were collected entirely via parent report. Self-reports can be prone to response bias. We aimed to reduce the saliency of the focus of our study by adding distractor items, but parents could have still inflated their responses to some of our items. Alternatively, it may have been difficult for parents to remember or recognize their own use of different types of math support, leading parents to underreport some types of support instead. Survey data in past studies have also had validity issues. Two recent studies have shown that parent report of home math activity engagement was not related to observations of parent math support (Missall, Hojnoski, & Moreano, 2017; Mutaf Yildiz, Sasanguie, De Smedt, & Reynvoet, 2018). However, it is unknown whether one-time home observations or parent reports of activities over time are more accurate reflections of the home math environment. Further, we did not ask about the frequency, duration, or quality of activities within a given day. Sustained and high-quality interactions may be more important than brief but daily interactions. Thus, we may have captured relations between support and child outcomes had we observed this support.

We also faced challenges with survey item generation. First, we had difficulty generating items about parents' beliefs about patterning tasks for themselves. These limitations prevented us from comparing across all different types of input and parents' related beliefs. We also encountered issues in measuring some aspects of the home math environment. As previously

mentioned, and others have discussed as well (Hart et al., 2016), we had difficulty generating spatial items relevant for children of the preschool age. For example, some of our spatial activity items were reportedly rarely done, suggesting that the activities may not be appropriate for 4-year-olds (doing mazes, connect the dots, drawing maps). This may have contributed to an overall lower reliability for our spatial input scale. Thus, more research may be needed to identify spatial activities that can be more reliably reported and valid for preschoolers.

Finally, the size and diversity of our sample constrains the generalizability of our findings. Parents were diverse socioeconomically, racially, and ethnically; however, most parents were mothers and had at least some college, limiting generalization of findings to fathers and families with less educated parents. Although more representative of the U.S. population at large than some past research, our sample size was not as large as those of other studies, giving us less power to detect significant effects that may have emerged with a larger sample.

Practical Implications

Next, we consider practical implications of our findings. Parents of preschoolers have reported being unclear about how to support their children's math development (Cannon & Ginsburg, 2008). We thus suggest providing guidance to parents about the nature of and ways to support young children's math knowledge more broadly. For example, suggestions to parents about spatial development and how to provide spatial input during puzzle play increased parent spatial input during block play, a separate spatial activity (Borriello & Liben, 2017). Similarly, providing one-on-one guidance to parents on the development of broad math skills and how to support them coupled with a school-based broad math intervention promoted Head Start children's numeracy and broad math knowledge (Starkey & Klein, 2000). Future studies should thus test the effectiveness (on both child outcomes and parent support) of educating parents

specifically on the broad nature of early mathematics and how to support it. These suggestions might also include additional non-numeracy related areas of early math, such as early geometry.

Conclusion

In conclusion, parents of preschoolers provided broad math input at home, though parents put more emphasis on numeracy activities than spatial or patterning activities. Parents' beliefs about their child's numeracy and patterning abilities were associated with home math support. Parents' beliefs about themselves were rarely related to the support they provided, although belief about their spatial abilities was related to the frequency of spatial input they provided. How much the early home math environment relates to preschool children's current and future broad math knowledge merits additional research.

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Tables

Table 1

Pattern, Spatial, and Numeracy Input Questionnaire Items

Activity Types and Items	Mean (SD)	α
Numeracy	3.35 (0.75)	.789
Count items	4.39 (0.81)	.794
Count out loud without objects	4.11 (0.95)	.759
Talk about written numbers (e.g., "That's a 7")	3.83 (1.24)	.758
Add simple sums or talk about number facts (e.g., 2+2=4)	2.94 (1.45)	.740
Compare quantities (e.g., when playing card games or serving food for dinner or sharing toys)	3.35 (1.39)	.733
Read books that show and talk about numbers	3.33 (1.07)	.750
Watch TV shows or videos that show and talk about numbers	3.46 (1.23)	.802
Play computer games, apps or visit interactive websites that include number games	2.75 (1.44)	.804
Play board games that involve counting (e.g., Chutes & Ladders)	2.02 (1.29)	.763
Spatial	2.45 (0.72)	.672
Use spatial words like: between, below, behind, next to, on, above, near, under, in	4.27 (1.19)	.710
Play with puzzles (e.g., picture puzzles, tangrams)	2.62 (1.17)	.598
Do mazes	1.46 (1.13)	.606
Do connect the dots activities	1.67 (1.12)	.585
Build with construction toys (e.g., Building blocks, Legos, Magnet sets, Lincoln logs)	3.70 (1.17)	.648
Play computer games, apps or visit interactive websites that involve building things	2.37 (1.69)	.663
Draw maps or plans	1.02 (1.14)	.636
Pattern	2.31 (0.97)	.886
Make or copy patterns with objects or sounds (e.g., putting blocks in a red-green-red-green pattern; clap-clap-snap pattern)	2.66 (1.45)	.877
Figure out what comes next in a pattern	2.46 (1.18)	.867
Describe patterns in words	2.29 (1.18)	.873
Copy a pattern by making the same kind of pattern, but with different materials (e.g., use circles and squares to make the same kind of pattern as in a red-blue pattern)	1.76 (1.42)	.864
Discuss patterns in days of the week, months of the year, or seasons.	2.63 (1.44)	.877
Watch TV shows or videos that show and talk about patterns	2.46 (1.35)	.880
Read books that show or talk about patterns	2.36 (1.17)	.876
Play computer games, apps or visit interactive websites that include pattern games	2.21 (1.39)	.877
Play hand or movement games that involve patterns (e.g., Miss Mary Mack, the hokey-pokey)	1.93 (1.45)	.873

