

Teachers' Views of Students' Mathematical Capabilities: Challenges and Possibilities for
Ambitious Reform

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1-2 Sentence Description of the Article:

This article reports on a qualitative analysis of interviews with 122 middle-grades teachers in two large, urban districts regarding their views of their students' mathematical capabilities in relation to ambitious instructional improvement efforts. Findings indicate that it is crucial to attend to teachers' views of students' mathematical capabilities in the context of reform along two dimensions: how teachers make sense of students' difficulty and how teachers can support students facing difficulty to participate substantially in rigorous mathematical activity.

Structured Abstract

Background: Research suggests that teachers' views of their students' capabilities matter when attempting to accomplish instructional reform, particularly in settings serving historically marginalized groups of students. However, to date, this issue has received minimal attention in the scholarship and practice of mathematics instructional reform.

Purpose: This study offers a large-scale snapshot of middle-grades teachers' views of their students' mathematical capabilities in the context of instructional reform. It contributes to the field's understanding of the learning demands for teachers inherent in achieving a vision of high-quality mathematics instruction, and therefore of what might constitute critical foci for professional learning opportunities.

Setting: The study took place in two large, urban districts pursuing ambitious reform in middle-grades mathematics.

Participants: Participants included 122 middle-grades mathematics teachers.

Research Design: The study consisted of a qualitative analysis of semi-structured interviews conducted with each of 122 teachers regarding their perspectives on the district's reform efforts, including their views of their students' mathematical capabilities in relation to the reform.

Conceptually, we approached our analysis of teachers' views of their students' mathematical capabilities by attending to how they framed a common problem of practice – students facing difficulty in mathematics – diagnostically (i.e., how they explained the source of students' difficulty) and prognostically (i.e., what they described doing to support students facing difficulty). Analysis also focused on patterns in the relations between teachers' diagnostic and prognostic framings.

Findings: On the whole, most teachers did not view all of their students as capable of participating in rigorous mathematical activity. Most teachers attributed at least some of their students' difficulty to inherent traits of students, or deficits in their families or communities, and most described lowering the cognitive demand of an activity if they perceived students were facing difficulty. Moreover, our analysis of the relations between teachers' diagnostic and prognostic framing revealed that even when teachers explained students' difficulty in terms of instructional opportunities, thereby taking responsibility for their students' learning, they did not necessarily respond in ways that would enable students to participate substantially in rigorous mathematical activity.

Conclusions: Findings suggest that a significant challenge in accomplishing ambitious reform entails supporting shifts in how teachers view their students' capabilities along two dimensions: how teachers explain the source of students' difficulties in mathematics and how they address such difficulties. Implications for designing professional learning opportunities to support productive shifts in teachers' views of their students' capabilities are discussed.

Executive Summary

The gulf between conventional mathematics teaching and the vision of mathematics teaching suggested by decades of research is vast. Thus, achieving a vision of high-quality mathematics instruction at some scale, as suggested in current policy documents like the National Council of Teachers of Mathematics' *Principles to Actions* or the Common Core State Standards for Mathematics (CCSS-M), requires most teachers to significantly reorganize their current practice. Efforts to support teachers to reorganize their practice tend to focus on the development of more sophisticated forms of mathematical knowledge for teaching, new perspectives on teaching and learning, and new forms of teaching that privilege eliciting and building upon student thinking.

In this article, we focus on an aspect of teaching that has received less attention, but appears crucial to achieving a vision of high-quality mathematics instruction: viewing students as capable of participating in rigorous mathematical activity. We report on a study of 122 middle-grades mathematics teachers' *views of their students' mathematical capabilities* across two large, urban districts that were engaging in ambitious instructional reform efforts. In doing so, we provide the field with a snapshot of teachers' views on a large scale. This kind of information contributes to the field's understanding of the learning demands for teachers inherent in achieving a vision of high-quality mathematics instruction at some scale, and therefore of what might constitute critical foci for professional learning opportunities.

Key Concepts

Prior literature suggests the value of using a lens of *problem framing* to conceptualize teachers' views of their students' capabilities, particularly in the context of ambitious instructional reform efforts. In Goffman's terms, frames function to organize people's experience

of any event or experience. We deliberately investigated how teachers framed the problem of students' differential success in order to gain insight into their views of their students' mathematical capabilities in relation to the goals of the reform. We drew specifically on sociologists Snow and Benford's concepts of *diagnostic and prognostic framing*. Diagnostic framing involves identifying the source of a problem, and prognostic framing involves articulating a solution to the problem. Together, information about a teacher's diagnostic and prognostic framing of a common problem of practice – students facing difficulty in mathematics – provides insight into whether a teacher views her students as mathematically capable, and the kind of mathematical activity in which a teacher thinks her students are capable of engaging.

We make a distinction between what we have termed productive and unproductive diagnostic and prognostic framings. We use the language of productive and unproductive to signal that invoking particular frames might encourage or discourage teachers from taking action to engage all students in rigorous activity. *Productive diagnostic framings* are those in which student difficulty is attributed to instructional or schooling opportunities, whereas *unproductive diagnostic framings* are those in which student difficulty is attributed to inherent traits of the student, or deficits in their family or community. *Productive prognostic framings* are descriptions of supports that aim to enable students who face difficulty to participate in rigorous mathematical activity. Conversely, *unproductive prognostic framings* are descriptions of supports that aim to reduce the rigor of the learning goals for students facing difficulty.

Methods

The study consisted of a qualitative analysis of semi-structured interviews, each approximately 45 minutes in length, conducted with 122 middle-grades mathematics teachers regarding their perspectives on the district's reform efforts, including their views of their

students' mathematical capabilities in relation to the reform. To elicit teachers' diagnostic framing, we asked questions such as, "When your students don't learn as expected, what do you find are typically the reasons?". To elicit teachers' prognostic framing, we followed up with questions about what they did to support students who were experiencing difficulty. We also asked teachers if they found they needed to adjust their instruction for different groups of students, and if so, why and what they did.

A team of coders coded turns of talk that were relevant to teachers' views of their students' mathematical capabilities with a coding scheme that distinguished between various kinds of explanations of sources of students' difficulties and supports that were provided. An overall code was then assigned at the level of a teacher for each of the diagnostic and prognostic dimensions (unproductive, productive, or mixed, the latter indicating that the teacher articulated both unproductive and productive framings). Procedures for maintaining sufficient inter-coder reliability were enacted.

Findings

Teachers' Diagnostic and Prognostic Framing

Nearly 30% of the teachers in our sample for whom we were able to assign an overall code for diagnostic framing (n= 100) attributed students' difficulty solely to inherent traits of the students, and/or deficits in their families and communities, thereby locating the responsibility for those students' learning as outside of their purview. Eighty-two percent of these same teachers attributed at least some of their students' difficulty to factors outside of their purview. Seventy percent of the teachers in our sample for whom we were able to assign an overall code for prognostic framing (n= 74) described lowering the cognitive demand of activities (e.g., showing students how to solve the problem) if they perceived students were facing difficulty.

Relations Between Teachers' Diagnostic and Prognostic Framing

Prior small-scale research suggests that unproductive explanations tend to go hand-in-hand with articulating unproductive supports, and that productive explanations tend to go hand-in-hand with productive supports. We were curious about the relation between teachers' explanations of student difficulties and the supports that they described providing for students they perceived as facing difficulty. We were able to code both diagnostic *and* prognostic framings for 56, or about 46%, of the 122 teachers. We found that the relationship between diagnostic and prognostic framings was not entirely straightforward.

Consistent with existing literature, we found that if a teacher articulated an unproductive explanation, it was more likely that she would articulate an unproductive prognostic framing (as compared to a mixed or productive prognostic framing). However, different from existing literature, we found that articulating a productive explanation did not go hand-in-hand with articulating a productive prognostic framing. Of the teachers who articulated productive explanations, nearly an equal number of teachers articulated unproductive supports as those who articulated productive supports. In other words, even when teachers viewed students' difficulty as related to instructional opportunities, and thus within their purview, they did not necessarily describe responding to those difficulties in ways that would enable students to participate substantially in rigorous mathematical activity. The lack of a straightforward relationship makes intuitive sense. A productive diagnostic framing suggests that teachers view their students as capable of participating in mathematical activity, given appropriate instructional support. However, a productive diagnostic framing, on its own, does not suggest *the kind of activity* in which students should be engaged.

Conclusions

One contribution of this study is that it supports the field in further specifying learning goals for teachers in relation to ambitious instructional reform efforts. Namely, professional learning opportunities are needed in which teachers are supported to shift how they make sense of the reasons behind differential success *and* in which teachers learn how to support students facing difficulty in ways that maintain rigorous goals for such students' learning. In light of our findings, we are skeptical of professional learning experiences that are focused generically on raising teachers' expectations of students, absent a focus on clarifying what "high expectations" entails with respect to a particular vision of instruction, and how to enable students to meet those expectations. Although our findings are specific to middle-grades mathematics, we argue that they have applicability to ambitious instructional reform efforts in other grade-levels of mathematics, as well as other subject areas.

A second contribution of this study concerns the value of the interview-based assessment we developed for eliciting teachers' views of their students' mathematical capabilities. This assessment could be a useful tool for prospectively accounting for teachers' views of students' capabilities prior to designing professional learning opportunities. Likewise, this assessment could be useful for monitoring and assessing ongoing instructional improvement efforts.

Future research should investigate designs that support teachers to develop more productive views of their students' mathematical capabilities. Extrapolating from extant research, we conjecture that a first step concerns supporting teachers to notice and focus on what students can do, as opposed to what they cannot do. In addition, we conjecture that it is crucial that teachers are provided facilitated opportunities to see their students engaging in different activity, and thus exhibiting different capabilities, than what they had previously come to expect.

Teachers' Views of Students' Mathematical Capabilities: Challenges and Possibilities for Ambitious Reform

The mathematics education research community has generally achieved broad consensus regarding a set of goals for students' mathematical learning, which are represented in policy documents like the National Council of Teachers of Mathematics' (NCTM; 2000) *Principles and Standards for School Mathematics*, the National Research Council's *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001), and the Common Core State Standards for Mathematics (CCSS-M; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These learning goals privilege developing procedural fluency and conceptual understanding of key mathematical ideas in a range of domains, mathematical reasoning, and the ability to communicate effectively about mathematical ideas.

