EXAMINING RELATIONS BETWEEN TEACHERS' EXPLANATIONS OF SOURCES OF STUDENTS' DIFFICULTY IN MATHEMATICS AND STUDENTS' OPPORTUNITIES TO LEARN

ABSTRACT

The nature of mathematical activity and discourse that teachers foster in classrooms is likely influenced by their explanations of sources of students' difficulty. Several small-scale qualitative studies suggest that how teachers make sense of student difficulty matters for whether they engage all of their students in rigorous mathematical activity. In this article we extend such work to a sample of 165 teachers in four large urban districts. In particular, we investigated the extent to which teachers' explanations of sources of students' difficulty in mathematics (as due to traits of students or the communities they come from, or as in relation to the opportunities to learn provided in classrooms) are related to students' participation in quality mathematical discourse. We found that they are significantly related and that they depend on the racial and linguistic classroom composition of students they teach.

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IN recent years, there have been significant advances in explicating instructional practices that have the potential for affording all students access to opportunities to engage in meaningful mathematics in the classroom (see Franke, Kazemi, & Battey, 2007; National Council of Teachers of Mathematics, 2014). For example, mathematics education research suggests that if students are to develop sophisticated understandings of mathematics, they need regular opportunities to "struggle with important mathematics" (Hiebert & Grouws, 2007, p. 387), likely by solving challenging, nonroutine tasks with multiple solution methods that require them to justify their thinking (Stein, Grover, & Henningsen, 1996). Students also need opportunities to discuss their thinking with classmates (Fraivillig, Murphy, & Fuson, 1999) in both small groups and in whole-class discussions. In small groups, it is important that students share and refine their thinking and that teachers monitor students' thinking in order to plan for productive whole-class discussions (Stein, Engle, Smith, & Hughes, 2008). In whole-class discussions, it is important that students reveal and compare their strategies in order to make connections between them and build shared understandings of key mathematical ideas (Franke et al., 2007).

Unfortunately, mathematics classrooms characterized by this kind of instruction are the exception rather than the rule (Boston & Wilhelm, 2015; Hiebert, 2013) and even rarer in schools serving historically underserved groups of students (Battey, 2013; Darling-Hammond, 2007). This reality, along with the widespread adoption of the Common Core State Standards for Mathematics (CCSSM), has spurred calls to action from leading professional organizations in mathematics education, including the National Councils of both Teachers (NCTM, 2014) and Supervisors (NCSM, 2014) of Mathematics. For example, in the recently released *Principles to Actions*, NCTM (2014) argued that "we must move from 'pockets of excellence' to 'systemic excellence' by providing mathematics education that supports the learning of all students" (p. 3).

A central issue facing the field, therefore, entails identifying what it takes to enact high-quality instruction, especially in schools serving historically underserved students, at some scale. Prior research has identified and attempted to measure a number of factors that influence teachers' enactment of high-quality mathematics instruction, including teachers' knowledge, conceptions, and experience (Baumert et al., 2010; Copur-Gencturk, 2015; Hill et al., 2008; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wilhelm, 2014). In addition, research has identified aspects of the institutional contexts in which teachers work (Cobb, McClain, Lamberg, & Dean, 2003) that influence instructional quality, such as curriculum (Stein & Kaufman, 2010), formal professional development opportunities (Boston & Smith, 2009; Franke, Carpenter, Levi, & Fennema, 2001), and teachers' access to expertise among their school colleagues (Sun, Wilhelm, Larson, & Frank, 2014). However, across this range of factors that have been identified as critical to teachers' enactment of high-quality instruction at scale, few have focused on equity-specific aspects of teachers' perspectives or practice.

One aspect of addressing equity concerns teachers' perspectives of their students' capabilities (Jackson, Gibbons, & Dunlap, in press). To this point, qualitative research focused on instructional reform efforts suggests that how teachers make sense of student difficulty matters for whether they engage all of their students in rigorous activity (Diamond, Randolph, & Spillane, 2004; Horn, 2007; Jackson, 2009; Sosa & Gomez, 2012; Sztajn, 2003). For example, in an ethnographic study of five urban elementary schools, Diamond et al. (2004) found that teachers they described as having low responsibility for student learning—that is, they explained student struggle as outside of their instructional responsibilities—also tended to enact instruction that communicated low expectations for students. For instance, a fifth-grade teacher suggested that students' disciplinary problems were related to their home environments and that they could not handle using materials such as manipulatives. Further, the researchers found that teachers tended to take greater responsibility for students' learning in schools that served greater percentages of White and economically advantaged students, as compared to schools that served more students of color and students from lower-income backgrounds.

