Teachers’ Views of Students’ Mathematical Capabilities: A Challenge for Accomplishing Ambitious Reform

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Abstract

Building on qualitative research that suggests teachers’ views of their students’ mathematical capabilities matter for the quality of students’ learning opportunities, we investigated the views of 122 middle-grades mathematics across two large, urban districts that were pursuing ambitious instructional reform. Our findings suggest that accomplishing ambitious reform (like the implementation of the Common Core State Standards) will require shifts in teachers’ views of their students’ capabilities. We approached our investigation through a lens of problem framing – we focused on teachers’ diagnoses of the source(s) of students’ difficulties in mathematics as well as their prognoses. Methodologically, we developed an interview-based assessment for eliciting and characterizing teachers’ diagnoses and prognoses. We report descriptive information regarding teachers’ diagnoses and prognoses, as well as on trends in the relations between the two. Our findings suggest that the majority of teachers viewed at least some of their students as incapable of engaging in rigorous mathematical activity, and that when they perceived students were facing difficulty, they tended to describe lowering the rigor of the work in which students were expected to engage. We conclude by considering the implications of our findings for designing and assessing improvement efforts as well as for future research.
Teachers’ Views of Students’ Mathematical Capabilities: A Challenge for Accomplishing Ambitious Reform

The adoption of the Common Core State Standards in Mathematics (CCSS-M; Common Core State Standards Initiative, 2010) by a majority of U.S. states, accompanied by the promise of conceptually oriented state assessments, represents a particular moment in the history of mathematics education reform. It appears possible that what U.S. teachers will be held accountable for teaching may reflect what decades of research on mathematics learning and teaching suggests supports students to develop robust, enduring understandings of mathematics. However, the task of actually achieving the vision suggested in the CCSS-M is ambitious, given what we know about the demands for teacher learning inherent in doing so (Cobb & Jackson, 2011a).

Namely, there is a stark contrast between current modal practice in teaching mathematics and what research suggests is necessary on the part of classroom practice, if students are to achieve the rigorous learning goals outlined in the CCSS-M, which privilege developing procedural fluency, conceptual understanding of key mathematical ideas, mathematical reasoning, and the ability to communicate effectively about mathematical ideas. Mathematics education research suggests that in order to support students to achieve such goals, students need regular opportunities to engage in solving challenging, non-routine tasks that can be solved in multiple ways and with multiple representations (Stein, Grover, & Henningsen, 1996). Further, they need regular opportunities to explain and justify their reasoning, as well as to make connections between solutions and to key mathematical ideas (e.g., Franke, Kazemi, & Battey, 2007; Stein, Smith, Henningsen, & Silver, 2000). More generally, it is important that students are provided ample opportunity to share mathematical authority with the teacher in assessing
what is (and what is not) mathematically acceptable, valid, and on what grounds (Lampert, 1990).

These forms of instructional practice, sometimes called ambitious teaching (Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010), contrast in important ways with the goals of typical mathematics instruction and the associated forms of practice. Most mathematics teaching emphasizes students developing procedural facility absence a focus on conceptual understanding. Accordingly, modal teaching practice engages students in procedural tasks with limited opportunities to engage in mathematical reasoning; students are expected to master known procedures to solve rather predictable sets of problems (Boston & Wilhelm, in press; Stigler & Hiebert, 1999). In addition, mathematical authority typically resides with the teacher and/or the text (Staples, 2007).

Therefore, achieving the vision for school mathematics represented by the CCSS-M will require, for most teachers, a radical reorganization of their practice (Cobb & Jackson, 2011a). Such a reorganization entails developing new perspectives on teaching and learning (e.g., Munter, 2014; Wilhelm, 2014) and new forms of mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008; Hill, Blunk, et al., 2008), in addition to skill in enacting forms of teaching that privilege eliciting and building upon student thinking (e.g., Franke et al., 2007). In this paper, we focus on an aspect of teaching that has received less attention, but, in our view, is essential to achieving the vision suggested by the CCSS-M – the importance of coming to view one’s students as capable of participating in rigorous mathematical activity. By rigorous mathematical activity, we intend what we described above as constituting ambitious teaching.

Small-scale qualitative studies suggest that teachers’ views of their students’ mathematical capabilities matter in terms of the kind of mathematical activity in which they
engage their students (e.g., Diamond, Randolph, & Spillane, 2004; Horn, 2007; Jackson, 2009). For example, both Diamond et al. (2004) and Jackson (2009) found that math teachers justified engaging students in activity solely aimed at developing procedural facility in terms of their perceptions of students’ capabilities. Developing a perspective that one’s students are capable of participating in rigorous mathematical activity is particularly important to achieving the vision embodied by the CCSS-M given that, at its core, ambitious teaching requires viewing one’s students as able to engage in sense-making and as having valuable ideas on which to build (Lampert, 2001). Relatedly, what it means for a student to be mathematically capable in a classroom oriented towards CCSS-M goals is decidedly different than in a classroom oriented towards more conventional learning goals (Gresalfi, Taylor, Hand, & Greeno, 2008; Lampert, 2001; Staples, 2007). This implies that it may be necessary for teachers to shift their perspectives on what it is that their students are capable of, as they work to implement forms of practice that reflect more rigorous goals for their student’s learning.

Furthermore, research suggests that the challenge of coming to view one’s students as capable of participating in rigorous mathematical activity is likely magnified in districts and schools serving historically underserved groups of students. In particular, anthropological and sociological research has demonstrated that it is common for teachers to articulate deficit-oriented views of such groups of students, as well as of their families and the communities in which they live (e.g., Anyon, 1981; Diamond et al., 2004; Jackson, 2009, 2011; Oakes, 1985; Rist, 1970). This body of work suggests that teachers are less likely to view historically disadvantaged groups of students as capable of participating in rigorous mathematical activity, as compared to wealthier, whiter populations of students.

That said, we know of no study that has attempted to provide a snapshot of teachers’
views of their students’ mathematical capabilities on a large scale. This kind of information is valuable, in our view, in contributing to the field’s understanding of the learning demands inherent in achieving the ambitious vision suggested in the CCSS-M, and therefore of what might constitute critical foci for professional learning opportunities. In this paper, we report on a study of 122 middle-grades mathematics teachers’ views of their students’ mathematical capabilities (hereafter, VSMC) across two large, urban districts that were in the midst of pursuing ambitious instructional reform. Conceptually, as we explain below, we approached our investigation of teachers’ VSMC through a lens of problem framing (e.g., Benford & Snow, 2000; Goffman, 1974). In particular, we answer the following research questions:

1. How do middle-grades math teachers across two districts pursuing ambitious reform diagnostically frame the problem of students’ difficulties in mathematics? That is, how do they explain the source(s) of students’ difficulties in mathematics?
2. How do the teachers prognostically frame the problem of students’ difficulties in mathematics? That is, how do they describe what they do to address the problem?
3. How are teachers’ diagnostic and prognostic frames related to one another?

In what follows, we first describe our conceptual framework, followed by our methods and then our findings. Our findings suggest that a significant challenge in accomplishing ambitious reform at some scale entails supporting the reorganization of how teachers view their students’ capabilities along two dimensions – in both how teachers diagnose the source of students’ difficulties in mathematics and how they address such difficulties. We conclude by considering the implications of our findings for designing and assessing improvement efforts as well as for future research. Although this analysis focuses on middle-grades mathematics
teachers, we contend that it has implications for accomplishing ambitious reform in other grade-bands in mathematics, and potentially in other subject matter areas as well.

**Conceptual Framework**

**Problem Framing**

As mentioned above, conceptually we approached our investigation of teachers’ VSMC by tapping into how they framed a common problem of practice – students facing difficulty in mathematics. Framing refers to how a particular situation is understood or interpreted (e.g., Benford & Snow, 2000; Goffman, 1974). Particular frames offer particular representations of a problem, and therefore “inevitably highlight certain aspects of the situation while deemphasizing others (Weiss, 1989)” (Coburn, 2006, pp. 343-344). Scholars across anthropology, sociology, and psychology have argued for the importance of attending to framing processes because how specific situations or problems are framed delimits what count as potential solutions (Benford & Snow, 2000; see also Hand, Penuel, & Gutiérrez, 2012; Horn, 2007).

Sociologists attend to at least two “framing tasks” when analyzing framing processes—*diagnostic framing* and *prognostic framing* (Benford & Snow, 2000). Diagnostic framing involves articulating the source of the problem, whereas “[p]rognostic framing involves articulating a proposed solution to the problem” (Coburn, 2006, p. 357). Further, as Coburn clarifies, the two tasks of framing are “intertwined, in that prognostic framing often rests implicitly on the problem definition and attribution that is part of diagnostic framing” (p. 357).

For example, consider these contrasting responses from two middle-grades mathematics teachers to the following interview question: *When your students don’t learn as expected, what do you find are typically the reasons?*
Mr. Williams:\ “I normally look first at me to see … is there something in the lesson that I didn't emphasize well enough or…I may talk to the teacher they had last year and say ‘When you went over this was this something that they struggled with?’”

Mr. Batsem: “Well I’m, you know, you’re not supposed to think necessarily but I, I believe there’s some innate, you know, ability in differences, …. math comes easier to some kids than others, you know.”