As described in NCTM's (2014) recent publication, *Principles to Actions*, decades of mathematics education research indicates key aspects of teaching that support students' attainment of such learning goals. For example, research suggests that to achieve such goals, students need to engage in solving challenging, non-routine tasks that can be solved in multiple ways and with multiple representations (Stein, Grover, & Henningsen, 1996). In addition, students need regular opportunities to explain and justify their reasoning, as well as to make connections between solutions and to key mathematical ideas (e.g., Franke, Kazemi, & Battey, 2007; Stein, Smith, Henningsen, & Silver, 2000). More generally, it is important that students are provided ample opportunity to share mathematical authority with the teacher in assessing what is mathematically acceptable, valid, and on what grounds (Lampert, 1990).

The learning goals and vision of instruction that are called for by the policy documents referenced above represent a radical departure from typical mathematics teaching practice. Conventional teaching engages students in procedural tasks with limited opportunities to engage in mathematical reasoning; students are expected to master known procedures to solve predictable sets of problems (Boston & Wilhelm, 2015; Stigler & Hiebert, 1999). Mathematical authority typically resides with the teacher and/or the text (Staples, 2007). As such, there are minimal opportunities for students to develop conceptual understanding of key mathematical ideas or disciplinary practices (Hiebert & Grouws, 2007).

In brief, the gulf between a vision of high-quality instruction and conventional teaching is vast. Thus, achieving such a vision for school mathematics will require, for most teachers, significant reorganization of their practice (Cobb & Jackson, 2011a). Extant research suggests it will require considerable opportunities to develop more sophisticated forms of mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008; Hill, Blunk, et al., 2008), new perspectives on teaching and learning (e.g., Munter, 2014; Wilhelm, 2014), and new forms of teaching that privilege eliciting and building upon student thinking (e.g., Franke et al., 2007).

In this article, we focus on an aspect of teaching that has received less attention, but appears crucial to achieving a vision of high-quality mathematics instruction: viewing students as capable of participating in rigorous mathematical activity. What it means for a student to be mathematically capable in a classroom oriented towards rigorous learning goals is decidedly different than in a classroom oriented towards more conventional learning goals (Gresalfi, Taylor, Hand, & Greeno, 2008; Lampert, 2001; Staples, 2007). Enacting high-quality instruction requires viewing students as able to engage in sense-making and as having valuable ideas on which to build (Lampert, 2001). In light of conventional practice, then, it may be necessary for

teachers to shift their perspectives on what it is that their students are capable of, as they work to implement forms of practice that reflect more rigorous goals for their students' learning.

In what follows, we report on a study of 122 middle-grades mathematics teachers' *views of their students' mathematical capabilities* across two large, urban districts that were in the midst of ambitious instructional reform efforts. In doing so, we provide the field with a snapshot of teachers' views on a large scale. This kind of information contributes to the field's understanding of the learning demands for teachers inherent in achieving a vision of high-quality mathematics instruction at some scale, and therefore of what might constitute critical foci for professional learning opportunities.

We first describe related literature and how we conceptually approached our investigation of teachers' views of students' mathematical capabilities. We then describe our methods, followed by our findings. Our findings suggest that a significant challenge in accomplishing ambitious reform entails supporting shifts in how teachers view their students' capabilities along two dimensions: how teachers diagnose, or explain, the source of students' difficulties in mathematics and how they address such difficulties. We conclude by considering the implications of our findings for designing and assessing improvement efforts as well as for future research. Although this analysis focuses on middle-grades mathematics teachers, we contend that it has implications for accomplishing ambitious reform in other grade-levels in mathematics and potentially in other subject matter areas as well.

Key Concepts and Related Literature

There is good reason to believe that teachers' views of their students' capabilities matter when attempting to accomplish instructional reform at some scale, particularly in settings serving historically marginalized groups of students. Small-scale anthropological and sociological

research has demonstrated that it is common for teachers to articulate deficit-oriented views of non-dominant groups of students, as well as of their families and the communities in which they live. Moreover, these views matter for the kind of learning opportunities teachers provide their students (e.g., Anyon, 1981; Diamond, Randolph, & Spillane, 2004; Jackson, 2009, 2011; McLaughlin & Talbert, 1993; Oakes, 1985; Rist, 1970). For example, Sztajn (2003), Diamond et al. (2004), and Jackson (2009) found that mathematics teachers justified engaging non-dominant groups of students in activity solely aimed at developing procedural facility in terms of their perceptions of students' capabilities.

Based on extensive study of secondary teaching across a number of schools and communities, McLaughlin and Talbert (1993) put forth, "Policy coherence as intended by reformers and policymakers ultimately is achieved or denied in the subjective responses of teachers – in teachers' social constructions of students" (p. 248). Specifically, they found that teachers' views of their students, especially historically marginalized groups of students, mattered for how teachers "structured their pedagogy and curriculum," and, ultimately, students' experiences of schooling and academic outcomes (p. 222). In other words, whether or not the target of an instructional policy was actually reached depended, in large part, on whether teachers' viewed their students as capable.

Moreover, McLaughlin and Talbert found that teachers often viewed students' capabilities in dramatically different ways across schools that served ostensibly similar populations demographically, as well as within schools. In light of this finding, they made a distinction between two aspects of teachers' views of their students: what they called an "objective reality" and a "subjective reality." An objective reality referred to factors such as, for example, a students' language background, or their "family circumstances ... [that] present

particular demands on teachers' instructional choices and classroom strategies" (p. 222). A subjective reality referred to the meaning teachers attributed to, or how they made sense of, an objective reality. As an example, teachers might interpret a student's language status in very different ways. One teacher might view a student identified as an English learner as incapable of participating in a mathematical discussion, whereas another teacher might view that same student as capable of doing so, when provided with targeted scaffolds. One can imagine, then, how these different subjective realities would lead teachers to engage in a discourse-focused reform effort, for example, in very different ways.

In what follows, we discuss how we conceptualized teachers' views of their students' mathematical capabilities. Prior literature suggests the value of using a lens of *problem framing* to provide insight into teachers' views of their students' capabilities, particularly in the context of ambitious instructional reform efforts (e.g., Bannister, 2015; Horn, 2007; Windschitl, Thompson, & Braaten, 2011). More specifically, as we describe below, we found the concepts of *diagnostic and prognostic framing* (Snow & Benford, 1988) particularly useful in teasing apart different dimensions of teachers' views of their students' mathematical capabilities that matter for enacting ambitious instructional reform.

Problem Framing

Most accounts of framing take Goffman's (1974) seminal work on frame analysis as their departure. In Goffman's terms, frames function to organize people's experience of any event or experience. They give particular meaning to an event or experience and enable people to answer the question of "what's going on here"? (p. 8). Depending on how an activity is framed impacts how people subsequently engage in that activity. As Goffman writes, "The elements and

processes [a person] assumes in [his/her] reading of the activity often are ones that the activity itself manifests” (p. 26).

Within education, scholars have taken up the concept of *frame* in interpreting teachers' engagement in ambitious reform efforts. For example, Horn (2007) studied two high school math departments' engagement in what she termed “equity-gear reforms” (p. 44). She analyzed teachers' talk in regularly scheduled workgroup meetings and identified that how teachers made sense of students facing difficulty mattered for how they engaged in the reforms. Specifically, Horn found that in one department the teachers framed the problem of differential success in terms of inherent traits of the students (e.g., students were fast, slow, lazy). These teachers tended to view mathematics as “a well-defined body of knowledge” with a rather fixed sequential order of topics (p. 43). Against this view of mathematics, the teachers aimed to cover the topics in a particular sequence to prepare students for subsequent coursework. And, given how they framed the problem of differential success, when students struggled, teachers placed the blame on the students and felt there was little they could alter about instruction.

However, in the other department, Horn found that the teachers tended to frame the problem of differential student success in terms of the learning opportunities provided in the classroom. For example, rather than attribute a student's difficulties to laziness, teachers considered the nature of learning opportunities that had been provided to the student. Students' engagement or disengagement depended, in part, on the nature of any given activity rather than some inherent characteristic of the student. Therefore, if students were not engaged, the teacher was more likely to consider how she might alter instruction. The teachers also tended to view mathematics as a connected and conceptual web of ideas. Because mathematics was viewed as a web of ideas, rather than a sequential ordering of topics, teachers felt more freedom in altering

curriculum to support students. More generally, Horn's work suggests that a key aspect of teachers' substantial participation in ambitious instructional reform entails framing differential student success as a problem of instruction.

The work of Windschitl, Thompson, and Braaten (2011) further supports the importance of framing differential student success as a problem of instruction. In an analysis of novice secondary science teachers' development of high-quality teaching, Windschitl et al. identified relations between how teachers framed what influenced students' learning of science and their instructional practice. Based on teachers' conversations in Critical Friends Groups, they identified two contrasting frames regarding what influenced students' learning: what they termed "problems with students" and "puzzles of practice." When considering what influenced student learning from a "problems with students" frame, novice teachers tended to suggest that the "responsibility for performance rested almost entirely with students" (p. 1323). On the other hand, novice teachers who considered what influenced student learning from a "puzzles of practice" frame engaged in discussions of students' performance that were "marked by a genuine sense of curiosity and intellectual challenge" (p. 1323). Windschitl et al. argued that within a "puzzles of practice" frame, "students were portrayed as capable of significant achievement under the right conditions" (p. 1324).

Furthermore, Windschitl et al. found that those teachers who tended to invoke the "problems with students" frame tended to enact instruction that reflected an acquisition theory of teaching and learning. In those classrooms, science teaching mainly consisted of teachers giving information about scientific facts and ideas to their students. On the other hand, teachers who tended to invoke the "puzzles of practice" frame tended to enact instruction that reflected a "sensemaking" theory of teaching and learning. In those classrooms, teachers engaged students

in activities in which they elicited students' ideas, attempted to build on those ideas, and pressed students to develop conceptual explanations of scientific phenomena.

Whereas Horn's (2007) analysis provides evidence of how teachers' problem framing links to their views of the discipline of mathematics, Windschitl et al.'s (2011) analysis illustrates how teachers' problem framing links to their theories of teaching and learning as well as their classroom practice. Both demonstrate the value of using a lens of framing to provide insight into teachers' views of their students' capabilities, and suggest that such views matter when engaging in ambitious instructional reform.