Similarly, studies of secondary mathematics teachers' talk in workgroups suggested that teachers' explanations of students' difficulty is an issue for achieving equity-oriented instructional reforms (Bannister, 2015; Horn, 2007). Specifically, Horn (2007) and Bannister (2015) distinguished between explaining student difficulty as a matter of instruction or educational opportunity as opposed to explaining student difficulty in terms of inherent traits of the student or as due to a deficit in the student's family or community. They argued that explaining student difficulty as a matter of instruction likely positioned teachers to enact more equitable, responsive instruction.

Studies such as these have provided important insights regarding the ways that teachers' explanations of student struggle in mathematics might relate to the quality of instruction, but they have generally been qualitative, small-scale studies (e.g., Diamond et al., 2004; Sosa & Gomez, 2012). In this article, we present a quantitative analysis of the relation between a large sample of middle-grades mathematics teachers' explanations of students' difficulty in mathematics and the quality of learning opportunities they provide for their students across large, urban districts pursuing reform. Our findings suggest that teachers' explanations of sources of students' difficulty in mathematics are significantly related to students' participation in quality mathematical discourse, and that the relation between teachers' explanations and student discourse is stronger in classrooms with higher percentages of historically underserved groups of students. By investigating this relation with a large sample of middle-school mathematics teachers, we confirm that the findings of ethnographic and case studies hold at scale, which suggests that teachers' explanations of sources of students' difficulty are critical to attend to when supporting reform efforts aimed at achieving "systemic excellence" (NCTM, 2014, p. 3). In addition, our study leverages the variation in teachers and students that only large-scale studies afford to examine how teachers' explanations of students' difficulty may be of extra consequence in schools serving primarily historically underserved groups of students.

The article is organized as follows. We first describe our key framing ideas, in particular, the opportunity-gap perspective, students' opportunity through participation in quality mathematical discourse, and teachers' explanations of students' difficulty in mathematics. We then present the method, followed by findings. Finally, we conclude by discussing our findings, offering possible interpretations and suggesting areas for further investigation.

Related Literature and Key Concepts

Opportunity Gap Perspective

Our conceptualization of teachers' explanations of student difficulty is broadly informed by a distinction scholars make between "achievement-gap" and "opportunity-gap" perspectives. As scholars such as Flores (2007), Martin (2009), and Milner (2011) have argued, the dominant way of understanding disparity in performance between historically underserved populations and historically advantaged populations is through a lens of the "achievement gap." This way of understanding student performance often ends up, explicitly and implicitly, as casting historically underserved populations as the problem in need of fixing (Martin, 2009); further, rarely are the root causes of the "achievement gap" discussed (Flores, 2007). An alternative way of understanding such disparity in performance is as an "opportunity gap" (Flores, 2007; Milner, 2011). An opportunity-gap perspective highlights the fact that current disparities in achievement are the product of long-standing structural inequities, such as access to highly-qualified teachers, resources, and so forth (Darling-Hammond, 2007; Flores, 2007). In other words, from this perspective, the "problem" does not rest with the individual students or the communities they come from, but with the opportunities that have (or have not) been provided to students.

Opportunity through Participation in Quality Mathematical Discourse

One key aspect of mathematics classrooms that an opportunity-gap lens brings to the fore—and the aspect that was the focus of our analysis—is whether students have opportunities to engage in high-quality discussion of mathematical ideas. Among the standards for mathematical practice outlined in the CCSSM (National Governors Association for Best Practices & Council of Chief State School Officers, 2010) is an emphasis on the need for students to engage in disciplinary, discursive practices, including justifying solutions, constructing viable arguments, and critiquing the reasoning of others. Supporting that emphasis is a significant body of research that has generated a robust understanding of what constitutes high-quality discourse in mathematics classrooms, and why it is crucial if students are to develop both conceptual understanding and procedural fluency (Franke et al., 2007).