Mr. Williams frames the problem of students not learning as expected as related to instructional opportunities, while Mr. Batsem frames this same problem as one due to inherent traits of students (e.g., some students are naturally better at mathematics than others). Mr. Williams’ diagnostic framing suggests the possibility for an instructionally focused solution. In contrast, given Mr. Batsem’s diagnostic framing of the problem, it is difficult to imagine that his prognostic framing would entail altering his instruction to support students who do not learn as expected. More generally, Mr. Williams suggests that his students are capable of engaging in classroom mathematical activity, when provided with the proper opportunities and support to do so. On the other hand, Mr. Batsem suggests that at least some of his students are inherently incapable of participating in classroom mathematical activity.

Prior qualitative research suggests that how teachers frame the problem of students not learning as expected matters, particularly when trying to understand teachers’ response to and participation in initiatives focused on ambitious teaching (Coburn, 2006; Horn, 2007; Windschitl, Thompson, & Braaten, 2011). As one example, Horn (2007) studied two high school math departments’ engagement in what she termed “equity-geared reforms” (p. 44). She focused on how the two departments framed a common problem of practice—the fact that not all students are academically successful in mathematics classrooms—in the context of regularly scheduled
teacher workgroups. Drawing on the concept of “category systems” (Bowker & Star, 1999), Horn (2007) documented the categories that teachers used to describe groups of students when explaining why some students succeeded in classrooms while others did not. She then related teachers’ ways of categorizing groups of students to the teachers’ own views of mathematics. She argued that both informed how teachers framed the problem of differential success in the mathematics classroom.

Specifically, Horn found that in one department the teachers tended to account for students’ performance in terms of inherent traits of the students (e.g., students were fast, slow, lazy). These same teachers also tended to view mathematics as “a well-defined body of knowledge” with a rather fixed sequential order of topics (Horn, 2007, p. 43). Against this view of mathematics, the teachers aimed to cover the topics in a particular sequence to prepare students for subsequent coursework. And, when students did not learn as expected, teachers placed the blame on the students, and felt there was little they could alter about instruction.

However, in the other department, Horn found that the teachers tended to account for students’ performance in terms of the learning opportunities provided in the classroom. For example, rather than attribute a student’s difficulties to “laziness,” they considered the nature of learning opportunities that had been provided to the student. Students’ engagement or disengagement depended, in part, on the nature of any given activity (rather than some inherent characteristic of the student). Therefore, if students were not engaged, the teacher was more likely to consider how she might alter instruction. The teachers also tended to view mathematics as a connected and conceptual web of ideas. Because mathematics was viewed as a web of ideas, rather than a sequential ordering of topics, teachers felt more freedom in altering curriculum to support students.
As a second example, Windschitl, Thompson, and Braaten (2011) focused on novice secondary science teachers’ development of ambitious teaching, and identified relations between how teachers framed what influenced students’ learning of science and their instructional practice. Based on teachers’ conversations in Critical Friends Groups, Windschitl et al. identified two contrasting frames regarding what influenced students’ learning—what they termed “problems with students” and “puzzles of practice.” The “problems with students” frame describes when novice teachers tended to suggest that the “responsibility for performance rested almost entirely with students” (p. 1323). On the other hand, a “puzzles of practice” frame describes when novice teachers engaged in discussions of students’ performance “marked by a genuine sense of curiosity and intellectual challenge” (p. 1323). Windschitl et al. argued that within a “puzzles of practice” framing, “students were portrayed as capable of significant achievement under the right conditions” (p. 1324).

Furthermore, Windschitl et al. found that those teachers who tended to invoke the “problems with students” frame tended to enact instruction that reflected an acquisition theory of teaching and learning (e.g., the teacher presented a set of ideas and activities, and expected students to acquire those same ideas). On the other hand, teachers who tended to invoke the “puzzles of practice” frame tended to enact instruction that reflected a “sensemaking” theory of teaching and learning (e.g., teachers elicited students’ ideas, attempted to build on those ideas, and engaged students in activities in which the teacher could then press their students to develop conceptual explanations of scientific phenomena).

Whereas Horn’s (2007) analysis provides evidence of how teachers’ problem framing link to teachers’ views of the discipline of mathematics, Windschitl et al.’s (2011) analysis illustrates how teachers’ problem framing links to teachers’ theories of teaching and learning as
well as their actual practice. Both suggest the value of using a lens of teachers’ framing as a window into their views of their students’ mathematical capabilities, and suggest that such views matter when attempting to implement ambitious teaching practices.

**Attending to Framing Processes In Light of a Particular Vision of Mathematics Instruction**

As discussed earlier, we were particularly concerned with understanding teachers’ VSMC in the context of ambitious instructional reform. In other words, we were not interested in understanding whether teachers viewed their students as capable of engaging in *any* mathematical form of classroom activity – rather, we were interested in whether they viewed their students as capable of engaging in what we referred to as *rigorous* mathematical activity. That is, we took a stance on what is worth knowing and doing mathematically; that stance is commensurate with the learning goals and the associated forms of practice represented by the CCSS-M and the National Council of Teachers of Mathematics’ (NCTM; 2000) *Principles and Standards for School Mathematics*.

As we illustrate in our discussion of our methods and findings, we operationalized our attention to a particular vision of mathematics instruction by focusing on both diagnostic and prognostic framing of students not learning as expected. In particular, we found that attending to teachers’ prognostic framing provided additional insight into the *kind of mathematical activity* a teacher thinks a student is capable of. In fact, attending only to diagnostic framing is similar in some respects to psychological research on teacher “self-efficacy” (Bandura, 1993; Gibson & Dembo, 1984; Sosa & Gomez, 2012; Tschannen-Moran & Hoy, 2001), which refers to a teacher’s belief that s/he “can influence learning” (Sosa & Gomez, 2012, p. 879). It usually emphasizes the extent to which teachers believe they “can help even the most difficult or unmotivated students” (Gibson & Dembo, 1984, p. 569). Teachers who are identified as having
high self-efficacy are those who perceive that they can overcome challenges, whereas teachers identified as having low self-efficacy are those who perceive they cannot overcome challenges.

Another related construct is teachers’ “responsibility” (e.g., Halvorsen, Lee, & Andrade, 2009; Lee & Smith, 1996), which Lee and Smith (1996) describe as including:

- teachers internalizing responsibility for the learning of their students, rather than attributing learning difficulties to weak students or deficient home lives;
- a belief that teachers can teach all students;
- a belief that teachers can teach all students;
- willingness to alter teaching methods in response to students’ difficulties and success; and feelings of efficacy in teaching. (p. 114).

Halvorsen et al. (2009) differentiate responsibility from teacher efficacy in the following way: “Responsibility focuses more on the teachers’ willingness to take responsibility for helping all students learn rather than on teachers’ beliefs about their professional effectiveness” (p. 183).

Similar to studies of self-efficacy and responsibility, we were interested in whether teachers appeared to take responsibility for supporting students to participate in mathematical activity. However, neither measures of self-efficacy or responsibility assume a particular vision of instruction (regardless of content area). That is, a teacher could be identified as having high self-efficacy or as taking responsibility for students’ learning, but with respect to a vision of traditional instruction. Within our analyses, we make a distinction between viewing one’s students as capable of participating in more conventional forms of school math activity but not necessarily in more rigorous forms of activity. As we discuss further below, our findings suggest this distinction is important when considering how to support teachers to develop ambitious teaching practices.

**Methods**
This study uses data gathered in two districts (Districts B and D) in Year 5 of a longitudinal study aimed at identifying what it takes to support instructional improvement in middle-grades mathematics at the scale of large, urban districts (Cobb & Jackson, 2011b; Cobb & Smith, 2008). We conjectured at the beginning of the study that it would be important to account for teachers’ VSMC in order to help us make sense of the ways in which teachers responded to ambitious reform efforts (e.g., how they took up, or not, specific instructional practices, how they made use of reform-oriented curricular materials). In what follows, we provide background on the research context and then describe the interview-based assessment we developed to analyze teachers’ VSMC – in particular, how they explained why students did not learn as expected (diagnostic framing) and how they described supporting students who did not learn as expected (prognostic framing).

**Research Context**

Districts B and D were both purposively selected to participate in the larger study for a few reasons. They were typical of large, urban districts in terms of the challenges they faced, including high teacher turnover and large numbers of students identified as low-performing. However, the districts were unusual in that they responded to high-stakes accountability pressures by attempting to achieve an ambitious vision of mathematics instruction, or instruction that, at the time, was broadly compatible with NCTM’s (2000) *Standards*, and is compatible with the CCSS-M. In both districts, teachers were provided with the second edition of the *Connected Mathematics Project* (CMP2) curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009), a text aimed at supporting rigorous learning goals for students, at the start of the study (2007-2008). Additionally, in order to assist teachers to develop ambitious instructional practices, each district provided a number of supports (e.g., curriculum frameworks, professional development,
regular time to collaborate with colleagues on issues of instruction, coaches, professional
development for principals).

Table 1 provides demographic information regarding Districts B and D student
populations in Year 5 of the study (2011-2012). In each district, approximately 60 teachers
located in a sample of 12-13 schools participated in the study. Schools were selected in
consultation with district leaders to represent a range in capacity for instructional improvement.