Diagnostic and Prognostic Framing

To focus our conceptualization of teachers' views of their students' mathematical capabilities, we drew on sociologists Snow and Benford's (1988, 1992) theorization of problem framing, which they generated in the study of how social movements mobilize people to engage in collective action. Building on Goffman's (1974) concept of frame (cf. Benford & Snow, 2000; Snow & Benford, 1988), Snow and Benford (1988) identified what they call three core framing tasks that mobilize people to develop consensus regarding a specific problem and to take action. The first two tasks – diagnostic and prognostic framing – are central to achieving consensus regarding how to make sense of a particular problem and what to do in response. “*Diagnostic framing* involves identification of a problem and the attribution of blame or causality,” whereas *prognostic framing* involves the identification of solutions to the problem, as well as “strategies, tactics, and targets” (p. 200). Snow and Benford clarify that coming to consensus regarding the source of a problem and how to respond is necessary but not sufficient for mobilizing action. They argue that a third task, *motivational framing*, which involves the identification of rationales for action, is essential to impel people to take action.

In this analysis, we focus on the first two framing tasks – diagnostic and prognostic framing – as they give us important insight into teachers' views of their students' capabilities in relation to ambitious reform efforts. We did not attend to motivational framing given that in this analysis we did not focus on how teachers might be motivated to engage in collective action.

Our use of diagnostic and prognostic framing is akin to that of Bannister (2015) and Coburn (2006). Bannister (2015) employed the concepts of diagnostic and prognostic framing in an analysis of secondary mathematics teachers' participation in one teacher workgroup focused on implementing equity-specific reforms. She identified changes in teachers' construction of the problem of "struggling students," in conjunction with changes in patterns of teachers' participation in the learning community, as a concrete way to account for teachers' learning. In doing so, she attended to changes in teachers' diagnostic frames, or how they "conceptualized the struggling student problem," and their prognostic frames, or "how teachers conceptualized interventions related to the struggling student problem" (p. 355). Over the course of an academic year, she found that teachers' diagnoses regarding why students struggled shifted from focusing on supposedly "fixed attributes of students" (e.g., laziness) to a consideration of students' "personal and systemic circumstances" to a more focused discussion regarding what support specific students needed (p. 357). Teachers' prognoses shifted in tandem, from initially focusing on what *students* needed to do to eventually focusing on what *teachers* could do in their classes to support struggling students. Although not an explicit focus of her analysis, her findings suggest that the developments in teachers' diagnoses and prognoses were a result of ongoing, generative conversation in the teacher workgroup.

Coburn (2006) employed the concepts of diagnostic and prognostic framing in a study of the implementation of an ambitious reading instruction policy in a California elementary school.

In particular, she focused on the role of the principal in supporting shifts in teachers' diagnoses of students' poor reading comprehension and prognoses. She found that the majority of teachers initially attributed the problem of their students' poor reading comprehension to "student or family deficits" (p. 352), while some suggested it was an issue of organizational features of the school, such as class size. Only a minority framed the problem as one of instruction, and they only articulated this privately. On the other hand, the principal framed the problem as one of instruction from the outset of the reform. The principal recognized the importance of teachers coming to frame reading difficulties as a problem of instruction, as otherwise it was unlikely they would work to implement the instructionally-focused reform. Coburn shows empirically how several months of principal-initiated conversation and activity supported the majority of the teachers to come to appreciate the problem as one of instruction, and therefore to begin to try out strategies that were associated with the reform in earnest.

Both Bannister's (2015) and Coburn's (2006) studies illustrate the utility in using the concepts of diagnostic and prognostic framing regarding why students face difficulty to understand teachers' engagement in instructional reform efforts. Just as importantly, their studies, in addition to that of Horn (2007) and Windschitl et al. (2011), illustrate that teachers' framings are dynamic and are shaped by professional learning opportunities as well as other aspects of the workplace, a point that we return to in the Discussion and Conclusion.

Productive and Unproductive Framings of the Problem of Differential Success in the Context of Ambitious Mathematics Reform

We anticipate that differential student success is a problem of practice that teachers face in nearly any mathematics classroom, regardless of the teacher's goals for student learning or the form of instruction. In this study, we deliberately investigated how teachers framed this common

problem of practice – diagnostically and prognostically – in order to gain insight into their views of their students' mathematical capabilities in relation to the goals of ambitious instructional reform. Specifically, we use the language of productive and unproductive diagnostic and prognostic framings to signal that invoking particular frames might encourage or discourage teachers from taking action to engage all students in rigorous activity (cf. National Council of Teachers of Mathematics, 2014), and therefore reflect whether a teacher views her students as capable of doing so.

Productive diagnostic framings are those in which student difficulty is attributed to instructional or schooling opportunities, whereas unproductive framings are those in which student difficulty is attributed to inherent traits of the student, or deficits in their family or community. For example, consider these contrasting responses from two middle-grades mathematics teachers in our study to the following interview question: *When your students don't learn as expected, what do you find are typically the reasons?*

Mr. Williams¹: "I normally look first at me to see ... is there something in the lesson that I didn't emphasize well enough or...I may talk to the teacher they had last year and say 'When you went over this was this something that they struggled with?'"

Mr. Batsem: "Well I'm, you know, you're not supposed to think necessarily but I, I believe there's some innate, you know, ability in differences ... math comes easier to some kids than others, you know."

Mr. Williams frames student difficulty in relation to instructional opportunities, while Mr. Batsem frames this same problem in terms of inherent traits of students (e.g., some students are naturally better at mathematics than others). We view Mr. Williams's diagnostic framing as productive, because it suggests the possibility for an instructionally focused solution. In contrast,

we view Mr. Batsem's diagnostic framing as unproductive, given that it is difficult to imagine that he would alter his instruction to support his students facing difficulty. More generally, Mr. Williams implicitly suggests that his students are capable of engaging in classroom mathematical activity, when provided with the proper opportunities and support to do so. On the other hand, Mr. Batsem suggests that at least some of his students are less capable of participating in classroom mathematical activity than others.

While attending to how teachers explain student difficulty provides insight into their views of their students' capabilities, it does not, by itself, provide insight into teachers' views of the *kind* of mathematical activity of which they are capable. Given our focus on teachers' engagement in ambitious reform, we were interested in whether they viewed their students as capable of engaging in *rigorous* mathematical activity. For that reason, we found it important to attend also to the nature of teachers' prognostic framing, that is, what they view as appropriate solutions to address problems of student learning. So, in addition to distinguishing between kinds of diagnostic framings, we make a distinction between what we term productive and unproductive prognostic framings.

Productive prognostic framings are descriptions of solutions that aim to support students who face difficulty to participate in rigorous mathematical activity. For example, when discussing how they address issues of student difficulty in the classroom, teachers might describe ensuring that all students have a shared understanding of the cultural features or mathematical relationships embedded in a complex task prior to students solving the task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Or, teachers might describe engaging students in explicit conversation regarding how to construct mathematical explanations and justifications (Boaler & Staples, 2008; Staples, 2007). Conversely, *unproductive prognostic framings* are descriptions of

solutions that aim to reduce the rigor of the learning goals for students facing difficulty. For example, teachers might describe “spoon-feeding” students the discrete steps to solve a complex task, or deciding to only focus on mastery of basic skills.

Table 1 provides a summary of the two dimensions of teachers' views of their students' mathematical capabilities. Together, information about a teacher's diagnostic and prognostic framing of a common problem of practice – students facing difficulty in mathematics –provides insight into whether a teacher views her students as mathematically capable, and the kind of mathematical activity in which a teacher thinks her students are capable of engaging.

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In the studies reviewed above, it appeared that what we would term unproductive diagnostic framings tend to hand in hand with unproductive prognostic framings, and productive diagnostic framings tend to go hand in hand with productive prognostic framings. That finding fits with Benford and Snow's (1988) finding in studies of social movements:

while proposed solutions to the problems may not necessarily follow directly from the causal attributions offered by a particular segment of a movement, more often than not there is a direct correspondence between diagnostic and prognostic framing efforts. (p. 201)

In light of the literature, we hypothesized that there would be strong relations between teachers' diagnostic and prognostic framings. However, we treated this as an empirical question. And, as we clarify in the Findings, we found that, in fact, the relations between diagnostic and prognostic framings were more varied than perhaps one might expect.

Research Questions

In order to provide a snapshot of teachers' views of students' mathematical capabilities

on a large scale, we asked the following research questions:

1. How do middle-grades math teachers across two districts pursuing ambitious reform explain the source(s) of students' difficulties in mathematics?
2. How do the teachers describe what they do to address student difficulty?
3. How are teachers' explanations of the source(s) of students' difficulties related to how they describe what they do to address student difficulty?

Methods

To answer these questions, we used data gathered in two districts (Districts B and D) in Year 5 of a longitudinal study aimed at identifying what it takes to support instructional improvement in middle-grades mathematics at the scale of large, urban districts (Cobb & Jackson, 2011b; Cobb & Smith, 2008). We conjectured at the beginning of the study that it would be important to account for teachers' views of their students' mathematical capabilities in order to help us make sense of the ways in which teachers responded to ambitious reform efforts (e.g., how they took up, or not, specific instructional practices and how they made use of reform-oriented curricular materials). In what follows, we provide background on the research context and then describe the interview-based assessment we developed to analyze teachers' views of their students' mathematical capabilities specific to the two dimensions described above.

Research Context

Districts B and D were both purposively selected to participate in the larger study for a few reasons. They were typical of large, urban districts in terms of the challenges they faced, including high teacher turnover and large numbers of students identified as low-performing. However, the districts were unusual in that they responded to high-stakes accountability pressures by attempting to achieve a vision of mathematics instruction that is broadly compatible

with NCTM's (2000) *Standards* and the CCSS-M. At the start of the study (2007-2008), teachers in both districts were provided with the second edition of the *Connected Mathematics Project* (CMP2) curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009), a text aimed at rigorous learning goals for students. Additionally, in order to assist teachers to develop high-quality instructional practices, each district provided a number of supports. For example, in both districts, teachers were provided with curriculum frameworks and regularly scheduled professional development. In District B, nearly all teachers in our sample reported meeting weekly to co-plan for instruction in either department or grade-level teams. In District D, principals were encouraged but not mandated to provide common planning time for mathematics teachers. Teachers in about one-third of the schools in our sample reported meeting in grade-level teams to plan for instruction either weekly or monthly.

Table 2 provides demographic information regarding Districts B and D student populations in Year 5 of the study (2011-2012). In each district, approximately 60 teachers located in a sample of 12-13 schools participated in the study. Schools were selected in consultation with district leaders to represent a range in capacity for instructional improvement. Approximately five teachers in each school were invited to participate in the study, on the basis of a random selection.