Generally, research on discussion-rich middle-grades mathematics teaching has promoted a "three-phase" lesson structure (Van de Walle, Karp, & Bay-Williams, 2012), in which the teacher (*a*) introduces a novel task by ensuring that students sufficiently understand key contextual features and requisite mathematical ideas to engage in the task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), (*b*) invites students to generate ideas and representations for solution strategies (typically in collaboration with each other), and (*c*) facilitates a whole-class discussion of those ideas. Within such a structure, concluding, whole-class discussions are crucial for advancing students' understanding, as they are the site in which teachers and students work together to explicate the mathematical concepts underlying the lesson's activity and draw connections between students' different strategies and representations for making sense of and solving problems (Stein et al., 2008).

Given our interest in equity, an important dimension of students' opportunities to learn is the distribution of participation in the whole-class discussion. Is it just one or two students who are discussing their thinking, or is it a larger percentage of the students in the class? Both the distribution and the quality of contributions make up what we refer to as participation in quality mathematical discourse. Teachers play a central role in ensuring students' participation in quality mathematics discourse. They must encourage widespread participation in order to ensure that variation in student thinking is represented and that all students have an opportunity to participate in this important disciplinary practice. Also, they must press and support students to explain and justify their reasoning in ways that other students can understand (Kazemi & Stipek, 2001; Wood, Cobb, & Yackel, 1991).

Teachers' Explanations of Student Difficulty

The likelihood that teachers will attempt to play such a role in every classroom depends on a number of factors, including teachers' mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004), conceptions of high-quality instruction (or "instructional vision"; Munter, 2015; Wilhelm, 2014), and years of experience (Charalambous, 2010; Wilhelm, 2014). However, given our focus on equity-specific aspects of teachers' practice, we were interested in the relation to instructional quality of a less examined teacher characteristic—teachers' explanations of the sources of students' difficulty (Jackson et al., in press).

As we indicated above, Horn (2007) and Bannister (2015) identified distinctions in teachers' explanations of students' difficulty in mathematics that positioned teachers as more or less likely to enact more equitable, responsive instruction. To elaborate, consider Horn's (2007) study of mathematics departments engaged in "equity-geared reforms" in two high schools serving economically and racially diverse student populations. In one department, Horn found that teachers tended to explain the source of students' success or difficulty in terms of inherent traits of the students (e.g., students were fast, slow, lazy). When students did not learn as expected, teachers placed the blame on the students. As such, they did not tend to focus on what they might do differently in instruction to support struggling students.

However, Horn found that the teachers in the other department framed the problem of varying levels of success in a different manner. As opposed to placing blame on the students, the teachers tended to explain the source of students' success or difficulty in terms of the learning opportunities provided in the classroom. Students' engagement or disengagement depended, in part, on the nature of any given activity (rather than some inherent characteristic of the student). Therefore, when students faced difficulty engaging in rigorous activity, teachers tended to focus on what they could do to alter instruction.

In our analysis, we make a distinction between what we have termed productive and unproductive explanations of why students face difficulty in mathematics. *Productive* explanations are ones that attribute student difficulty to instructional and/or schooling opportunities, whereas *unproductive* explanations are ones that attribute student difficulty to inherent traits of the student, or their family or community. Consistent with NCTM's (2014) *Principles to Actions*, we use the language of productive and unproductive to signal perspectives that influence whether teachers take action to engage all students in rigorous activity.

In light of the existing literature, we conjectured that teachers' explanations of students' difficulty in mathematics might differentially relate to the learning opportunities provided to their students. We also conjectured that this relation might vary depending on the composition of students in the classroom, given that research has found that mathematics teachers justify engaging underserved groups of students in activity solely aimed at developing procedural facility in terms of their perceptions of students' capabilities (Diamond et al., 2004; Jackson, 2009; Sztajn, 2003). The research questions that guided our analysis were therefore: (1) how are teachers' explanations of sources of students' difficulty related to students' participation in whole-class discussion? and (2) does the relation between teachers' explanations of sources of students' difficulty and students' participation in whole-class discussion vary depending on student-level characteristics of the classroom?