Note. Rated on a 6-point Likert-type scale, where 0 = never, 1 = once a month or less, 2 = 2- to 3-times a month, 3 = 1-to 2-times a week, 4 = 3- to 4-times a week, 5 = daily. Column 3 indicates the Cronbach's alpha reliability of the scales in bold as well as the reliability of the relevant scale if each item was removed from the larger scale.

Table 2

Parent Academic Belief Survey Items, Child Abilities, and Partial Correlations with Parent Math Support

Parent Academic Beliefs	Mean (SD)	Partial Correlations with Parent Support Type			
		Broad Math	Numeracy	Spatial	Pattern
Parent-Specific Academic Beliefs					
Importance of being good at ^a					
<i>Math</i> tasks	6.14(1.06)	.125	-.004	.007	.252
<i>Spatial</i> tasks	5.35(1.63)	.127	.165	.113	.065
Parent ability ^b					
<i>Math</i> ability	5.32(1.49)	.170	.120	.227	.119
<i>Spatial</i> ability	5.18(1.44)	.304	.206	.393**	.229
Nervous or anxious about ^c					
<i>Math</i> tasks	3.02(2.06)	-.147	-.133	-.188	-.086
<i>Spatial</i> tasks	2.61(1.71)	-.259	-.277	-.275	-.152
Liking of ^d					
<i>Math</i> tasks	4.80(1.87)	.027	.000	.111	-.009
<i>Spatial</i> tasks	5.20(1.81)	.234	.118	.229	.250
Child-Specific Academic Beliefs					
Importance of ^e					
<i>Numeracy</i> activities	6.75(.46)	-.002	.007	-.064	.027
<i>Spatial</i> activities	6.33(1.00)	.000	.142	-.011	-.104
<i>Pattern</i> activities	6.48(.79)	.057	.102	.045	.011
Child ability ^f					
<i>Numeracy</i> tasks	5.87(.83)	.326**	.355**	.229	.255
<i>Spatial</i> tasks	5.90(.87)	.298	.280	.300	.214
<i>Pattern</i> tasks	5.54(.94)	.327**	.314	.149	.335**
Child interest in ^d					
<i>Numeracy</i> activities	6.13(1.13)	.299	.264	.183	.297
<i>Spatial</i> activities	6.34(1.00)	.209	.197	.203	.154
<i>Pattern</i> activities	5.57(1.36)	.243	.196	.175	.241
Child Current, Later Ability^g					
Broad Math Knowledge T1	-.84(.92)	.172	.175	.174	.120
Numeracy Knowledge Only T1	-.75(1.85)	.196	.310*	.157	.074
Broad Math Knowledge T2	.25(.96)	.047	.097	.086	-.028
Numeracy Knowledge Only T2	.68(1.63)	-.126	-.030	-.071	-.197
Spatial Skills Time 1	.05(.80)	.098	.107	.157	.028
Patterning Skills Time 1	.07(.93)	.022	.042	.116	-.056

Notes. * $p < .05$, ** $p < .01$. $n = 63$ for survey items. Analyses with child abilities control for age at time of testing, child language ability, and paternal education. ^a 1 = Not at all important, 7 = Very important. ^b Mean ability estimates when in school and currently (1 = Not good at all, 7 = Very good). ^c 1 = Not at all anxious, 7 = Very anxious. ^d 1 = Not at all, 7 = Very much. ^e Includes mean ratings of importance of being good at (1 = Not very important, 7 = Very important) and usefulness of activities for child's future (1 = Not Very useful, 7 = Very useful). ^f Average of current ability (1 = Not good at all, 7 = Very good), ability in Kindergarten (1 = Not good at all well, 7 = Very good), and innate ability compared to other children (1 = Much less than other children, 7 = Much more than other children). ^g $n = 61$ for child skills and $df = 56$ for correlations involving child skills due to two missing Time 2 math measures, and because we controlled for paternal education in addition to child age at test time and child language ability.