Data Collection

Each year, several types of data were collected to test and refine a series of hypotheses
and conjectures about district and school organizational arrangements, social relations, and
material resources that might support mathematics teachers’ development of high-quality
instructional practices at scale (Cobb & Jackson, 2011b; Cobb & Smith, 2008). For the purposes
of this analysis, we focus on semi-structured interviews (Merriam, 2009) conducted with
teachers each January – and, in particular, teachers’ responses to interview questions specific to
their VSMC. The audio-recorded interviews, which lasted approximately 45 minutes, focused on
the formal and informal supports they received, to whom and for what they perceived themselves
to be accountable, their vision of high-quality mathematics instruction (Munter, 2014), as well as
their VSMC.

We elicited how teachers diagnosed the problem of students’ difficulties in mathematics,
as well as their descriptions of their prognosis. To elicit their diagnostic frames, we asked
teachers questions like, “When your students don’t learn as expected, what do you find are
typically the reasons?” We also asked teachers to describe the challenges they face, which often
provided us with insight into how they framed students’ difficulties. Furthermore, interviewers
were trained to press on teachers’ explanations of why students might not learn as expected. To elicit teachers’ prognostic frames, we followed up on diagnostic frames with questions like, “What do you do to address that challenge?” We also systematically asked teachers if they found they needed to adjust their instruction for different groups of students, and if so, why and how they did so.

**Methods of Analysis**

Our first step in analysing interview data involved developing a coding scheme specific to VSMC. Our approach was both deductive and inductive, although primarily inductive. Our initial coding scheme included a list of provisional categories based on existing literature (Miles, Huberman, & Saldaña, 2014), however, it was heavily informed by reading approximately one-third of the Year 1 interview transcripts from all teachers in the larger study (n=132). The primary codes (Campbell, Quincy, Osserman, & Pedersen, 2013) for our initial coding scheme included 1) the categories teachers used to describe groups of students and the characteristics they ascribed to the categories; 2) the pedagogical actions teachers described that they took to meet the needs of groups of students; 3) extent to which a teacher took responsibility for groups of students’ learning; 4) teachers’ views about learning mathematics and the curriculum; and 5) teachers’ reports on instructional leaders’ expectations regarding supporting all students. After having achieved a stable coding scheme, we then coded interview transcripts for the remaining teachers in Year 1 of the study. The first two authors individually coded all of the transcripts, and came to consensus on the particular codes we assigned to each instance in a transcript.

We found that this initial coding scheme was too unwieldy, however it proved to be an important step in helping us clarify what, specifically, about teachers’ VSMC we might feasibly and reliably assess. To do so, the first and second authors created analytic memos (Hammersley
& Atkinson, 1995) for approximately two-thirds of the schools in the study that described the patterns and variations across teachers, based on our initial coding scheme. We then looked across the analytic memos and found that it appeared possible to consistently characterize participants’ VSMC along the two framing dimensions described above – teachers’ explanations of the source(s) of students’ difficulties in mathematics (hereafter, referred to explanations) and teachers’ descriptions of how they support students facing difficulty (hereafter, referred to supports).

The first author then drafted a coding scheme organized according to the two dimensions. She worked with a team of coders in each of the summers of 2009-2012 to refine this scheme in the practice of coding interviews collected each January. (The coding process is described in detail below.) The coding scheme presented here is in its penultimate form, and has since been used to code all teacher interview data collected for the first seven years of the project.

**Coding scheme for explanations (diagnostic frames).** Table 2 provides an abbreviated version of the coding scheme used to code teachers’ diagnostic frames, or explanations. We make the following distinction between kinds of explanations. Does the teacher frame problems of student difficulty in terms of the nature of instruction, or learning opportunities? Or, does the teacher ascribe student difficulty to inherent traits of the child (e.g., laziness, lack of motivation) or to perceived deficits in their families or communities? We termed the former kind of explanation *productive* and the latter *unproductive*. We use these terms to signal that the former suggests the teacher is positioned to examine and perhaps alter her instruction, whereas the latter suggests the teacher is unlikely to do so. As illustrated in Table 2, we assigned a code of *mixed explanations* for those instances in which a teacher wavered between attributing students’ difficulty to a problem of instruction and suggesting it was due to something inherent in the child
or to perceived deficits in their families or communities. We decided to make a distinction of “mixed” because we conjectured that a teacher who articulated mixed explanations might be in a better position to examine her instruction in relation to students’ difficulties than a teacher who only articulated unproductive explanations.

**Coding scheme for supports (prognostic frames).** Table 3 provides an abbreviated version of the coding scheme to assess teachers’ descriptions of supports for students who are not identified as English learners (ELs) or identified as receiving special education services. We use a different coding scheme to code for prognostic framing specific to ELs, which we developed with the support of an expert in EL education. Given space limitations, in this paper, we focus on teachers’ talk of supports for non-EL students. In the larger study, we generally did not include special education teachers in our sample and therefore did not code for talk of supports specific to students receiving special education services.

In general, we aim to make a distinction between whether a teacher describes supports aimed at supporting students to participate in rigorous mathematical activity. In assigning a code for supports, we draw on the form-function distinction that Saxe, Gearhart, Franke, Howard, and Crockett (1999) and Spillane (2000) have made specific to teachers’ and district leaders’ understandings of mathematics reform, respectively. For example, when asked how they support students facing difficulties, participants may say they “use manipulatives” or “put students in groups.” Such descriptions do not suggest the function of the particular pedagogical forms teachers use. For example, “form-only” talk about manipulatives does not detail what mathematical objectives teachers have when using manipulatives or why manipulatives might enable students to develop a more robust understanding of a particular mathematical idea or concept. The supports coding scheme elaborated in Table 3 is only used if we can infer the
function, or goal, of such supports; otherwise, we flag “form-only” talk of supports. We made this decision because we felt unable to determine whether the support being described was aimed at supporting students to participate in rigorous mathematical activity unless we could infer the function of the support.

If we were able to infer the function of the support described, as illustrated in Table 3, we assigned codes aimed at identifying whether teachers’ prognostic framing was aimed at supporting students to participate in rigorous activity. For example, imagine that a student faces difficulties in solving a rigorous math task in a classroom where ostensibly the goal is to develop both conceptual understanding and procedural fluency. On the one hand, the teacher might suggest that the best way to support that student is to ensure that the student has an opportunity prior to a specific lesson to develop an understanding of some material needed to participate in that lesson at a rigorous level (Boaler & Staples, 2008), sometimes called “pre-teaching.” This may indicate that the teacher views that student as capable of participating in rigorous mathematical work, albeit with targeted support. On the other hand, if a teacher suggests that the best way to support that student is to show the child how to solve the problem, it may indicate that the teacher does not view that particular student as capable of achieving the ostensible learning goals. We coded the former productive supports and the latter unproductive supports to indicate the extent to which the nature of supports described appear to frame the student(s) facing difficulty as capable of participating in rigorous activity. In addition to categories of productive and unproductive, we assign a code of mixed to indicate that the teacher describes actions that are aimed at supporting struggling students to participate in rigorous mathematical activity (high-cognitive demand activity), but some of what the participant suggests is aimed at conventional, low-cognitive demand activity. As illustrated in Table 3, a code of “mixed
supports” tends to reflect the view that students must first master “basics” before (and separate from) being provided opportunities to engage in conceptually-oriented activity.

Coding process. All transcripts were coded within a qualitative software package, NVIVO. Coders were trained to search for specific questions and keywords pertaining to teachers’ explanations and supports within interview transcripts. One coding decision we faced entailed determining how to unitize (Campbell et al., 2013), or chunk the text into meaningful, code-able parts. In semi-structured interviews, it is often the case that relevant ideas (e.g., an explanation regarding a source of difficulty or description of a support) unfold over the course of multiple turns of talk (Campbell et al., 2013). Therefore, in an effort to ensure coders were coding the same unit of text, we aimed to code at the unit of a turn of a talk, however, within NVIVO, we also captured whatever relevant text we used to make sense of that particular turn of talk. Coders were also strongly encouraged to use NVIVO to add linked annotations to any text to record their rationale for a particular coding decision.

An overall code was then assigned at the level of a teacher for each dimension. To do so, we looked across all coded passages of text for a specific dimension of VSMC (explanations, supports). If all passages for a specific dimension were coded as unproductive, the overall code for that VSMC dimension for the interview assigned was unproductive. If all passages for a dimension were coded as productive, the overall code assigned for the dimension was productive. And, if all passages were scored as mixed, or there was a combination of codes (e.g., unproductive for a relevant unit of talk and productive for another relevant unit), the overall code assigned for the dimension was mixed.
A team of four coders (including the first and third author) coded the data reported on in this analysis. The team was composed of the first author (“anchor coder”), another experienced coder, and two additional coders who were new to the process. The first author led a two-day training. This training was followed by an introductory, training phase of coding in which each coder individually coded 20 transcripts selected to represent a variety of coding queries that could arise. All four coders discussed and came to consensus for each relevant unit of talk for the 20 transcripts. The coders were then assigned 10 randomly selected transcripts that the anchor coder and experienced coder had coded independently and on which they had reached consensus. The coders were required to achieve 80% intercoder reliability (Campbell et al., 2013) for every overall code in the set of 10 interviews before being allowed to code independently. After this initial phase of coding, coders (including the experienced coder) were then provided with a list of randomly assigned interviews. Each week, approximately 20% of the transcripts being coded were randomly chosen to be double coded by either the anchor coder or the experienced coder, and the anchor coder coded 20% of the experienced coder’s coding to ensure she maintained reliability. In all cases, coders were required to maintain 80% reliability for each overall code, both cumulatively and for the 10 transcripts most recently coded. The overall percent agreement (calculated as average percent agreement with the consensus code per code) was 92%. In addition, we calculated Kappas for each of the relevant dimensions of VSMC; the Kappa for Explanations was 0.78 and the Kappa for Supports was 0.65, both of which are generally considered to indicate high levels of agreement (Gwet, 2010).