Study Context

The data we analyzed were collected in the first 4 years of the Middle School Mathematics and the Institutional Setting of Teaching (MIST) project, which sought to address the question of what is needed to improve the quality of middle-grades mathematics teaching and thus student achievement in large urban school districts (Cobb & Jackson, 2011; Cobb & Smith, 2008). The research team collaborated with the leaders of four large urban school districts located in three states. The four collaborating districts provided settings in which we could investigate our relations of interest within the context of four distinct, at-scale reform efforts. The school districts were typical of large urban districts in that they had limited resources, large numbers of historically underserved populations of students, high teacher turnover, and disparities among subgroups of students in their performance on state standardized tests (Darling-Hammond, 2007). The districts were atypical, however, in their response to high-stakes accountability pressures: they responded by focusing primarily on improving the quality of instruction rather than exclusively on raising student test scores. Namely, each district was attempting to achieve a vision of mathematics instruction compatible with the NCTMs' (2000) *Principles and Standards for School Mathematics* and (2014) *Principles to Actions*—one in which all students would have regular opportunities to collaboratively make sense of and solve challenging mathematical tasks, and, in discussing their solutions, develop robust understandings of key mathematical ideas.

In each of the four districts, the research team and district leaders selected 6 to 10 middle-grades schools that reflected variation in student performance and in capacity for improvement in the quality of instruction across the district. Within each school, up to five mathematics teachers were selected to participate in the study, for a total of approximately 30 teachers per district. The schools remained constant throughout the study, but, as is typical, some of the teachers changed schools or roles during the 4 years. In each case, we recruited replacements in order to maintain a representative and consistently sized sample.

Method

Sample and Primary Measures

Our primary analytic sample included 165 middle-school mathematics teachers pooled over the 4 years of the study, resulting in a total of 275 (statistical) observations (9 teachers with 4 years of data, 20 teachers with 3 years, 43 teachers with 2 years, and 93 teachers with 1 year of data). Because we did not have information about student-level characteristics for some of those teachers, we used a reduced sample of 156 teachers with 238 observations to answer the second research question. We drew on a number of sources of data collected during the MIST project as we tried to understand relations between teachers' explanations of sources of students' difficulty and students' participation in whole-class discussion, including videorecordings of teachers' classroom instruction, interviews with teachers, and an assessment of mathematical knowledge for teaching (Hill et al., 2004). Additionally, our analyses included student demographic information collected and provided by the districts. The number of students served in the four districts ranged from approximately 35,000 to 160,000 students. On average, 29% of the students were White, 33% of the students were Black, and 36% of the students were Hispanic. Approximately 20% of the students were classified as limited English proficient, and 68% of the students were eligible for free or reduced-price lunch. In the following paragraphs, we describe each of our measures, with descriptive statistics for each provided in Table 1.

Students' participation in quality mathematical discourse. The two outcome variables in our analyses were generated through the coding of videorecordings of teachers' classroom instruction. In each year of the project, teachers' instruction in two consecutive lessons with the same class was videorecorded in February or March. Teachers were asked to engage in a problem-solving lesson with a related whole-class discussion and knew when the research team would come to videorecord. Each videorecorded lesson was scored using the Instructional Quality Assessment (IQA; Boston, 2012). We took the higher set of scores across the 2 days to represent each teacher's best attempt at enacting high-quality mathematics in-

Table 1. Descriptive Statistics

	Primary Sample (n = 275)				Reduced Sample (n = 238)			
	Mean	*SD*	Min	Max	Mean	*SD*	Min	Max
Providing	.968	.750	0	4	.959	.750	0	4
Linking	.633	.513	0	3	.619	.505	0	3
Task	2.631	.558	1	4	2.641	.547	1	4
MKT	−.069	.803	−1.848	2.013	−.104	.794	−1.848	2.013
VHQMI	2.300	.685	0	3.833	2.280	.713	0	3.833
Teaching experience	9.193	8.460	1	42	9.080	8.573	1	42
Mean prior SA					−.569	.581	−2.862	1.217
% FRL					.724	.222	0	1
% LEP					.127	.148	0	1
% SoC					.784	.245	.143	1

Note.—Providing and linking refer to "students' participation in providing" and "students' participation in linking," respectively.

struction (rather than representative of that teacher's typical instruction), and used those scores in our analysis.

The IQA consists of a series of rubrics for assessing the "academic rigor" and the quality and distribution of Accountable Talk® in instruction. The three rubrics pertaining to academic rigor are based on the Mathematical Tasks Framework (Stein et al., 1996) and aim to assess the level of cognitive demand over the course of the lesson. Of the five rubrics that attend to Accountable Talk®, we focused on the three pertaining to students' contributions to the whole-class discussion: the extent to which (*a*) students provide accounts of their reasoning (*student providing*), (*b*) students link to and build on each other's ideas (*student linking*), and (*c*) all students participate in class discussion (*participation*). With respect to reliability, the average percent agreement across the three student-focused Accountable Talk® rubrics for the 4 years was 72.8%, with an average kappa score (J. Cohen, 1960) of 0.49. Prior work suggests that these levels are sufficient to discern differences in learning opportunities for students (Hartmann, Barrios, & Wood, 2004), especially given the measures' complexity.