=====INSERT TABLE 2 AROUND HERE=====

Data Collection

Data for this analysis comes from semi-structured interviews (Merriam, 2009) conducted with teachers in January. The audio-recorded interviews, which lasted approximately 45 minutes and were transcribed, focused on the formal and informal supports teachers received, to whom and for what they perceived themselves to be accountable, their vision of high-quality

mathematics instruction (Munter, 2014), as well as their views of their students' mathematical capabilities. For the purpose of this analysis, we focused on teachers' responses to interview questions specific to their views of their students' mathematical capabilities.

To elicit teachers' diagnostic framing regarding the source(s) of students' difficulties, we asked questions like, "When your students don't learn as expected, what do you find are typically the reasons?" Interviewers were trained to press on teachers' explanations of why students might not learn as expected. We also asked teachers to describe the challenges they face, which often provided us with insight into how they framed the source(s) of students' difficulties. To elicit teachers' prognostic framing, we followed up with questions like, "What do you do to address that challenge?" We also systematically asked teachers if they found they needed to adjust their instruction for different groups of students, and, if so, why and what they did.

Methods of Analysis

Our first step in analysing interview data involved developing a coding scheme specific to views of students' mathematical capabilities.² Our approach was primarily inductive. To generate an initial coding scheme, we read approximately one-third of the Year 1 interview transcripts from all teachers in the larger study (n = 132). The primary codes (Campbell, Quincy, Osseman, & Pedersen, 2013) for our initial scheme included 1) the categories teachers used to describe groups of students and the characteristics they ascribed to the categories; 2) the pedagogical actions teachers described that they took to meet the needs of groups of students; 3) the extent to which a teacher took responsibility for groups of students' learning; 4) teachers' views about learning mathematics and the curriculum; and 5) teachers' reports on instructional leaders' expectations regarding supporting all students. After having achieved a stable coding

scheme, we then coded interview transcripts for the remaining teachers in Year 1 of the study. The first two authors individually coded all of the transcripts, and came to consensus on the particular codes we assigned to each instance in a transcript.

Although comprehensive, we found that this initial coding scheme was too unwieldy to code the remaining years of data. To narrow what we might feasibly and reliably assess about teachers' views of their students' mathematical capabilities on a large scale, the first and second authors created analytic memos (Hammersley & Atkinson, 1995) for approximately two-thirds of the schools in the study in Year 1 that described the patterns and variations across teachers for each of the primary codes listed above. We then looked across the analytic memos and found that it appeared possible to consistently characterize participants' views of their students' mathematical capabilities along two dimensions: teachers' explanations of the source(s) of students' difficulties in mathematics (hereafter, referred to as *explanations*) and teachers' descriptions of how they support students facing difficulty (hereafter, referred to as *supports*).

The first author then drafted a coding scheme organized according to the two dimensions. She worked with a team of coders in each of the summers of 2009-2012 to refine this scheme in the context of coding interviews collected each January. (The coding process is described in detail below.) The coding scheme presented here is in its ultimate form, and has since been used to code all teacher interview data collected for the first seven years of the project.

Coding scheme for explanations (diagnostic framing). Table 3 provides an abbreviated version of the coding scheme used to code teachers' diagnostic framing, or explanations. As described earlier, we distinguish between productive explanations (teacher frames student difficulty in terms of the nature of instruction, or learning opportunities) and unproductive explanations (teacher ascribes student difficulty to inherent traits of the child, or to

perceived deficits in their families or communities). As illustrated in Table 3, we assigned a code of *mixed explanations* for those instances in which a teacher wavered between productive and unproductive explanations. We did so because we conjectured that a teacher who articulated mixed explanations might be in a better position to examine her instruction in relation to students' difficulties than a teacher who only articulated unproductive explanations.

Coding scheme for supports (prognostic framing). Table 4 provides an abbreviated version of the coding scheme to assess teachers' descriptions of supports for students who are not identified as English learners (ELs) or identified as receiving special education services. We used a different coding scheme to code for prognostic framing specific to ELs, which we developed with the support of an expert in EL education. Given space limitations, we focus on teachers' talk of supports for non-EL students. In the larger study, we generally did not include special education teachers in our sample and therefore did not code for talk of supports specific to students receiving special education services.

In assigning a code for supports, we drew on the form-function distinction that Saxe, Gearhart, Franke, Howard, and Crockett (1999) and Spillane (2000) have made specific to teachers' and district leaders' understandings of mathematics reform, respectively. For example, when asked how they support students facing difficulties, participants may say they "use manipulatives" or "put students in groups." Such descriptions do not suggest the *function* of the particular pedagogical *forms* teachers use. For example, "form-only" talk about manipulatives does not detail what mathematical objectives teachers have when using manipulatives or why manipulatives might enable students to develop a more robust understanding of a particular mathematical idea or concept. We only used the supports coding scheme if we were able to infer the function, or goal, of such supports.

If we were able to infer the function of the support, we assigned codes aimed at identifying whether teachers' prognostic framing was aimed at supporting students to participate in rigorous activity. In addition to categories of productive and unproductive supports, which we described earlier, we assigned a code of *mixed* to indicate that the teacher described actions that were aimed at supporting students facing difficulty to participate in rigorous mathematical activity (high-cognitive demand activity), but *some* of what the participant suggests was aimed at conventional, low-cognitive demand activity. As illustrated in Table 4, a code of "mixed supports" tends to reflect the view that students must first master "basics" before (and separate from) being provided opportunities to engage in conceptually-oriented activity.

=====INSERT TABLES 3 AND 4 AROUND HERE=====

Coding process. All transcripts were coded within a qualitative software package, NVIVO. Coders were trained to search for specific questions and keywords pertaining to teachers' explanations and supports within interview transcripts. One coding decision we faced entailed determining how to unitize (Campbell et al., 2013), or chunk the text into meaningful, code-able parts. In semi-structured interviews, it is often the case that relevant ideas unfold over the course of multiple turns of talk (Campbell et al., 2013). Therefore, in an effort to ensure coders were coding the same unit of text, we aimed to code at the unit of a turn of a talk, however, within NVIVO, we also captured whatever relevant text we used to make sense of that particular turn of talk. Coders were strongly encouraged to record their rationale for particular coding decisions as linked annotations in their personal copies of the NVIVO project. Coders could then refer to their rationales when engaging in consensus conversations.

An overall code was then assigned at the level of a teacher for each dimension. To do so, we looked across all coded passages of text for a specific dimension (explanations, supports) of

their views of their students' mathematical capabilities. If all passages for a specific dimension were coded as unproductive, the overall code for that dimension for the interview assigned was unproductive. If all passages for a dimension were coded as productive, the overall code assigned for the dimension was productive. And, if all passages were scored as mixed, or there was a combination of codes (e.g., unproductive for a relevant unit of talk and productive for another relevant unit), the overall code assigned for the dimension was mixed.

A team of four coders (including the first and third author) coded the data reported on in this analysis, which included transcripts from interviews with each of the 122 teacher participants in the study in Year 5. The team was composed of the first author ("anchor coder"), another experienced coder, and two additional coders who were new to the process. The first author was an assistant professor of mathematics education, and the other coders were research assistants on the larger project. The experienced coder was a doctoral student in leadership and policy and had worked in a school district office of research and evaluation; the other two coders were doctoral students in either leadership and policy or mathematics education and had taught secondary mathematics. The first author led a two-day training. This training was followed by a training phase of coding in which each coder individually coded 20 transcripts selected to represent a variety of coding queries that could arise. All four coders discussed and came to consensus for each relevant unit of talk for the 20 transcripts. The coders were then assigned 10 randomly selected transcripts that the anchor coder and experienced coder had coded independently and on which they had reached consensus. The coders were required to achieve 80% intercoder reliability (Campbell et al., 2013) for every overall code in the set of 10 interviews before being allowed to code independently. After this initial phase of coding, coders (including the experienced coder) were then provided with a list of randomly assigned

interviews. Each week, approximately 20% of the transcripts being coded were randomly chosen to be double coded by either the anchor coder or the experienced coder, and the anchor coder coded 20% of the experienced coder's coding to ensure she maintained reliability. In all cases, coders were required to maintain 80% reliability for each overall code, both cumulatively and for the 10 transcripts most recently coded. The overall percent agreement (calculated as average percent agreement with the consensus code per code) was 92%. In addition, we calculated Kappas for each of the relevant dimensions; the Kappa for Explanations was 0.78 and the Kappa for Supports was 0.65, both of which are generally considered to indicate high levels of agreement (Gwet, 2010).

Positionality. Each of the three authors identify as mathematics education researchers, teacher educators, and as former secondary mathematics or elementary teachers. As such, we identified with the challenges teachers described regarding students facing difficulty in mathematics. Our position when interpreting findings regarding teachers' views of their students' mathematical capabilities was to assume positive intentions. However, we also viewed it as our responsibility to describe both what we have termed unproductive and productive ways of making sense of students' difficulty and of supporting students. As stated earlier, our hope is that in doing so, we identify key foci for professional learning opportunities which will, in turn, support teachers to improve their practice and, ultimately, improve students' access to high-quality instruction.

Limitations. Before sharing our findings, we discuss the limitations of our methods. First, as illustrated in our discussion of Coburn (2006), Horn (2007), Windschitl et al. (2011), and Bannister (2015), framing problems of practice is an inherently social process (see also Snow & Benford, 1992). Given the scope and design of the larger project, we were unable to

investigate on a large scale how teachers framed students' difficulty in conversations with colleagues. Instead, we were able to elicit how an interviewee framed a specific problem of practice in the context of an interview.

Second, in the case of any interview-based assessment that is being administered by a team of researchers, undoubtedly, interviewers were inconsistent in how they asked questions and did not always probe in the same ways. This problem improved over the course of the larger study, as we improved the quality of training we provided to the team of interviewers. As shown in Table 5, in Year 5, we were able to assign an overall code for explanations for 82% ($n = 100$) of the 122 interviews. However we were only able to assign an overall code for supports for 61% ($n = 74$) of the 122 interviews. The most common reason why we were not able to code for explanations was that when teachers described a challenge students faced (e.g., "students lacked basic skills"), the interviewer did not probe on why that was a challenge (e.g., the teacher was not asked *why* students might lack basic skills). Such probing was necessary to identify the source of the difficulty. A reason for the smaller percentage of teachers who were coded for supports is that, even with targeted probing, participants often did not articulate the *goal* (or the function) of their supports. We conjecture that teachers' challenge in articulating how they support students facing difficulty is indicative of a lack of professional learning experiences focused on this very issue.