**Limitations.** Before sharing our findings, we discuss the limitations of our methods. First, as illustrated by the Horn (2007) and Windschitl et al. (2011) studies, framing problems of practice is an inherently social process (Benford & Snow, 2000; Coburn, 2006; Goffman, 1974;
Hand et al., 2012). Given the scope and design of the larger project, we were unable to investigate on a large scale how teachers framed students’ difficulty in conversations with colleagues. Instead, we were able to elicit how an interviewee chose to share how she framed a specific problem of practice in the context of an interview. More generally, the overall codes we generated (and report on below) are specific to an interview event.

Second, in the case of any interview-based assessment that is being administered by a team of researchers, undoubtedly, interviewers are inconsistent in how they ask questions and do not always probe in the same ways. This problem improved over the course of the larger study, as we improved the quality of training we provided to the team of interviewers. As shown in Table 4, in Year 5, we were able to assign an overall code for explanations for 82% of the 122 interviews, however we were only able to assign an overall code for supports for 61% of the interviews. The most common reason why we were not able to code for explanations was that when teachers described a challenge students faced (e.g., “students lacked basic skills”), the interviewer did not probe on why that was a challenge (e.g., the teacher was not asked why students might lack basic skills). Such probing was necessary to identify the source of the challenge. A reason for the smaller percentage of teachers who were coded for supports is that, even with targeted probing, participants often did not articulate the goal (or the function) of their supports. We return to this point in the Discussion and Conclusion section, as we view teachers’ challenge in articulating how they support students facing difficulty as indicative of, more generally, a lack of professional learning experiences focused on this very issue.

Findings

In what follows, we first provide descriptive information regarding how teachers across the two districts diagnostically and prognostically framed the problem of students’ difficulties in
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mathematics. Such information provides us with a large-scale snapshot of teachers’ VSMC, as articulated within interviews. We then explore how teachers’ diagnostic and prognostic frames relate to one another – and argue that both matter in understanding teachers’ uptake of ambitious reform. Throughout the findings, we have combined results from District B and District D, given that the distributions in how we coded the data were extremely similar and that our purpose here is not to investigate district-level differences.

**Descriptive Information Regarding Teachers’ Diagnostic and Prognostic Framing**

As shown in Table 4, coders were able to assign an overall code specific to teachers’ diagnostic framing for 100 of the 122 interviews. The findings regarding teachers’ explanations indicate that the overwhelming majority of teachers expressed views that for at least some of their students, the sources of their difficulties are distinct from classroom instruction. Only 28 percent of the 100 teachers offered purely productive explanations regarding why their students did not learn mathematics as expected – that is, they suggested that the source of the difficulty was likely due to instructional or schooling opportunities. Eighteen percent of the teachers offered purely unproductive explanations, attributing the source of the difficulty to individual traits of the students or the families or communities in which they resided. The majority of teachers (54%) wavered in some respect, articulating both productive and unproductive explanations.

Given that the majority articulated mixed explanations (and that we have previously provided examples of productive and unproductive explanations, via Mr. Williams and Mr. Batsem, respectively), we provide a couple of examples here to illustrate what mixed explanations tend to sound like. For example, consider Mr. Hopkins’ response when asked why his students might not learn as expected:
Well, I think … a big part of our battle every day is … if we have students that come to school who don’t get support at home, those students tend to think that this is just … a place where you can come and try to have as much social fun as possible during the day and that’s a big challenge for us, the lack of parental involvement on this campus, the lack of educated parents on this campus. … The other reason students are not learning is because you have teachers that are not teaching with fidelity to the [district’s curriculum framework], and that the students are, they don’t know what the day’s objective is. It’s not made clear to them. They’re not made to write it down necessarily. They’re not … made to share aloud with a partner. … There’s too much of this … reinventing of the wheel instead of sticking to the script with the [district’s curriculum framework]. And so when you have only ninety minutes a day, time is precious. And … the other reason that I think some students are not learning is because teachers’ classroom management is not what it should be… [S]o that would be my biggest reasons, lack of parental involvement, classroom management, concerns with new teachers, and a kind of a disregard for teaching with fidelity to [the district’s curriculum framework].

Within his response, Mr. Hopkins wavered between what we considered an unproductive explanation (lack of parental involvement and what he refers to as “educated parents”) and what we considered productive explanations (issues of teaching).

Whereas we coded Mr. Hopkins’ explanation as mixed at the level of a turn of talk, we coded other teachers’ explanations as mixed because they articulated a productive explanation in one part of the interview and an unproductive explanation in another part of the interview. For example, when asked why his students do not learn as expected, Mr. Dawkins responded with a productive explanation:
I usually assume there’s something I could do to do better. That’s what I really try to focus on doing first, that I needed to have done something else, and I think one weakness of mine is classroom management and that ties in to participation, because … I think if the students were doing the things that I’ve prepared for them to do then they would be doing better, but I’ve got to get a hold on … being positive with them and being emotionally empowering with them but also being strict about this is what work in the classroom looks like and these are your expectations. That’s something that I’ve really needed to work on a lot.

However, during a different part of the interview, Mr. Dawkins suggested that the adopted textbook, CMP2, was not appropriate for what he referred to as his school’s “demographic” of students because “it’s assuming that the students can do a level of thinking that they cannot do.” We coded this excerpt as unproductive because it suggested that his views of his students’ capabilities were relatively static. Thus, when we characterized Mr. Dawkins’ explanations across the interview, we assigned an overall code of mixed.

Sixty-one percent of the 122 teachers (n = 74) articulated forms of supports used to address students’ difficulties in mathematics and the functions they expected those forms to serve. Of those, only about one-fifth (n=14) described productive supports, or those aimed at enabling students to participate in rigorous mathematical activity. Eleven percent of the 74 teachers (n= 8) articulated mixed supports; most teachers whose supports were characterized as mixed prioritized drilling basic skills as a necessary prerequisite to supporting students to engage in rigorous mathematical activity. Notably, an overwhelming majority of teachers (70%) described unproductive supports, meaning they solely described lowering the cognitive demand
for students facing difficulties, which usually entailed demonstrating procedures for how to solve problems absent a focus on why procedures work.

Relations Between Teachers’ Diagnostic and Prognostic Framing

The descriptive information regarding how we coded teachers’ VSMC suggests that on the whole, teachers did not view all of their students as capable of participating in rigorous mathematical activity. The majority of teachers did not frame student difficulty in mathematics as a relation between the student and instructional opportunities, and the majority of teachers (for whom we were able to code supports) described lowering the cognitive demand of activities for students that faced difficulties. However, we were curious as to the relation between teachers’ explanations of student difficulties and the supports that they described for those students. For example, did articulating unproductive explanations tend to go hand-in-hand with articulating unproductive supports? Did articulating productive explanations tend to go hand-in-hand with articulating productive supports? We were able to code both diagnostic and prognostic framings for 56, or about 46%, of the 122 teachers we interviewed. Table 5 presents descriptive statistics regarding the relations between their diagnostic and prognostic framings.

Unproductive diagnostic framings. We first report on relations for those teachers who articulated unproductive diagnostic framings of the problem of student performance. As shown in Table 5, we found that if a teacher articulated an unproductive diagnostic framing, it was more likely that s/he would articulate an unproductive prognostic framing (as compared to a mixed or productive prognostic framing). We had conjectured this was the case, given that unproductive explanations indicate the view that at least some students are incapable of
succeeding in mathematics because of factors outside the teacher’s locus of control. Thus, it seemed likely that any attempt at supporting such students would, at best, aim at supporting them to participate in less challenging mathematical work – work for which they might be perceived as able to engage in.

For example, when asked about the biggest challenges of teaching mathematics, Mr. Gomez said, “The … apathy from the kids, [they are] completely apathetic, they could care less what they’re learning ….” The interviewer then probed, “What do you attribute the kids’ apathy to?” to which Mr. Gomez responded:

… I think the kids see the world as, that we live in right now, as a place where they’re not gonna be successful … I mean they see that a lot of people don’t have jobs … I was in the military in 2001, you know… during September 11th. [T]hese kids … haven’t had a conscious life that doesn’t involve war, doesn’t involve conflict, doesn’t involve a depression of some sort, so I think that they’re just down, they’re just depressed and I think they … see a pretty dim future.