Supplemental Materials

Table S1

Participants' Demographic Information

Demographics	Frequency	Percent (%)
Parent Race or Ethnicity ^a		
African-American or Black	25	40.3
Caucasian or White	28	45.2
American Indian or Alaska Native	1	1.6
Asian or Pacific Islander	2	3.2
Biracial or Mixed Race	6	9.7
Parent Identification as Hispanic or Latino		
Hispanic or Latino	3	4.8
Mother's Highest Level of Education		
Some high school	2	3.2
High school diploma or GED	4	6.3
Some college or 2-year degree	21	33.3
Bachelor's degree	16	25.4
Some graduate work	3	4.8
Master's, professional, or doctoral degree	17	27.0
Father's Highest Level of Education ^b		
Elementary school	1	1.7
Some high school	3	5.0
High school diploma or GED	12	20.0
Some college or 2-year degree	17	28.3
Bachelor's degree	13	21.7
Some graduate work	2	3.3
Master's, professional, or doctoral degree	12	20.0
Child Race or Ethnicity		
African-American or Black	27	42.9
Caucasian or White	27	42.9
Hispanic	2	3.2
Asian	1	1.6
Middle Eastern	2	3.2
Multiple races/ethnicities	4	6.3
Child Financial Assistance ^c		
None	32	52.5
Some	9	14.8
Full	20	32.8
Child Attended Preschool During Previous Year	42	66.7
Language(s) Spoken at Home		
English Only	58	92.1
English and/or another language	5	7.9
Child Special Education Services		
Receives services	32	52.5
Does not receive services	29	90.5

Note. n = 63

^aOne parent did not report her race or ethnicity. ^bData on father's highest level of education were missing for 3 participants. ^cFinancial data were missing for two participants.

Table S3

Parent's Non-Math Academic Beliefs Alongside Math-Related Beliefs

Parent Academic Beliefs	Mean (SD)
Parent-Specific Beliefs	
Importance of being good at ^a	
<i>Math</i> tasks	6.14 (1.06) ^g
<i>Spatial</i> tasks	5.35 (1.63)
<i>Writing</i> tasks	6.13 (1.29)
Parent ability ^b	
<i>Math</i> ability	5.32 (1.49)
<i>Spatial</i> ability	5.18 (1.44)
<i>Writing</i> ability	5.59 (1.36) ^g
Nervous or anxious about ^c	
<i>Math</i> tasks	3.02 (2.06)
<i>Spatial</i> tasks	2.61 (1.71)
<i>Writing</i> tasks	2.76 (1.83)
Liking of ^d	
<i>Math</i> tasks	4.80 (1.87)
<i>Spatial</i> tasks	5.20 (1.81)
<i>Writing</i> tasks	5.34 (1.60)
Child-Specific Beliefs	
Importance of ^e	
<i>Numeracy</i> activities	6.75 (.46)
<i>Spatial</i> activities	6.33 (1.00)
<i>Pattern</i> activities	6.48 (.79)
<i>Language</i> activities	6.63 (.68) ^h
Child Ability ^f	
<i>Numeracy</i> tasks	5.87 (.83) ^h
<i>Spatial</i> tasks	5.90 (.87) ^h
<i>Pattern</i> tasks	5.54 (.94)
<i>Language</i> tasks	5.92 (1.19) ^h
Child Interest in ^d	
<i>Numeracy</i> activities	6.13 (1.13) ^h
<i>Spatial</i> activities	6.34 (1.00) ^h
<i>Pattern</i> activities	5.57 (1.36)
<i>Language</i> activities	6.13 (1.41) ^h

^a 1 = Not at all important, 7 = Very important. ^b Mean ability estimates when in school and currently (1 = Not good at all, 7 = Very good). ^c 1 = Not at all anxious, 7 = Very anxious. ^d 1 = Not at all, 7 = Very much. ^e Includes mean ratings of importance of being good at (1 = Not very important, 7 = Very important) and usefulness of activities for child's future (1=Not Very useful, 7=Very useful). ^f Average of current ability (1 = Not good at all, 7 = Very good), ability in Kindergarten (1 = Not good at all well, 7 = Very good), and innate ability compared to other children (1 = Much less than other children, 7 = Much more than other children). ^g Significantly different from spatial tasks, ^h Significantly different from pattern tasks and activities.