Findings

In what follows, we first provide descriptive information regarding how teachers across the two districts diagnostically and prognostically framed the problem of students' difficulties in mathematics. We then explore how teachers' diagnostic and prognostic framings relate to one another. Together, these findings provide the field with a large-scale snapshot of teachers' views

of their students' mathematical capabilities, as articulated within interviews. Throughout the findings, we have combined results from Districts B and D, given that the distributions in how we coded the data were extremely similar and that our purpose here is not to investigate district-level differences.

Teachers' Explanations of the Source(s) of Student Difficulty

The overwhelming majority of teachers for whom we could code explanations expressed that for at least some of their students, the sources of their difficulties were distinct from classroom instruction. As shown in Table 5, 18% of the 100 teachers offered entirely productive explanations regarding why their students did not learn mathematics as expected; that is, they suggested that the source of their students' difficulty was likely due to instructional or schooling opportunities. Twenty-eight percent of the teachers offered wholly unproductive explanations, attributing the source of the difficulty to individual traits of the students or to deficits in the families or communities in which they resided. About half of the teachers (54%) explained student difficulty by both appealing to instructionally-focused reasons and suggesting that difficulty was, at least in part, due to deficits in their students, families, or their communities.

In an earlier section, we provided representative examples of productive and unproductive explanations, via Mr. Williams and Mr. Batsem, respectively. Here, we provide a couple of illustrative examples of instances in which teachers offered a mix of both unproductive and productive explanations. For example, when Mr. Hopkins was asked why his students might not learn as expected, he identified a couple of reasons: lack of parental involvement and the quality of his students' prior instruction. He first described his students' families in deficit terms. Namely, he attributed some students' behavior to a "lack of educated parents" who did not support their children at home:

Well, I think ... a big part of our battle every day is ... we have students that come to school who don't get support at home, those students tend to think that this is just ... a place where you can come and try to have as much social fun as possible during the day and that's a big challenge for us, the lack of parental involvement on this campus, the lack of educated parents on this campus.

However, he also explained students' difficulties in terms of the quality of his students' prior instruction.

The other reason students are not learning is because you have teachers that are not teaching with fidelity to the [district's curriculum framework], and ... the students ... don't know what the day's objective is. It's not made clear to them. They're not made to write it down necessarily. They're not ... made to share aloud with a partner.

Mr. Hopkins is a case, then, of a teacher who explained students' difficulty both in terms of deficits in their families as well as a matter of instruction. Notably, though, he did not explain his students' difficulties as in relation to his *own* instructional practice.

As another example, consider Mr. Dawkins. Early in the interview, he explained student difficulty as in relation to his own instruction. When asked why his students do not learn as expected, he responded:

I usually assume there's something I could do to do better. That's what I really try to focus on doing first, that I needed to have done something else, and I think one weakness of mine is classroom management and that ties in to participation, because ... I think if the students were doing the things that I've prepared for them to do then they would be doing better, but I've got to get a hold on ... being positive with them and being emotionally empowering with them but also being strict about this is what work in the

classroom looks like and these are your expectations. That's something that I've really needed to work on a lot.

Here, Mr. Dawkins clearly situates students' learning in relation to his instruction. Specifically, he connects his own challenges with classroom management to students' participation in class.

However, during a different part of the interview, Mr. Dawkins suggested that the adopted textbook, CMP2, was not appropriate for what he referred to as his school's "demographic" of students because "[the text is] assuming that the students can do a level of thinking that they cannot do." We coded this excerpt as an unproductive explanation because it was deficit-oriented and suggested that his views of his students' capabilities were relatively static. More generally, across the interview, Mr. Dawkins articulated both productive and unproductive explanations of why students might not learn as expected.

Teachers' Descriptions of How They Support Students Facing Difficulty

As we suggested previously, attending to how teachers describe supporting students who face difficulty is critical to assessing teachers' views of their students' mathematical capabilities because it provides valuable insight into the *kind* of mathematical activity in which teachers think their students are capable of engaging. As shown in Table 5, of the 74 teachers for whom we could code supports, only about one-fifth ($n = 14$) described exclusively productive supports, or those aimed at enabling students to participate in rigorous mathematical activity. Eleven percent of the 74 teachers ($n = 8$) articulated mixed supports; most of these teachers prioritized drilling basic skills as a necessary prerequisite to engaging in rigorous mathematical activity. Notably, an overwhelming majority of teachers (70%; $n = 52$) described unproductive supports, meaning they solely described lowering the cognitive demand for students facing difficulties,

which usually entailed demonstrating procedures for how to solve problems absent a focus on why procedures work.

=====INSERT TABLE 5 AROUND HERE=====

Relations Between Teachers' Diagnostic and Prognostic Framing

Our findings discussed thus far suggest that, on the whole, teachers did not view all of their students as capable of participating in rigorous mathematical activity. The majority of teachers suggested, at least at one point in the interview, that students' difficulty could be explained in terms of inherent traits of the students, and/or deficits in their families and communities. And, of the teachers who articulated the function of their supports, the majority described lowering the cognitive demand of activities for students that faced difficulties.

However, we were curious about the relation between teachers' explanations of student difficulties and the supports that they described for those students. For example, did, as Bannister (2015) found, articulating unproductive explanations tend to go hand-in-hand with articulating unproductive supports? Did articulating productive explanations tend to go hand-in-hand with articulating productive supports? We were able to code both diagnostic *and* prognostic framings for 56, or about 46%, of the 122 teachers we interviewed. Table 6 presents descriptive statistics regarding the relations between their diagnostic and prognostic framings. As we elaborate below, we found that the relationship between diagnostic and prognostic framings was not entirely straightforward. In what follows, we unpack these findings. We first focus on unproductive explanations in relation to supports, and then on productive explanations in relation to supports.

=====INSERT TABLE 6 AROUND HERE=====

Unproductive diagnostic framings (explanations). As shown in Table 6, we found that *if a teacher articulated an unproductive explanation, it was more likely that s/he would*

articulate an unproductive prognostic framing (as compared to a mixed or productive prognostic framing). We had conjectured this was the case, given that unproductive explanations indicate the view that at least some students are incapable of succeeding in mathematics because of factors outside the teacher's locus of control. Thus, it seemed likely that any attempt at supporting such students would, at best, aim at supporting them to participate in less challenging mathematical work – work for which they might be perceived as able to engage in.

As an example of someone who articulated both unproductive diagnostic and prognostic framings, consider Mr. Gomez. When asked about the biggest challenges of teaching mathematics, Mr. Gomez said, “The ... apathy from the kids, [they are] completely apathetic, they could care less what they're learning ...” The interviewer then probed, “What do you attribute the kids' apathy to?” to which Mr. Gomez responded:

... I think the kids see the world as, that we live in right now, as a place where they're not gonna be successful ... I mean they see that a lot of people don't have jobs ... I was in the military in 2001, you know... during September 11th. [T]hese kids ... haven't had a conscious life that doesn't involve war, doesn't involve conflict, doesn't involve a depression of some sort, so I think that they're just down, they're just depressed and I think they ... see a pretty dim future.

Mr. Gomez diagnosed the problem of students not performing (or learning) as an issue of apathy, and suggested that the source of this apathy is due to historical conditions and students' outlook on their future. While it may be objectively true that the historical conditions of his students are challenging, attributing apathy to factors outside of instruction obscures the possibility that the apathy he perceives is, in part, produced in instruction.

When asked how he addresses what he perceives as apathy, he said: “Unfortunately our kids, because of their background, they like somebody to tell them what to do, they like to take notes. They like ... teacher led work and then independent work.” He contrasted his approach to what district leaders explicitly suggested teachers should do: “[District leaders] don’t like proceduralization, but it works for... these kids.” Here, we see that Mr. Gomez justifies proceduralizing mathematics (i.e., demonstrating procedures for solving problems) for his students in terms of “their background” – and presumably, what he has interpreted as apathy. Proceduralizing mathematics, as Mr. Gomez acknowledges, was antithetical to the goals of the ambitious reform the district was pursuing.

As another example, consider how Mr. Beaumont framed both the source of student difficulty in mathematics and a solution in unproductive ways. His school had created two tracks of eighth grade teachers and students – what he referred to as the “good team” and the “bad team.” The “good team” consisted of high-performing students on the state mathematics assessment, whereas the “bad team” consisted of low-performing students. Mr. Beaumont was assigned to the “bad team,” which he resented: “The teachers ... that are on the bad team realize that this is the hand that we got dealt, we’re stuck with it, and it’s terrible, it’s not fair.”

Similar to Mr. Gomez, and many other teachers we interviewed who articulated unproductive prognostic framings, Mr. Beaumont suggested that “low-level kids” were not able to engage in the forms of activity required by the district’s ambitious curriculum. For example, one form of activity emphasized by district leaders involved grouping students together for the “explore” phase of a CMP2 lesson, in which it was expected that students would share their thinking with one another in an effort to collaboratively solve a complex task. Regarding this expectation, Mr. Beaumont said, “These kids you put them in a group of two it’s play time....A

group of four, 'oh boy, we're not going to do anything today.' [District leaders] insist they want us to use ... grouping strategies. It doesn't work with this level kid in mathematics."

Both Mr. Gomez' and Mr. Beaumont's talk are illustrative of more general patterns in our data regarding how unproductive diagnostic and unproductive prognostic framings tended to co-occur. Teachers who tended to attribute students' difficulties in mathematics to inherent traits of the students or to deficits in their families or communities also tended to suggest that such students were incapable of participating in activity aimed at rigorous goals for their learning. Similarly, as evidenced in Table 6, teachers who articulated both unproductive and productive explanations for why students faced difficulties (mixed explanations) tended to articulate an unproductive prognostic framing.

Although extremely rare, there were exceptions to these patterns. We identified two cases in which teachers articulated an unproductive explanation of student difficulty coupled with a description of productive supports. For example, when asked about the biggest challenges of teaching mathematics, Mr. Carter responded that a number of his students were not motivated. He elaborated: "They just don't care and ... their parents you know they're not there staying on 'em and so ... that's probably the worst, one of the hardest things." When asked why some students are motivated, while others are not, he suggested it was due to the students' "culture," which appeared to map onto assumptions about his students' family lives:

I know it sounds crazy, but I mean I've talked to parents who don't even know that report cards [have] come out, they have no clue, like, "Oh, really?" I talked to a parent who told me one time, "Well when he's 15 he's gonna start working for the family company, so I don't care." I've had a kid tell me he was gonna drop out before this year was over. I mean it's just [a] different culture.