Mr. Gomez diagnosed the problem of students not performing (or learning) as an issue of apathy, and suggested that the source of this apathy is due to students’ outlook on their future. We coded this as an unproductive framing regarding why students are not performing because it suggests that the source of the problem (students’ apathy) is located outside the classroom or school, and thus does not likely position him to consider instructional supports as potential interventions.

How, then, does Mr. Gomez describe addressing what he perceives as apathy? He described his solution as follows: “Unfortunately our kids, because of their background, they like somebody to tell them what to do, they like to take notes. They like … teacher led work and then independent work.” He contrasts his approach to what district leaders explicitly suggested
teachers should do: “They [district leaders] don’t like proceduralization, but it works for… these kids.” Mr. Gomez justifies proceduralizing mathematics (i.e., demonstrating procedures for solving problems) for his students in terms of “their background” – and presumably, what he has interpreted as “apathy.” Proceduralizing mathematics, as Mr. Gomez acknowledges, was antithetical to the goals of the ambitious reform the district was pursuing.

As another example, consider how Mr. Beaumont framed both the source of student performance in mathematics and a solution in unproductive ways. His school had created two tracks of eighth grade teachers and students – what he referred to as the “good team” and the “bad team.” The “good team” consisted of high-performing students on the state mathematics assessment, whereas the “bad team” consisted of low-performing students. Mr. Beaumont was assigned to the “bad team,” which he resented: “The teachers … that are on the bad team realize that this is the hand that we got dealt, we’re stuck with it, and it’s terrible, it’s not fair.” He suggested that students were inherently at particular levels in mathematics – low and high. And, similar to Mr. Gomez, and many other teachers we interviewed who articulated unproductive prognostic framings, he suggested that “low-level kids” were not able to engage in the forms of activity required by the district’s ambitious curriculum. For example, one form of activity emphasized by district leaders involved grouping students together for the “explore” phase of a CMP2 lesson, in which it was expected that students would share their thinking with one another in an effort to collaboratively solve a complex task. Regarding this expectation, Mr. Beaumont said, “These kids you put them in a group of two it’s play time….A group of four, ‘oh boy, we’re not going to do anything today.’ [District leaders] insist they want us to use … grouping strategies. It doesn’t work with this level kid in mathematics.”
Both Mr. Gomez’ and Mr. Beaumont’s talk are illustrative of more general patterns in our data regarding how unproductive diagnostic and unproductive prognostic framings tended to fit together. Teachers who tended to attribute students’ difficulties in mathematics to sources outside the classroom also tended to suggest that such students were incapable of participating in activity aimed at rigorous goals for their learning. Similarly, as evidenced in Table 5, teachers who tended to waver between articulating unproductive and productive explanations for why students faced difficulties (mixed explanations) tended to articulate an unproductive prognostic framing.

However, there were exceptions to these patterns. Although extremely rare, we did identify two cases out of 56 in which teachers articulated an unproductive diagnostic framing coupled with a productive prognostic framing regarding students’ performance. For example, when asked about the biggest challenges of teaching mathematics, Mr. Carter responded that a number of the students were not motivated. He elaborated: “They just don’t care and … their parents you know they’re not there staying on ‘em and so … that’s probably the worst, one of the hardest things.” When asked why some students are motivated, while others are not, he suggested it was due to the students’ “culture,” which appeared to map onto assumptions about his students’ family lives:

I know it sounds crazy, but I mean I’ve talked to parents who don’t even know that report cards [have] come out, they have no clue, like, oh, really? I talked to a parent who told me one time, ‘Well when he’s 15 he’s gonna start working for the family company, so I don’t care.’ I’ve had a kid tell me he was gonna drop out before this year was over. I mean it’s just [a] different culture. And a lot of it too is you know they’ve got one parent and they’re working three jobs cause they’ve got three kids and they gotta do what they
gotta do, you know I understand that. So I mean I understand if you know you’re babysitting your brother and sister all night long, your math homework isn’t probably on the top of your list of things to do and that’s not anything that these kids are at fault for, it’s just the life they’ve gotten put into.

As illustrated above, Mr. Carter attributes students’ low-performance in mathematics to not being motivated, which, he suggests, stems from students’ family backgrounds. However, different from Mr. Gomez and Mr. Beaumont, Mr. Carter expresses some empathy with his students for what he assumes is a difficult, demanding home life. This framing may explain why his prognostic framing was different from that of Mr. Gomez and Mr. Beaumont; namely, when asked what he does to address challenges in teaching mathematics, he described taking measures to support students to participate in high-cognitive demand activity. In particular, he had been provided with professional development specific to “launching,” or introducing, cognitively demanding tasks (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). He discussed how he worked to ensure that his students understood the context associated with particular CMP2 tasks, especially those in which the context was likely to be unfamiliar to his students. For example, he described a CMP2 investigation organized around renting bicycles and taking a bicycle trip in New England:

Man, I spent almost a day just explaining the concept and how it worked and I mean it, it was like, …I launched the crap out of it, cause I was like, I’m bound and determined for these kids to understand it and they did, most of them, I would say 80-90% of ‘em could’ve said, “Yeah they were trying to open this bike shop and rent these and make some money” … but I won’t be able to do that next year. I spent a whole day on that just explaining and we went off on you know business tangents and different things, but it
was like then they seemed more engaged in that unit. But I had to waste an entire [day] to
get them there. You know not waste a day, I didn’t, it wasn’t wasteful, … to me it was
great because it was like then I had 70 or 80% of them that was really involved in [the
investigation].

Mr. Carter’s response to his students’ difficulties was markedly different from strategies
aimed at lowering the cognitive demand of expected activity (like that which Mr. Gomez
described). However, even though Mr. Carter recognized how ensuring his students understood
the context of the bike tour supported the majority of his students to be “engaged in the unit,” he
also suggested that he could not afford such time the following year. Thus, although he
articulated productive supports during this interview, he appeared somewhat skeptical of doing
so in the future.

**Productive diagnostic framings.** We now turn our attention to those teachers who
articulated productive explanations, meaning those who attributed the source of students’
difficulties to instructional opportunities. As shown in Table 5, the relationship between
articulating a productive diagnostic framing and a particular kind of prognostic framing is not
straightforward. The lack of a straightforward relationship makes intuitive sense. A productive
diagnostic framing suggests that teachers view their students as capable of participating in
mathematical activity, given appropriate instructional support. However, a productive
diagnostic framing, on its own, does not suggest *the kind of activity* students should be engaged in. This is
evidenced in the fact that, as shown in Table 5, of the nine teachers who articulated productive
explanations, nearly an equal number of teachers articulated unproductive supports (n = 3) as
those who articulated productive supports (n = 5). We consider those two cases here.
Ms. Jacobi, an eighth grade Algebra teacher, provides an illustrative example of a teacher who articulated a productive explanation while articulating unproductive supports. She described her goal as having all of her students develop a “foundation” in Algebra, because, in her view, “if they have the foundation, they’re not going to have a problem for … high school math classes.” In doing so, she at least implicitly conveyed an orientation to viewing students’ performance as dependent on the quality of instructional opportunities – that if students were provided with a strong foundation in mathematics, they were capable of succeeding in high school mathematics. When asked what she attributed students’ difficulties to, she suggested instructional reasons. Yet, when asked how she tended to adjust her instruction for students facing difficulties within mainstream instruction, similar to Mr. Gomez, Ms. Jacobi described proceduralizing instruction.

I can tell a lot of them … are not where they should be. And that means they [need] practice. Now after [giving them the] problem on the board, I’ll go around, I can figure out if they haven’t started – that means they are still behind. And then I just give them a hand for the first step. I do the first step with them and I’m asking them for the next step.

Then I can go back over there, if they didn’t hear me I have to repeat it.

Ms. Jacobi’s prognostic framing reveals that when she perceives that students “are not where they should be,” she tells them how to solve the problem. Although this could be perceived as supporting such students to succeed at solving the immediate task, it does not support such students to develop conceptual understanding or practices of mathematical reasoning. More generally, examining Ms. Jacobi’s diagnostic and prognostic framings serves to illustrate that attributing the source of students’ difficulty to instruction does not guarantee that teachers then
take measures that aim to support such students to substantially participate in rigorous mathematical activity.

On the other hand, some teachers who articulated productive diagnostic framings also articulated productive prognostic framings. For example, consider Ms. Baker. When asked why students do not learn as expected, Ms. Baker replied, “I think a lot of it is when they don’t know why they got where they got with their answer. So, if they’re just solving and they’re not having to explain or think about the process they’re not going to retain it. They’re just going to memorize it for that day and then, you know, later down the road they’re not going to retain that knowledge.” Moreover, she described discussing the issue of students “not retaining” information with her math department:

So we’ve had that struggle, you know, we’ve all talked as a department, “Why are they not retaining it? Why are we, you know, still scoring low on our [interim assessment] scores when in class they seem to be getting it?” And we kind of came to the consensus that we are not letting them think the problem through. When they struggle, we’re sometimes just telling them how to get there.

It is clear that Ms. Baker views students’ success and difficulty as produced in relation to the quality of instruction. When describing what would support students who are not learning as expected, she maintained a focus on providing students with the means to participate in rigorous mathematical activity. She explained:

I think letting the kids spend time on … a problem and not rushing them though something and then being accountable for what they learned, you know, “Explain to your friend why you got that,” and … “How did you get to your answer?” Cause if we’re just
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zipping through problems all day they’re not learning it. They’re memorizing it and then they’re leaving class.