Table S4

Correlations Between Child-Specific Parent Belief Measures and Child Ability Measures

Child-Specific Academic Beliefs	Child-Specific Academic Beliefs				Child Ability					
	Numeracy	Spatial	Pattern	Language	Broad Math Time 1	Broad Math Time 2	Numeracy Time 1	Numeracy Time 2	Spatial	Pattern
Importance of										
<i>Numeracy</i>	1									
<i>Spatial</i>	.435**	1								
<i>Pattern</i>	.584**	.727**	1							
<i>Language</i>	.461**	.619**	.345**	1						
Child Ability										
<i>Numeracy</i>	1				.292*	.341**	.286*	.238	.322*	.171
<i>Spatial</i>	.579**	1			-.064	-.013	-.034	-.119	.025	-.079
<i>Pattern</i>	.730**	.527**	1		.134	.261*	.161	.192	.172	.134
<i>Language</i>	.220	.257*	.357**	1	-.003	.035	.087	-.023	.028	-.005
Child Interest in										
<i>Numeracy</i>	1									
<i>Spatial</i>	.388*	1								
<i>Pattern</i>	.561**	.478**	1							
<i>Language</i>	.143	.273*	.255*	1						

Notes. * $p < .05$. ** $p < .01$. $n = 63$ for survey items. Correlations are zero-order for all analyses.

Supplemental Results for Parents' Academic Beliefs Survey

Comparisons of parents' academic beliefs. As shown in Table S3, parents had generally positive views about their abilities and did not rate their abilities, interests in, valuing of, and anxiety about spatial and math tasks significantly differently, $t(62)$'s < 2.05 , p 's $> .15$, with the exception that parents believed being good at math tasks was more important than being good at spatial tasks $t(62) = 4.248$, $p < .01$, $d = .57$. Additionally, parents' math and spatial beliefs did not differ on average from their beliefs about writing, with the exception that parents rated being good at writing tasks as more important than being good at spatial tasks $t(62) = 3.71$, $p < .01$, $d = .48$. Overall, parent-specific academic beliefs were similar for math, spatial and writing tasks, except they viewed math and writing tasks as more important than spatial tasks.

Considering child-specific academic beliefs, parents believed their children to be comparable in their abilities in numeracy and spatial tasks, $t(62) = .33$, $p = .74$, but believed their children were better at both numeracy and spatial tasks than patterning tasks $t_{number\ ability}(62) = 3.98$, $p < .01$, $d = .51$; $t_{spatial\ ability}(62) = 3.26$, $p < .01$, $d = .41$. Parents also rated children's interests in numeracy and spatial tasks comparably high $t(62) = 1.38$, $p < .17$, but believed their preschooler was more interested in numeracy and spatial tasks than patterning tasks, $t_{number}(62) = 3.76$, $p < .01$, $d = .48$; $t_{spatial}(62) = 4.90$, $p < .01$, $d = .63$. As shown in Table S3, parents' child-specific beliefs on these topics did not differ from their beliefs for language, with the exception that parents rated their children as having more interest and ability in language than patterning tasks $t_{interest}(62) = 2.64$, $p = .01$, $d = .33$, $t_{ability}(62) = 2.48$, $p < .05$, $d = .32$, and rated language tasks as more important for their child than spatial tasks $t(62) = 3.09$, $p < .01$, $d = .42$. Overall, child-specific academic beliefs were similar for math, spatial, patterning and language tasks, except they rated patterning tasks less positively than other tasks.

Evidence for Validity. Some evidence existed for the validity of the survey. As shown in Table S4, parents' child-specific beliefs about the three math-related components were often more strongly related to each other than to their child-specific beliefs about language skills (a distractor item), providing some evidence of discriminant validity. Further, parents' beliefs about their child's numeracy abilities were correlated significantly with several of our measures of children's math knowledge, as shown in Table S4, providing some evidence of convergent validity. Parents' beliefs about their child's patterning skills were related to Time 2 broad math knowledge, showing some evidence of predictive validity, although child-specific pattern ability beliefs were not related to children's concurrent patterning skills. Parents' beliefs about their child's spatial abilities were not related to our measure of children's spatial ability. Finally, parents' ratings of their child's language abilities did not correlate with their child's math skills, which provides additional evidence of discriminant validity. Overall, there was evidence for discriminant and convergent validity of the parent survey, although the relation between parents' beliefs about their child's spatial and patterning ability and children's measured patterning and spatial ability, respectively, was not established.