As illustrated above, Mr. Carter attributed students' low-performance in mathematics to a lack of motivation, which, he suggests, stems from students' family backgrounds. However, he also expressed some empathy with his students for what he assumes is a difficult, demanding home life:

And a lot of it too is you know they've got one parent and they're working three jobs cause they've got three kids and they gotta do what they gotta do, you know I understand that. So I mean I understand if you know you're babysitting your brother and sister all night long, your math homework isn't probably on the top of your list of things to do and that's not anything that these kids are at fault for, it's just the life they've gotten put into.

When asked what he does to address challenges in teaching mathematics, he described taking measures to support students to participate in high-cognitive demand activity. In particular, he had been provided with professional development specific to launching, or introducing, cognitively demanding tasks (Jackson et al., 2013). He discussed how he worked to ensure that his students understood the context associated with particular CMP2 tasks, especially those in which the context was likely to be unfamiliar to his students. For example, he described a CMP2 investigation organized around renting bicycles and taking a bicycle trip in New England³:

Man, I spent almost a day just explaining the concept and how it worked. ... I launched the crap out of it, cause I was like, I'm bound and determined for these kids to understand it and they did, most of them, I would say 80-90% of 'em could've said, "Yeah they were trying to open this bike shop and rent these and make some money" ... but I won't be able to do that next year. I spent a whole day on that just explaining and we went off on you know business tangents and different things, but it was like then they seemed more

engaged in that unit. But I had to waste an entire [day] to get them there. You know not waste a day, I didn't, it wasn't wasteful, ... to me it was great because it was like then I had 70 or 80% of them that was really involved in [the investigation].

Mr. Carter's response to his students' difficulties was markedly different from strategies aimed at lowering the cognitive demand of expected activity (like that which Mr. Gomez described). However, even though Mr. Carter recognized how ensuring his students understood the context of the bike tour supported the majority of his students to be "engaged in the unit," he also suggested that he could not afford such time the following year. So, although he articulated productive supports during this interview, he appeared skeptical of doing so in the future.

Productive diagnostic framings (explanations). We now turn our attention to those teachers who articulated productive explanations. As shown in Table 6, *the relationship between articulating a productive explanation and a particular kind of prognostic framing is not straightforward.* The lack of a straightforward relationship makes intuitive sense. A productive diagnostic framing suggests that teachers view their students as capable of participating in mathematical activity, given appropriate instructional support. However, a productive diagnostic framing, on its own, does not suggest *the kind of activity* in which students should be engaged. This is evidenced in the fact that, as shown in Table 6, of the nine teachers who articulated productive explanations, nearly an equal number of teachers articulated unproductive supports ($n = 3$) as those who articulated productive supports ($n = 5$). We consider those two cases here.

Ms. Jacobi, an eighth grade Algebra teacher, provides an illustrative example of a teacher who articulated a productive explanation while describing unproductive supports. She described her goal as having all of her students develop a "foundation" in Algebra, because, in her view, "if they have the foundation, they're not going to have a problem [in] ... high school math classes."

In doing so, she at least implicitly conveyed an orientation to viewing students' performance as dependent on the quality of instructional opportunities; if students were provided with a strong foundation in mathematics, they were capable of succeeding in high school mathematics. When asked what she attributed students' difficulties to, she suggested instructional reasons. Yet, when asked how she tended to adjust her instruction for students facing difficulties within mainstream instruction, similar to Mr. Gomez, Ms. Jacobi described proceduralizing instruction.

I can tell a lot of them ... are not where they should be. And that means they [need] practice. Now after [giving them the] problem on the board, I'll go around, I can figure out if they haven't started – that means they are still behind. And then I just give them a hand for the first step. I do the first step with them and I'm asking them for the next step. Then I can go back over there, if they didn't hear me I have to repeat it.

Ms. Jacobi's prognostic framing reveals that when she perceives that students "are not where they should be," she tells them, step by step, how to solve the problem.⁴ Although this could be perceived as supporting such students to succeed at solving the immediate task, it does not support such students to develop conceptual understanding or practices of mathematical reasoning. More generally, examining Ms. Jacobi's diagnostic and prognostic framings serves to illustrate that attributing the source of students' difficulty to instruction does not guarantee that teachers then take measures that aim to support such students to substantially participate in *rigorous* mathematical activity.

On the other hand, some teachers who articulated productive diagnostic framings also articulated productive prognostic framings. For example, consider Ms. Baker. When asked why students do not learn as expected, Ms. Baker replied:

I think a lot of it is when they don't know why they got where they got with their answer. So, if they're just solving and they're not having to explain or think about the process they're not going to retain it. They're just going to memorize it for that day and then, you know, later down the road they're not going to retain that knowledge.

She clearly suggests that students' difficulty is a product of instructional opportunity. Moreover, she indicated that she and her colleagues had collectively come to view students' difficulty with "retaining knowledge" as a problem of instruction:

We've all talked as a department, "Why are they not retaining it? Why are we ... still scoring low on our [interim assessment] scores, when in class [students] seem to be getting it?" And we kind of came to the consensus that we are not letting them think the problem through. When they struggle, we're sometimes just telling them how to get there.

When describing supports, she maintained a focus on providing students with the means to participate in rigorous mathematical activity. She explained:

I think letting the kids spend time on ... a problem and not rushing them through something and then being accountable for what they learned, you know, "Explain to your friend why you got that," and ... "How did you get to your answer?" Cause if we're just zipping through problems all day they're not learning it. They're memorizing it and then they're leaving class.

Different from Ms. Jacobi, who told students how to solve problems step by step, Ms. Baker expected students to take time engaging in problems and emphasized routines that focused on students being able to explain to one another why particular methods made sense.

Knowing *how* to support students facing difficulties to participate substantially in rigorous mathematical activity is not an easy task. Most of the teachers who articulated both productive explanations and productive supports acknowledged the challenge inherent in altering instruction to support students who typically face difficulties in ways that maintain rigorous goals for students' mathematical learning. For example, when asked how she goes about adjusting her instruction for students who face difficulties, Ms. Newman described how much effort it takes on her part:

I feel like if I put the work into it to really understand the concepts myself, and then just give it a shot, then it works well, and has worked well. I mean, it's the time ... When I sat down and I've really worked on it, like I really wanted them to understand the meaning behind dividing fractions because that was definitely a standard, and ... I was really doubtful that [my students who typically have difficulties] were going to get it, but ... I tweaked the lesson here and there, [and] they got it. [My students who typically have difficulties] definitely impressed me [with] what they could do.

Here, Ms. Newman describes the significant time it took for her to work through the mathematics and to figure out how to enable her students who face difficulties to make conceptual sense of a difficult mathematical idea. In addition, she describes the reward of having done so. As a product of the time and effort she dedicated to lesson planning, Ms. Newman's students who typically faced difficulty exhibited understanding. This illustrates that teachers' views of their students' mathematical capabilities might shift as a result of seeing their students successfully engage in rigorous activity, a point to which we return in what follows.

Discussion and Conclusion

This analysis provides the field with a snapshot of middle-grades mathematics teachers' views of their students' mathematical capabilities across two large, urban districts that were in the midst of ambitious instructional reform. Our findings suggest that on the whole, teachers did not view all of their students as capable of participating in rigorous mathematical activity. Our focus on teachers' diagnostic framing revealed that nearly 30 percent of the teachers in our explanations sample attributed students' difficulty solely to inherent traits of the students, and/or deficits in their families and communities, thereby locating the responsibility for those students' learning as outside of their purview. And, about 85% of the teachers in our explanations sample attributed at least some of their students' difficulty to factors outside of their purview. That said, one way to interpret our findings regarding mixed and productive explanations is that about 70% of teachers articulated issues of instruction as at least one factor in understanding students' difficulties. This could be viewed as promising, in that viewing student learning in relation to instruction is an important and necessary step in engaging substantially in instructional improvement efforts (Gresalfi & Cobb, 2011).

Our focus on teachers' prognostic framing revealed that 70% of the teachers in our supports sample described lowering the cognitive demand of an activity (e.g., showing students how to solve the problem) if they perceived students were facing difficulty. Moreover, our analysis of the relations between teachers' explanations of students' difficulty and descriptions of supports illustrates that even when teachers viewed students' difficulty as related to instructional opportunities, and thus within their purview, they did not necessarily respond in ways that would enable students to participate substantially in rigorous mathematical activity.

On the whole, these findings are concerning. However, we do not intend for these findings to be interpreted as disparaging of teachers. Rather, these findings suggest to us that

teachers have not been supported sufficiently in coming to view all of their students as capable of participating substantially in rigorous mathematical activity. Against this background, in what follows we first consider what it might take to support teachers to come to view all of their students as mathematically capable in the context of ambitious reform efforts. Next, we discuss the value of an assessment like the one we developed for both designing for and monitoring instructional improvement efforts. Finally, we consider the applicability of these findings for ambitious instructional reform efforts in other grade-levels of mathematics, as well as other subject areas. Throughout, our discussion points to areas in need of empirical research.

Supporting Shifts in Teachers' Views of Students' Mathematical Capabilities

A key contribution of this study is that it supports the field in further specifying learning goals for teachers in relation to ambitious instructional reform efforts. Extant research has suggested that substantially engaging in ambitious reform requires teachers to develop stronger mathematical knowledge for teaching (e.g., Hill, 2010), to develop more sophisticated visions of high-quality mathematics instruction (Munter, 2014; Wilhelm, 2014), and to develop particular forms of instructional practice (e.g., Boston & Wilhelm, 2015). Identification of such learning goals for teachers informs the foci of professional learning opportunities. This study indicates that it is likely teachers also need to shift their views of who is capable of engaging in rigorous forms of activity; and thus shifting such views should be a focus of professional learning.

What would it take to support teachers to come to view students' difficulty as an issue of instruction, and to learn to respond to such difficulty by enacting supports that enable students to participate more fully in rigorous mathematical activity? On our read, the literature is thin regarding how to support teachers to develop more productive views of their students' mathematical capabilities (cf. Battey & Franke, 2013). In what follows, we extrapolate from

relevant literature to identify potentially important aspects of designs to support shifts in teachers' views of their students' mathematical capabilities that could become the object of empirical study.