Knowing how to support students facing difficulties to substantially participate in rigorous mathematical activity is not an easy task. Most of the teachers who articulated both productive explanations and productive supports acknowledged the challenge inherent in altering instruction to support students who typically face difficulties in ways that aim at rigorous goals for students’ mathematical learning. For example, when asked how she goes about adjusting her instruction for students who face difficulties, Ms. Newman described how much effort it takes on her part:

I feel like if I put the work into it to really understand the concepts myself, and then just give it a shot, then it works well, and has worked well. I mean, it's the time … when I sat down and I've really worked on it, like I really wanted them to understand the meaning behind dividing fractions because that was definitely a standard, and … I was really doubtful that [my students who typically have difficulties] were going to get it, but … I tweaked the lesson here and there, [and] they got it. [My students who typically have difficulties] definitely impressed me [with] what they could do.

Here, Ms. Newman describes the significant time it took for her to work through the mathematics and to figure out how to enable her students who face difficulties to make conceptual sense of a difficult mathematical idea. In addition, Ms. Newman highlights the potential for shifting views of students’ capabilities when one has the capacity to design and enact lessons that take into account supporting students who typically face difficulty, a point which we return to in what follows.

Discussion and Conclusion
Our assessment of teachers’ VSMC across two large, urban districts pursuing ambitious reform in middle-grades mathematics supports the field in further specifying learning goals for teachers, if they are to enact ambitious teaching. Not only is it likely that teachers need to develop stronger mathematical knowledge for teaching (e.g., Hill, 2010), more sophisticated visions of high-quality mathematics instruction (Munter, 2014; Wilhelm, 2014), and skills in enacting specific forms of practice (e.g., Boston & Wilhelm, in press), it is likely they also need to shift their views of who is capable of engaging in rigorous forms of activity. More specifically, this entails shifting how one makes sense of the source of student difficulty in mathematics, but also learning how to more productively support students who face difficulty in mathematics.

In what follows, we consider what our findings, as well as related literature, suggest in terms of what it would take to support such positive shifts in teachers’ VSMC. We first make the argument that efforts to focus on teachers’ expectations for all students are likely necessary, but insufficient if teachers are to enact ambitious teaching. Second, we specify what a more sufficient design for supporting shifts in teachers’ VSMC might take into consideration. Third, we discuss the value of an assessment like the one we developed in both designing for and monitoring improvement efforts. Finally, we consider areas for future research in light of our discussion.

**High Expectations – Necessary But Not Sufficient**

In our work with large, urban districts pursuing ambitious reform in middle-grades mathematics, it is quite common to hear district leaders, school leaders, and coaches suggest that one reason for disparities in performance on measures of student achievement between groups of students is because teachers do not have “high expectations” for particular groups of students. In
response, teachers are often provided with professional development focused on raising their expectations for all students; this professional development is usually not specific to a content area.

Our findings suggest that a large number of teachers in our sample did not hold high expectations for all students. However, our findings call into question the utility of professional learning experiences that are focused generically on raising teachers’ expectations, absent a focus on clarifying what “high expectations” entails with respect to a particular vision of instruction, and how to enable students to meet those expectations. In particular, our findings regarding teachers’ prognostic framings suggested that most teachers did not articulate supports that would enable students to develop conceptual understandings of mathematics. Instead, most teachers (52 of the 74 teachers for whom we were able to code prognostic framing) described lowering the cognitive demand of an activity, if they perceived students were facing difficulty. Furthermore, our analysis of the relations between teachers’ diagnostic and prognostic framings supports the claim that even if teachers were to view students’ performance as dependent on instructional opportunities, it did not mean that teachers described responding to students’ difficulties in ways that would enable them to develop conceptual understandings of mathematics. Recall Ms. Jacobi, the eighth grade Algebra teacher who we identified as articulating a productive diagnostic framing yet an unproductive prognostic framing. She could be identified as having high expectations for her students, in that she wanted them to succeed in high school mathematics classes. And, she viewed students’ difficulty in relation to instructional opportunities. However, when discussing how she addressed students’ difficulties, she described proceduralizing instruction, or showing students how to solve the problems, thereby taking the thinking away from the students. So although she articulated high expectations for her students,
the supports she described enacting in instruction were unlikely to enable students’ development of robust, enduring understandings of mathematics.

Findings such as these make us wary that attending to high expectations absent how to enable students to meet those expectations is a viable strategy for addressing long-standing disparities in students’ opportunities to learn. Instead, it appears that high expectations may be necessary but insufficient for enacting instruction that positions all students as capable of engaging in rigorous mathematical activity. As Sosa and Gomez (2012) argued, “along with high expectations, teachers must provide the support necessary to meet those high goals” (p. 894). Our findings suggest that at present, it is unlikely that most teachers are equipped to enable students to meet high expectations in light of the learning goals and associated forms of practice called for in the CCSS-M. We return to this point when we discuss areas for future research below.

Conjectured Key Aspects of Designs to Support Positive Shifts in Views of Students’ Mathematical Capabilities

What would it take to support teachers to come to view students’ difficulty as an issue of instruction, and to learn to respond to such difficulty by enacting supports that enable students to more fully participate in rigorous mathematical activity? On our read, the literature is scarce regarding how to shift teachers’ VSMC. In what follows, we extrapolate from relevant literature to identify potentially important aspects of designs to support shifts in teachers’ VSMC. As we elaborate below, this is another area in need of further research.

The research of Horn (2007) and Windschitl et al. (2011) suggests that views of the target discipline (in our case, mathematics) as well as theories of teaching and learning impact the nature of how student performance is framed. This suggests that professional learning
opportunities focused on shifting VSMC must be tightly integrated with a focus on other key aspects of teaching and learning mathematics, like, for example what it means to engage in the discipline of mathematics or how it is that students learn. In other words, focusing on VSMC absent other aspects of an instructional system is unlikely to be fruitful.

Gresalfi and Cobb’s (2011) analysis of a professional development collaboration between middle-school math teachers and researchers focused on statistical reasoning is relevant in this regard. At the start of the collaboration, teachers tended to articulate what we would have coded as unproductive diagnostic framings regarding why some students succeeded while others faced difficulty in mathematics. However, over the course of the first two years of the five-year collaboration, they documented that “deficit language about students was gradually displaced by talk about why students thought or performed in particular ways” (p. 289). This shift was accompanied by understandings of “how to support the development of all students’ mathematical reasoning” (p. 296). It appeared that a shift in views of students’ capabilities went hand-in-hand with a shift in coming to value eliciting and making sense of students’ reasoning. As teachers gained an appreciation for the value of investigating students’ thinking, presumably they enacted practices that supported and encouraged students to make their thinking apparent. This, then, made visible that students who the teachers may have previously characterized in deficit terms were indeed capable of participating in more rigorous forms of mathematical activity.

More generally, this case highlights two issues that are important to consider when addressing teachers’ VSMC. First, we would conjecture that the relationship between shifting views of students’ capabilities and improving instructional practice is bidirectional. By this we mean that VSMC potentially shifts as teachers take on new forms of practice, and vice versa. It is
unlikely that teachers’ views of students’ capabilities will shift absent a context in which they can see their students engaging in different activity, and thus exhibiting different capabilities. We would therefore be doubtful of efforts to target VSMC separate from (or before) instructional practice. In cases where a teacher struggles to enact more rigorous forms of practice, it may be useful to have someone more accomplished in the targeted form of practice (e.g., a coach) model for the teacher, in the hopes the teacher will see his students exhibit “new” capabilities (Gibbons & Cobb, 2013). That said, it is important to couple the modeling with explicit discussion of what the modeler did and why, and what the teacher noticed in terms of his students’ capabilities as well as targeted support aimed at the teacher being able to enact similar forms of practice and/or investigating particular students’ subsequent thinking (Gibbons & Cobb, 2013).

Second, studies such as those discussed in this paper, and more generally, research on teacher noticing (e.g., Sherin, 2007), suggest the importance of someone (e.g., a professional development leader, a formal or informal teacher leader) making explicit teachers’ often tacit frames and pressing teachers to consider alternative diagnoses and prognoses. For example, in Gresalfi and Cobb’s (2011) analysis, teachers expressed frustration with students who wanted the teacher to tell them how to solve particular problems. A member of the research team leading the work suggested students “are not born that way” and offered an alternative (in our terms, productive) diagnostic frame, namely that students have learned such behavior through prior instruction, which then was taken up by the teachers (p. 290). Similarly, in Horn’s (2007; see also Horn & Little, 2010) analysis of the more productive math department, the co-chair of the department often offered alternative (what we would have coded as productive) diagnoses of the problems of practice that teachers brought to their workgroups, which helped shift the framing of the problem.
Another aspect of designing to support positive shifts in teachers’ VSMC concerns the fact that the influence of professional learning opportunities on teachers’ practice is mediated by the institutional setting in which teachers work (Cobb, McClain, Lamberg, & Dean, 2003). How problems of practice are identified and framed likely depend on teachers’ histories of working in a particular context, including relations of accountability and support, for example, instructional leaders’ expectations (Coburn, 2006; Gresalfi & Cobb, 2011), available tools (e.g., curriculum), as well as historical discourses regarding specific groups of students and why they succeed or not in mathematics (Jackson, 2009). In this particular analysis, we did not attend to the relations between teachers’ VSMC and aspects of the school context. However, we view such relations as central to account for when designing professional learning opportunities that intend to influence teachers’ VSMC. For example, in the context of professional learning aimed at shifting teachers’ VSMC, it would be important to ensure that teachers had curricular materials that aligned with the goal of supporting all students to engage in rigorous mathematical activity. Similarly, it would be important to coordinate with any existing additional supports for students facing difficulty in mathematics (e.g., tutoring, on-line computer resources) – and instructional leaders’ expectations for the use of such supports.