Research suggests a first step concerns the importance of supporting teachers to notice and focus on what students can do, as opposed to what they cannot do (Battey & Franke, 2013; Gresalfi & Cobb, 2011). In addition to noticing student thinking, we conjecture that shifting views of students' capabilities entails being supported to enact more productive forms of instructional practice. It is unlikely that teachers' views of students' capabilities will shift absent a context in which they can see their students engaging in different activity, and thus exhibiting different capabilities. Recall, for example, Ms. Newman's description of how students who typically had difficulty "impressed her" when she was able to take the time to carefully plan for their participation in an upcoming lesson on understanding the meaning behind the procedure for dividing fractions. She was able to see those students' capabilities in a new light, as a result of having reorganized how she typically approached her teaching.

Gresalfi and Cobb's (2011) analysis of a professional development collaboration between middle-school math teachers and researchers focused on statistical reasoning is relevant in this regard. At the start of the collaboration, teachers tended to articulate what we would have identified as unproductive diagnostic framings regarding why some students succeeded while others faced difficulty in mathematics. In the context of professional development, teachers were supported to analyse records of student thinking in an effort to identify students' reasoning, and why it might make sense, in light of their instruction. Over the course of the first two years of the five-year collaboration, teachers came to identify as teachers who valued student reasoning, and "deficit language about students was gradually displaced by talk about why

students thought or performed in particular ways” (p. 289). In addition, teachers developed a commitment to understand “how to support the development of all students’ mathematical reasoning” (p. 296). Moreover, in an analysis of teachers’ practices in subsequent years of the collaboration, Visnovska (2009) found that as teachers gained an appreciation for the value of investigating students’ thinking, they enacted practices that supported and encouraged students to make their thinking apparent. It appeared that a shift in views of students’ capabilities went hand-in-hand with a shift in coming to value eliciting and making sense of students’ reasoning, and learning how to do so in practice.

Relatedly, the research of Horn (2007) and Windschitl et al. (2011) suggests that views of the target discipline (in our case, mathematics) as well as theories of teaching and learning impact the nature of how student performance is framed. This suggests that professional learning opportunities focused on shifting views of students’ mathematical capabilities must be tightly integrated with a focus on other key aspects of teaching and learning mathematics like, for example, what it means to engage in the discipline of mathematics or how it is that students develop deep understandings of mathematics.

In light of these findings, we are skeptical of professional learning experiences that are focused generically on raising teachers’ expectations of students, absent a focus on clarifying what “high expectations” entails with respect to a particular vision of instruction, and how to enable students to meet those expectations (cf. Sosa & Gomez, 2012). Recall Ms. Jacobi, the eighth grade Algebra teacher who articulated a productive explanation regarding the sources of students’ difficulty yet described unproductive supports. She had high expectations for her students in that she wanted them to succeed in high school mathematics classes. However, when discussing how she addressed students’ difficulties, she described proceduralizing instruction, or

showing students how to solve the problems, thereby taking the thinking away from the students. So although she articulated high expectations for her students, the supports she described enacting in instruction were unlikely to enable students' development of robust, enduring understandings of mathematics.

In addition, we conjecture that when designing to support shifts in teachers' views of their students' mathematical capabilities, it is crucial to take account of the contexts in which teachers work (Cobb, McClain, Lamberger, & Dean, 2003; McLaughlin & Talbert, 1993). Relations between teachers' views of their students' mathematical capabilities and aspects of the school and district context were beyond the scope of this analysis. However, when considering how to support teachers, it is crucial to recognize that teachers' views of their students' mathematical capabilities are socially constructed and develop and circulate in interaction with colleagues (Bannister, 2015; Horn, 2007; McLaughlin & Talbert, 1993; Talbert & McLaughlin, 1994). In fact, McLaughlin and Talbert (1993) found that how teachers constructed students or their roles as teachers had "little to do with formal aspects of the school and much to do with the character of the professional community that defines the school (or department) culture" (p. 223). Within secondary schools, the key unit of influence was the department to which a teacher belonged (e.g., mathematics department), and constructions of students could vary dramatically by department.

These findings suggest that, at the least, efforts to shift teachers' views of their students' mathematical capabilities should take account of the prevailing norms for talking about students and making sense of their performance within a school, and likely within the units that teachers are organized (e.g., departments, grade-level teams). At present, this is especially important, given the frequency with which teachers are expected to collaborate with their colleagues on

issues of instruction. As illustrated in Horn's (2007) and Bannister's (2015) analyses, regular meetings can serve as a place to challenge one another's views and develop more productive ways of explaining student difficulty and supporting students. However, regular meetings can also serve as a setting in which unproductive views are reinforced (Talbert & McLaughlin, 1994).

Assessing Teachers' Views of Their Students' Mathematical Capabilities

Another contribution of this study concerns the value of the interview-based assessment we developed for eliciting teachers' views of their students' mathematical capabilities. Specifically, an assessment like the one we have described can play at least three functions in designing and accounting for teachers' learning in the context of improvement efforts. First, our findings suggest that prior to designing specific efforts, it is likely important to take account, prospectively, of how teachers view their students' capabilities in mathematics. If teachers diagnostically frame the problem of student difficulty in mathematics in unproductive ways, this would indicate the need for deliberate attention to shifting the framing. Similarly, if teachers tend to describe unproductive supports for students facing difficulty in mathematics, this would indicate the need for targeted professional learning on how to enable students to participate in rigorous mathematical activity. Second, an assessment like what we have described can be useful in monitoring improvement efforts. It is very unlikely that if unproductive views of students' mathematical capabilities continue to dominate the work context that all students are going to be provided with the necessary support to participate in rigorous mathematical activity. It is important to be able to assess on an ongoing basis how teachers' views are changing, or not; this information can then be used to inform revisions to the ongoing improvement efforts.

To be clear, we are not suggesting that accounting for views of students' mathematical capabilities either prospectively or as part of monitoring ongoing improvement efforts requires as extensive and formal an interviewing and coding process like the one we described. Certainly, using the instrument on a smaller scale than we did would make it less demanding in terms of time and effort. We could also imagine researchers and practitioners developing and using alternative methods of assessing teachers' views of students' capabilities, perhaps in the context of common planning time or professional development sessions. We view the two dimensions of teachers' views of students' mathematical capabilities that we identified as providing useful guidance regarding what would be important to attend to in such settings.

Third, we suggest that having an assessment like the one described here is useful to include in research analyses of instructional improvement efforts. It would be fruitful to include assessments of teachers' views of students' capabilities in analyses investigating the relations between aspects of teacher knowledge, perspectives, and practice, and in relation to specific classroom, school, and district settings. Doing so can contribute to the field's existing knowledge of what impacts teachers' enactment of high-quality teaching, including in classrooms serving historically underserved populations, and thus inform our understanding of how to support teachers.

As an example, Wilhelm, Munter, and Jackson (in press) created a quantitative variable based on the qualitative coding of teachers' views of their students' mathematical capabilities in Years 1-4 of the larger study, and conducted quantitative analyses focused on the relations between teachers' explanations of student difficulty and the distribution and quality of students' discourse in whole-class discussions. (The measures of instructional quality were based on coding video-recorded observations of teachers' instruction.) Wilhelm et al. found a positive,

statistically significant relationship between teachers' articulation of productive explanations of students' difficulty and the quality and distribution of students' contributions in whole-class discussions, when controlling for teachers' mathematical knowledge for teaching, vision of high-quality instruction, choice of mathematical task, years of experience, and setting. Students were, on average, more likely to have opportunities to participate in discussions in which they provided reasoning for their solutions if their teacher articulated productive diagnoses of sources of their difficulty. And, the results of an interaction analysis suggested that this relation was strongest in classrooms composed (almost) entirely of students of color.

As another example, also using data from Years 1-4 of the larger study, Wilhelm (2014) found a positive, statistically significant relation between teachers' descriptions of the supports they provided to students facing difficulty and whether the cognitive demand of a high-level task was maintained over the course of a lesson. Specifically, the "odds of maintaining the cognitive demand of a high-level task for teachers who espoused productive [supports] were 2.92 times the odds for teachers who described unproductive or mixed [supports]" (p. 660). Findings such as these help the field to further pinpoint important aspects of accomplishing ambitious reform at scale and to identify areas deserving of future investigation.

Applicability to Ambitious Reform Efforts in Other Grade Levels and Subject Areas

Last, we consider the applicability of these findings for ambitious instructional reform efforts in other grade-levels of mathematics, as well as other subject areas. We imagine that regardless of the grade band or subject area, if the targeted forms of practice associated with a reform represent a significant change in teachers' current practice, it is likely that they will also need to reconsider what it means to be capable in that subject area. And, given that some students are bound to face difficulty in any instructional regime, teachers will likely need

assistance regarding how to support students who face difficulty to participate substantially in more rigorous activity. That said, there may be important differences in how teachers explain the sources of student difficulty or in how they support students facing difficulty, depending on the level of schooling or subject matter.

Based on extant research (Bannister, 2015; Horn, 2007), we suspect that what we found at the middle-grades may be similar to what one would find in a similar study of high school mathematics teachers. However, in light of adolescent development, it may be that secondary mathematics teachers are more likely to position issues of student difficulty as outside of their locus of control, as compared to elementary teachers. In addition, given the sequential nature of mathematics curriculum (Stodolsky & Grossman, 1995), and the fact that what happens in secondary mathematics builds on at least five years of prior schooling, it may be that secondary teachers place more value on students having “mastered” particular skills prior to engaging in conceptually oriented activity than in elementary school. Thus, it may be that secondary mathematics teachers tend to proceduralize instruction for students facing difficulty more often than elementary teachers do, as they view that as the fastest way to help students catch up.

Coburn's (2006) study of the implementation of an ambitious elementary reading reform initiative and Windschitl et al.'s (2011) study of novice science teachers' development support our hypothesis that attending to teachers' views of their students' capabilities when implementing any ambitious instructional reform is necessary, regardless of the subject area. However, it is likely that there are some discipline-specific aspects regarding how teachers tend to view their students' capabilities. For example, as Stodolsky and Grossman (1995) clarified, mathematics teachers tend to perceive mathematics as static and hierarchal, and report feeling pressure to cover topics, whereas, for example, English teachers tend to perceive their curriculum

as much more fluid and dynamic. It may be the case, then, that mathematics teachers are more likely to explain student difficulty in unproductive ways than English teachers, especially given the prevalence of the idea that there are “math people” and “non-math people” (Battey & Stark, 2009; see also Battey & Franke, 2013).

More generally, it appears that a significant challenge in accomplishing ambitious reform at some scale entails the reorganization of how teachers view their students' capabilities. For that reason, we are hopeful that others will take up the issues we identified above. Absent deliberate attention to teachers' views of students' capabilities in improvement efforts, we are doubtful that such efforts to improve teaching and learning will take hold on a large scale.

Endnotes

¹ All teacher names are pseudonyms.

² In this article, we only report on teachers' views of their students' mathematical capabilities. However, within the larger project, we also assessed coaches', school leaders', and district leaders' views of students' mathematical capabilities. Interested readers can email the first author for information about the questions we asked of the various role groups.

³ Task 1.1, “Preparing for a bicycle tour,” in *Variables and Patterns: Introducing Algebra* by Lappan et al. (2009).

⁴ As a reviewer usefully pointed out, giving students “a hand for the first step” might not necessarily involve telling students how to solve the problem. We cannot be certain that Ms. Jacobi's descriptions of her supports indicated that she proceduralized tasks when students faced difficulty. However, we viewed a video-recording of her instruction that was collected as part of the larger research project in Year 5 (the same year as the interview data we analyzed for this analysis) as a point of triangulation. The video-recording indicated that when students hesitated

to complete the assigned task, she did as we presumed; she intervened and asked them step by step what they should do, and if they did not answer immediately or answered incorrectly, she showed them what to do, step by step.

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Table 1

Two Dimensions of Teachers' Views of Students' Mathematical Capabilities

	Unproductive	Productive
Diagnostic Framing: Explanations of the source(s) of students' difficulties in mathematics	Explanations suggest student difficulty is due to inherent traits of the student, or deficits in their family or community.	Explanations suggest student difficulty is produced in relation to instructional and/or schooling opportunities.
Prognostic Framing: Descriptions of how to support students who face difficulties in mathematics	Supports aim at reducing the rigor of the learning goals for students.	Supports aim at enabling students to participate in rigorous mathematical activity.

Table 2

Demographic Information Regarding the Districts' Student Populations, 2011-2012

District	# of students	Limited English Proficient	White	African American	Hispanic	Asian	Native American	Free or Reduced Price Lunch
B	83,000	28%	14%	23%	60%	1.9%	<1%	77.5%
D	95,000	6%*	52%	36%	7%	3%	<1%	60%

Note. *Information was not available regarding percent of students classified as Limited English

Proficient (LEP) in District D in 2011-2012. The most recent data we have available is from

2009-2010, which indicated that 6% of students were classified as LEP.

Table 3

Abbreviated Version of Coding Scheme to Assess the Nature of Teachers' Explanations Regarding the Source(s) of Students' Difficulties in Mathematics

Code and Definition	Example of coded transcript
<p>PRODUCTIVE Student performance (e.g., failure, success, engagement, interest) is described as a relation between student(s) and instructional and/or schooling opportunities.</p>	<p>Interviewer: So, in your own classroom when students don't learn as expected, what do you usually find are the reasons? Teacher: Why a kid didn't learn? Because I didn't make him. Interviewer: How do you make a kid learn? Teacher: I don't know. That's always the problem, isn't it? I, I do, and also again I, I might be different on that, but I, I really feel like if a kid's not learning in a classroom, it's my fault. That it's something that I'm not doing. There has to be a reason. I mean, I, you know, especially in the 8th grade, I mean, they can learn something. There is, there's something they can be doing. There's some way they can be doing it. And so, I mean, if a kid's just flat out not learning then there's something that I need to do better to make him learn and I don't always know what that is, but I mean, I do put most of the emphasis back on me.</p>
<p>MIXED Participant wavers between explaining student performance (e.g., failure, success, engagement, interest) 1) as a relation between student(s) and instructional and/or schooling opportunities and 2) as due to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional opportunities (e.g., parents don't value education, therefore students don't).</p>	<p>Interviewer: In your classrooms, when the students do not learn as expected, what do you find are the typical reasons? Teacher: Probably me...I don't put blame on the students. I mean, I think it's a combination. They have to do their part, and I have to do mine, so if they're not getting it, it may, and this, this may not be the best way, but I'll be honest, I look to the students that are consistently successful, and if they don't understand something, I know I'm doing something wrong, so I need to go back, and I need to think it through again or come up with a different strategy or a way of showing them to do the problem. You know, if it's a kid that is consistently off task and playing around or something, then I might just kind of think that, "Well, they're not paying attention," so, it's just kind of like what the majority of the class is doing, and I kind of judge off that.</p>
<p>UNPRODUCTIVE Student performance (e.g., failure, success, engagement, interest) is attributed to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional and/or schooling opportunities (e.g., parents don't</p>	<p>Interviewer: So what are some of the major challenges ... of teaching mathematics in this school? Teacher: The kids already don't want to learn math. They have this notion of not caring for it and usually it's instilled by their parent's cause their parents didn't get it, so they think its okay that they didn't get it.</p>

Explanation presents students' mathematical capabilities as relatively stable (i.e., they are unlikely to change).	
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Table 4

Abbreviated Version of Coding Scheme to Assess Teachers' Descriptions of How They Support Students Who Face Difficulties in Mathematics

Code and Definition	Example of coded transcript
<p>PRODUCTIVE Description of instructional actions one takes to support students who are facing difficulties are aimed at rigorous learning goals.</p> <p>Below is a list of instructional actions that are generally aimed at supporting students facing difficulty to participate in rigorous activity in the context of mainstream instruction. This is not an exhaustive list; the coder will need to make judgments regarding the nature of what participants describe.</p> <ul style="list-style-type: none"> • <i>Pre-teach particular skills to students prior to mainstream instruction that are necessary for engaging in the targeted mathematical idea at a conceptual level; this is sometimes done in the context of a 2nd math class or intervention.</i> • <i>Focus on how the task is introduced, or set-up. Ensure students are familiar with the context in a problem-solving scenario.</i> • <i>Use tasks with multiple entry points.</i> • <i>Focus on norms of participation.</i> • <i>Assign competence to students (e.g., strategically mark students' contributions as important to attend to).</i> • <i>Group students in ways that aim to maximize each student's participation (e.g., assigning roles, assigning near-peers).</i> 	<p>Interviewer: In terms of ... having kids in the same room with a wide range of knowledge, what are like some of the strategies you use to address that ...? Teacher: One thing is [to] re-write the tasks from the books and Interviewer: From ... the [text] or? Teacher: Right. Trying to make sure that they maintain the rigor, but then ... are there multiple entry points into this particular task? Can I make sure that my student who struggles the most can find a way to engage in this task and my student who has the most skills in this classroom is still gonna be challenged?</p> <p>***</p> <p>Interviewer: [D]o you make changes within that class to provide different types of instruction? Teacher: ... I just make sure that there's lots of accountable talk, lots of group work. Making sure that kids are staying on task and it's been kind of a handful all year to get it going. I think I'm making some progress with them... Interviewer: Hmm hmm, so what do you do? Teacher: I keep the bar high. I have high expectations for them and I keep telling them that they can learn and they can be smart.</p>
<p>MIXED Clearly articulates learning goals and instructional supports for students who are facing difficulties that are aimed at rigorous activity however, some of what participant says indicates that some instructional actions are aimed at conventional learning goals. Supports typically aim at first ensuring that students develop "basic skills" before engaging in more rigorous activity.</p>	<p>Teacher: I'm always afraid to go ahead because I don't feel my kids are mastering things and I try to challenge my kids and use a lot of word problems, use a lot of words and a lot of real world settings because that's what they're going to, you know, they're not going to sit in some room doing a hundred adding fractions problems, but at the same time some of my kids actually need to do a hundred addition problems with fractions just so it sticks in their head that they've got to get a common denominator.</p>
<p>UNPRODUCTIVE Description of instructional actions one takes to support students</p>	<p>Teacher: In the longer classes you can get a little bit more done but as far as the ability wise, there's always going to be a class that can do more, so you going to give them</p>

<p>who are facing difficulties are generally aimed at lowering the cognitive demand of activity (e.g., proceduralizing a task).</p> <p>Below is a list of instructional actions that are generally aimed at lowering the cognitive demand of activity. This is not an exhaustive list; the coder will need to make judgments regarding the nature of what participants describe.</p> <ul style="list-style-type: none"> • <i>Remove any prompts that ask students to explain their thinking.</i> • <i>Shorten problems.</i> • <i>Show students how to complete a similar problem.</i> • <i>Provide examples.</i> • <i>“Drill,” “Use direct instruction.”</i> • <i>Assign fewer problems.</i> 	<p>more to chew on than you would the class that’s not quite capable.</p> <p>Interviewer: Okay so you would be adjusting, would you be adjusting the kind of tasks that you give them or would you be adjusting the pace or maybe how, how you would group the, the kinds of students? ...</p> <p>Teacher: ... [A] little bit of both. The ... pacing would be a little bit slower in the longer classes and then faster in the shorter classes. And then the, the tasks for the kids who are in the more capable classes they would get more independent practice where as the one in the less capable class they would get more modelling and guided practice.</p>
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Table 5

Teachers' Views of Students' Mathematical Capabilities in Two Districts Pursuing Ambitious Reform in Middle-grades Mathematics, 2011-2012

	Assigned a Code	Unproductive	Mixed	Productive
Diagnostic Framing (Explanations)	100 (82%)	28 (28%)	54 (54%)	18 (18%)
Prognostic Framing (Supports)	74 (61%)	52 (70%)	8 (11%)	14 (19%)

Note. Percentages are rounded to the nearest per cent, and therefore totals may not equal 100%.

Percentages for “assigned a code” refer to total number of teachers interviewed (n = 122).

Percentages for unproductive, mixed, and productive refer to the number of teachers who were assigned a code in that category (n = 100 for Explanations; n = 74 for Supports).

Table 6

Relations Between Teachers' Diagnostic and Prognostic Framing of Students' Difficulty in Mathematics (n = 56 teachers)

Prognostic Framing (Supports)	Diagnostic Framing (Explanations)		
	Unproductive	Mixed	Productive
Unproductive	10 (18%)	25 (45%)	3 (5%)
Mixed	3 (5%)	3 (5%)	1 (2%)
Productive	2 (4%)	4 (7%)	5 (9%)

Note. Percentages are rounded to the nearest per cent, and therefore totals may not equal 100%.