The Value of Assessing Teachers’ Views of Students’ Mathematical Capabilities

We suggest that the interview-based assessment we developed could serve as a useful tool for both researchers and practitioners to use and/or adapt in designing for ambitious reform in mathematics as well as accounting for teachers’ learning in the context of improvement efforts. Specifically, we suggest that an assessment like the one we have described can play at least three different functions in designing and assessing reform efforts in mathematics education.
First, we suggest that prior to designing specific efforts, it is important to take account, prospectively, of how teachers view their students’ capabilities in mathematics. If teachers diagnostically frame the problem of student difficulty in mathematics in unproductive ways, this would indicate the need for deliberate attention to shifting the framing. Similarly, if teachers tend to describe unproductive supports for students facing difficulty in mathematics, this would indicate the need for targeted professional learning on how to enable students to participate in rigorous mathematical activity. Second, an assessment like what we have described can be useful in monitoring improvement efforts. It is very unlikely that if unproductive framings continue to dominate the work context that all students are going to be provided with the necessary support to participate in rigorous mathematical activity. It is important to be able to assess on an ongoing basis how framings are changing, or not – which can then be used to inform subsequent improvement efforts.

To be clear, we are not suggesting that accounting for VSMC either prospectively or as part of monitoring ongoing improvement efforts requires as extensive and formal an interviewing and coding process like the one we described. Certainly, using the instrument on a smaller scale than we did would make it less demanding in terms of time and effort. However, we could also imagine researchers and/or practitioners developing and using alternative methods of assessing teachers’ VSMC, perhaps in the context of common planning time or professional development sessions. We view the two dimensions of VSMC that we identified as providing useful guidance regarding what it would be important to attend to in such settings. For example, we could imagine working with coaches or school leaders to develop routines to intentionally listen for how teachers frame students’ capabilities in the context of common planning time, with the goal of then devising ways to support teachers to shift framings, if necessary.
Third, we suggest that having an assessment like the one described here is useful to include in research analyses of instructional improvement efforts. It would be fruitful to include assessments of VSMC in analyses investigating the relations between aspects of teacher knowledge, perspectives, and practice, and in relation to specific classroom, school, and district settings. Doing so can contribute to the field’s existing theories of what impacts teachers’ enactment of ambitious teaching, including in classrooms serving historically underserved populations, and thus inform our understanding of how to support teachers to develop ambitious teaching.

As an example, Wilhelm, Munter, and Jackson (under review) created a quantitative variable based on the qualitative coding of teachers’ VSMC in Years 1-4 of the larger study, and conducted quantitative analyses focused on the relations between teachers’ explanations and the distribution and quality of students’ discourse in whole-class discussions. (The measures of instructional quality were based on coding video-recorded observations of teachers’ instruction.) Wilhelm et al. found a positive, statistically significant relationship between teachers’ articulation of productive explanations and the quality and distribution of students’ explanations in whole-class discussions, when controlling for teachers’ mathematical knowledge for teaching, vision of high-quality instruction, choice of mathematical task, years of experience, and setting. Students were, on average, more likely to have opportunities to participate in discussions in which students provided reasoning for their solutions if their teacher articulated productive diagnoses of sources of their difficulty. And, the results of an interaction analysis suggested that this relation was strongest in classrooms composed (almost) entirely of students of color. As another example, also using data from Years 1-4 of the larger study, Wilhelm (2014) found a positive, statistically significant relation between supports and maintenance of the
cognitive demand of a high-level task over the course of a lesson. Specifically, the “odds of maintaining the cognitive demand of a high-level task for teachers who espoused productive [supports] were 2.92 times the odds for teachers who described unproductive or mixed [supports]” (p. 660).

Findings such as these help the field to further pinpoint important aspects of accomplishing ambitious reform at scale and identify areas deserving of future investigation. For example, based on the relation that Wilhelm et al. (under review) identified between teachers’ explanations and students’ classroom discourse, we could imagine designing a qualitative study to investigate further the contexts in which teachers develop productive diagnoses regarding their students’ difficulties and how the teachers support their students to more fully engage in classroom discussion.

**Future Research**

We conclude by elaborating on what our findings suggest as important areas for future research regarding teachers’ VSMC. First, we consider our finding that the majority of the teachers in our sample described supports that involved lowering the cognitive demand of the overall activity itself. We view this as indicative of a lack of professional learning experiences focused on this very issue, which we conjecture is related to the lack of relevant research. To date, we know of little research that has investigated what supports teachers might enact to enable students facing difficulties to participate in rigorous mathematical activity (for exceptions, see Boaler & Staples, 2008; Horn, 2012; Jackson et al., 2013). In our view, this is a critical area for future research, especially given the widespread adoption of the CCSS-M and associated assessments.
Methodologically, we see value in two kinds of studies aimed at contributing knowledge regarding the forms of practice and/or supports that enable students, particularly those facing difficulties, to substantially participate in rigorous mathematical activity. Qualitative studies of teachers who are particularly accomplished in supporting students facing difficulty could be of great value. We imagine the study would focus on identifying the specific forms of practice the teachers enact, and specifying why they result in (or not) increased forms of participation for students facing difficulties. Of course, this requires identifying teachers who are particularly adept at doing this kind of work ahead of the study.

Design studies are a second kind of study that we see great promise in for addressing this knowledge gap. Design studies are particularly useful when the phenomenon of interest is unlikely to occur in situ, the existing research base is thin, and when the goal is to develop, test, and revise conjectures about learning processes and the means of supporting that learning (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). In this case, the goals would be to generate and refine particular forms of practice and/or supports that enable students who are facing difficulty to participate in rigorous mathematical activity; and to generate theory regarding why, when, and under what conditions those particular forms of practice and/or supports result in improved access for students facing difficulty.

Second, as we indicated above, it is a rather open research question as to how to support teachers to positively shift their VSMC. Here, again we see value in design studies. A professional development design study would ideally result in the development of a practice-specific professional development theory that would consist of a substantiated learning process that culminates with teachers’ positive shift in their VSMC, and the demonstrated means of supporting that learning (Cobb, Jackson, & Dunlap, in press). Above, we indicated some
conjectures regarding what might be important to account for in the design of professional learning aimed at shifting teachers’ VSMC. Moreover, we imagine that a classroom design study focused on identifying productive supports could inform a professional development design study focused on shifting teachers’ VSMC, in that the forms of practice and/or supports identified in the classroom design study could be a focus of the professional development design study.

We are hopeful that researchers will take up the issues we identified above, given that it appears that a significant challenge in accomplishing ambitious reform at some scale entails the reorganization of how teachers view their students’ capabilities. Absent deliberate attention to teachers’ views of students’ capabilities in improvement efforts, we are doubtful that such efforts to improve mathematics teaching will take hold on a large scale.

**Endnotes**

1 All teacher names are pseudonyms.

2 In this paper, we only report on teachers’ VSMC. However, within the larger project, we also assessed the VSMC of coaches, school leaders, and district leaders. Interested readers can email the first author for information about the questions we asked of the various role groups.


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Postdoctoral Fellowship Program supported Kara Jackson’s contributions to the manuscript. The opinions expressed do not necessarily reflect the views of either Foundation. The empirical data that we present in this article are based on research conducted in collaboration with Paul Cobb, Thomas Smith, Erin Henrick, Ilana Horn, Mollie Appelgate, Dan Berebitsky, Jason Brasel, Glenn Colby, Sarah Green, Adrian Larbi-Cherif, Christine Larson, Britnie Kane, Karin Katterfeld, Charles Munter, Mahtab Nazemi, Jessica Rigby, Brooks Rosenquist, Rebecca Schmidt, Megan Webster, Anne Garrison Wilhelm, and Jonee Wilson. We are especially grateful to Rebecca Schmidt and Brooks Rosenquist for their contributions to coding the data on which this analysis is based; to Jennifer Bauman, Mahtab Nazemi, and Anne Garrison Wilhelm for coding additional years of data (that informed the development of the coding scheme); to Charles Munter for providing a model of how to approach the development of an interview-based assessment; and to Heather Hebard and Katherine Lewis for providing insightful feedback on a previous draft.
References


Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class
discussions in middle-grades mathematics instruction *Journal for Research in Mathematics Education, 44*(4), 646-682.


Table 1

Demographic information regarding the districts’ student populations, 2011-2012

<table>
<thead>
<tr>
<th>District</th>
<th># of students</th>
<th>Limited English Proficient</th>
<th>White</th>
<th>African American</th>
<th>Hispanic</th>
<th>Asian</th>
<th>Native American</th>
<th>Receiving Free or Reduced Lunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>83,000</td>
<td>28%</td>
<td>14%</td>
<td>23%</td>
<td>60%</td>
<td>1.9%</td>
<td>&lt;1%</td>
<td>77.5%</td>
</tr>
<tr>
<td>D</td>
<td>95,000</td>
<td>6%*</td>
<td>52%</td>
<td>36%</td>
<td>7%</td>
<td>3%</td>
<td>&lt;1%</td>
<td>60%</td>
</tr>
</tbody>
</table>

*Note: Information was not available regarding percent of students classified as Limited English Proficient (LEP) in District D in 2011-2012. The most recent data we have available is from 2009-2010 (and indicated that 6% of students were classified as LEP).
Table 2

Abbreviated version of coding scheme to assess the nature of participants’ explanations regarding the source(s) of students’ difficulties in mathematics

<table>
<thead>
<tr>
<th>Code and Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PRODUCTIVE</strong></td>
<td>Interviewer: So, in your own classroom when students don’t learn as expected, what do you usually find are the reasons? Teacher: Why a kid didn’t learn? Because I didn’t make him. Interviewer: How do you make a kid learn? Teacher: I don’t know. That’s always the problem, isn’t it? I, I do, and also again I, I might be different on that, but I, I really feel like if a kid’s not learning in a classroom, it’s my fault. That it’s something that I’m not doing. There has to be a reason. I mean, I, you know, especially in the 8th grade, I mean, they can learn something. There is, there’s something they can be doing. There’s some way they can be doing it. And so, I mean, if a kid’s just flat out not learning then there’s something that I need to do better to make him learn and I don't always know what that is, but I mean, I do put most of the emphasis back on me.</td>
</tr>
<tr>
<td>Student performance (e.g., failure, success, engagement, interest) is described as a relation between student(s) and instructional and/or schooling opportunities.</td>
<td></td>
</tr>
<tr>
<td><strong>MIXED</strong></td>
<td>Interviewer: In your classrooms, when the students do not learn as expected, what do you find are the typical reasons? Teacher: Probably me... I don’t put blame on the students. I mean, I think it’s a combination. They have to do their part, and I have to do mine, so if they’re not getting it, it may, and this, this may not be the best way, but I’ll be honest, I look to the students that are consistently successful, and if they don’t understand something, I know I’m doing something wrong, so I need to go back, and I need to think it through again or come up with a different strategy or a way of showing them to do the problem. You know, if it’s a kid that is consistently off task and playing around or something, then I might just kind of think that, “Well, they’re not paying attention,” so, it’s just kind of like what the majority of the class is doing, and I kind of judge off that.</td>
</tr>
<tr>
<td>Participant wavers between explaining student performance (e.g., failure, success, engagement, interest) 1) as a relation between student(s) and instructional and/or schooling opportunities and 2) as due to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional opportunities (e.g., parents don’t value education, therefore students don’t).</td>
<td></td>
</tr>
<tr>
<td><strong>UNPRODUCTIVE</strong></td>
<td>Interviewer: So what are some of the major challenges ... of teaching mathematics in this school? Teacher: The kids already don’t want to learn math. They have this notion of not caring for it and usually it’s instilled by their parent’s cause their parents didn’t get it, so they think its okay that they didn’t get it.</td>
</tr>
<tr>
<td>Student performance (e.g., failure, success, engagement, interest) is attributed to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional and/or schooling opportunities (e.g., parents don’t value education, therefore students don’t).</td>
<td></td>
</tr>
<tr>
<td>Explanation presents students’ mathematical capabilities as relatively stable (i.e., they are unlikely to change).</td>
<td></td>
</tr>
</tbody>
</table>
Table 3

Abbreviated version of coding scheme to assess the nature of how participants describe supporting students who face difficulties in mathematics (specific to students not identified as English learners or as receiving special education services)

<table>
<thead>
<tr>
<th>Code and Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PRODUCTIVE</strong></td>
<td>Interviewer: In terms of ... having kids in the same room with a wide range of knowledge, what are like some of the strategies you use to address that ...? Teacher: One thing is [to] re-write the tasks from the books and Interviewer: From ... the CMP [text] or? Teacher: Right. Trying to make sure that they maintain the rigor, but then ... are there multiple entry points into this particular task? Can I make sure that my student who struggles the most can find a way to engage in this task and my student who has the most skills in this classroom is still gonna be challenged? ***</td>
</tr>
</tbody>
</table>
| Description of instructional actions one takes to support students who are facing difficulties are aimed at rigorous learning goals. Below is a list of instructional actions that are generally aimed at supporting struggling students to participate in rigorous activity in the context of mainstream instruction. Note that this is not an exhaustive list and that the coder will need to make judgments regarding the nature of what participants describe.  
  - **Pre-teach particular skills to students prior to mainstream instruction that are necessary for engaging in the targeted mathematical idea at a conceptual level; this is sometimes done in the context of a 2nd math class or intervention.**  
  - **Focus on how the task is introduced, or set-up.** Ensure students are familiar with the context in a problem-solving scenario.  
  - **Use tasks with multiple entry points.**  
  - **Focus on norms of participation.**  
  - **Assign competence to students (e.g., strategically mark students’ contributions as important to attend to).**  
  - **Group students in ways that aim to maximize each student’s participation (e.g., assigning roles, assigning near-peers).** | |
| **MIXED** | Teacher: I’m always afraid to go ahead because I don’t feel my kids are mastering things and I try to challenge my kids and use a lot of word problems, use a lot of words and a lot of real world settings because that’s what they’re going to, you know, they’re not going to sit in some room doing a hundred adding fractions problems, but at the same time some of my kids actually need to do a hundred addition problems with fractions just so it sticks in their head that they’ve got to get a common denominator. |
| Clearly articulates learning goals and instructional supports for students who are facing difficulties that are aimed at rigorous activity however, some of what participant says indicates that some instructional actions are aimed at conventional learning goals. Supports typically aim at first ensuring that students develop “basic skills” before engaging in more rigorous activity. | |
UNPRODUCTIVE

Description of instructional actions one takes to support students who are facing difficulties are generally aimed at lessening the cognitive demand of activity (e.g., proceduralizing a task).

Below is a list of instructional actions that are generally aimed at lowering the cognitive demand of activity. Note that this is not an exhaustive list and that the coder will need to make judgments regarding the nature of what participants describe.

- Remove any prompts that ask students to explain their thinking.
- Shorten problems.
- Show students how to complete a similar problem.
- Provide examples.
- “Drill,” “Use direct instruction.”
- Assign fewer problems.

Teacher: In the longer classes you can get a little bit more done but as far as the ability wise, there’s always going to be a class that can do more, so you going to give them more to chew on than you would the class that’s not quite capable.

Interviewer: Okay so you would be adjusting, would you be adjusting the kind of tasks that you give them or would you be adjusting the pace or maybe how, how you would group the, the kinds of students? ...

Teacher: ... [A] little bit of both. The ... pacing would be a little bit slower in the longer classes and then faster in the shorter classes. And then the, the tasks for the kids who are in the more capable classes they would get more independent practice where as the one in the less capable class they would get more modeling and guided practice.
Table 4

*Teachers’ Views of Students’ Mathematical Capabilities in Two Districts Pursuing Ambitious Reform in Middle-Grades Mathematics, 2011-2012 (n = 122 teachers)*

<table>
<thead>
<tr>
<th></th>
<th>Coded</th>
<th>Unproductive</th>
<th>Mixed</th>
<th>Productive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diagnostic Framing</strong> (Explanations)</td>
<td>100 (82%)</td>
<td>28 (28%)</td>
<td>54 (54%)</td>
<td>18 (18%)</td>
</tr>
<tr>
<td><strong>Prognostic Framing</strong> (Supports)</td>
<td>74 (61%)</td>
<td>52 (70%)</td>
<td>8 (11%)</td>
<td>14 (19%)</td>
</tr>
</tbody>
</table>

Note: Percentages are rounded to the nearest per cent, and therefore totals may not equal 100%. Percentages for coded / not coded refer to total number of teachers interviewed (n = 122). Percentages for unproductive, mixed, and productive refer to the number of teachers who received a code in that category (n = 100 for Explanations; n = 74 for Supports).
Table 5

*Relations Between Participants’ Diagnostic and Prognostic Framing of Students’ Performance in Mathematics (n = 56 teachers)*

<table>
<thead>
<tr>
<th>Prognostic Framing (Supports)</th>
<th>Diagnostic Framing (Explanations)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unproductive</td>
<td>Mixed</td>
<td>Productive</td>
</tr>
<tr>
<td>Unproductive</td>
<td>10 (18%)</td>
<td>25 (45%)</td>
<td>3 (5%)</td>
</tr>
<tr>
<td>Mixed</td>
<td>3 (5%)</td>
<td>3 (5%)</td>
<td>1 (2%)</td>
</tr>
<tr>
<td>Productive</td>
<td>2 (4%)</td>
<td>4 (7%)</td>
<td>5 (9%)</td>
</tr>
</tbody>
</table>

Note: Percentages are rounded to the nearest per cent, and therefore totals may not equal 100%.