The IQA's student providing and linking rubrics have five levels each, scored 0–4: (0) no whole-class discussion; (1) no student providing or linking; (2) providing procedural accounts of reasoning (e.g., describing steps taken or calculations performed) or superficial linking; (3) a few strong efforts to provide conceptual accounts of reasoning or link ideas; and (4) consistent strong efforts to provide conceptual reasoning or link ideas. The Participation rubric accounts for the distribution of participation by representing the percentage of students who participated in the whole-class discussion: (0) no whole-class discussion; (1) at most 25% of the students participating; (2) 26%–50% of students participating; (3) 51%–75% of students participating; and (4) 76%–100% of students participating.

Because we were interested in students' participation in quality mathematical discourse, we weighted each of the student-providing and student-linking scores by the participation score, which accounted for the distribution of students' discourse. We call these weighted measures "students' participation in providing" and "students' participation in linking." Figure 1 shows the frequencies for each of the weighted scores. As demonstrated in the figure, scores for students' participation in providing were more normally distributed than scores for students' participation in linking, which were more skewed to the right. However, the most frequent score for both measures was 0.5.

Cognitive demand of classroom activity. Certain types of tasks (e.g., those that are cognitively demanding and provide opportunities for sense-making) likely lend themselves to better whole-class discussion (Franke et al., 2007). We controlled for potential differences in cognitive demand of classroom activity by including the mean of scores on the two IQA academic rigor rubrics pertaining to the cognitive demand of tasks: *task potential* (the task "as written") and *task implementation* (the task "as enacted"). We refer to the combined variable simply as "Task." Scores on both rubrics range from 1 to 4, with lower levels representing memorization or using procedures without conceptual connections and higher levels representing genuine inquiry or connecting procedures to conceptual ideas (Boston, 2012; Stein et al., 1996).

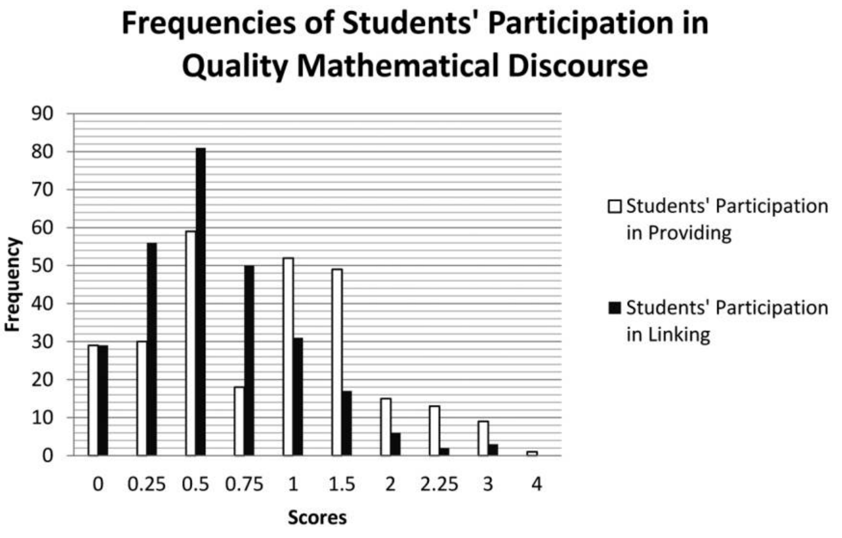

Figure 1. Frequencies of students' participation in quality mathematical discourse: combined, weighted scores for students' participation in providing and students' participation in linking.

Explanations of sources of students' difficulty in mathematics. Our assessment of how teachers explained sources of students' difficulty in mathematics (hereafter referred to as "explanations of difficulty") was interview based (Jackson et al., in press). As part of the interviews in the larger study we asked questions like, "When your students don't learn as expected, what do you find are typically the reasons?" We also asked teachers to describe the challenges they face, which often provided insight into how they explained student difficulty. For example, they often described students' behavior, a lack of basic skills, students' home lives, or a lack of student motivation.

All interviews were transcribed, and all transcripts were coded using a qualitative data analysis program, NVIVO. Coders searched transcripts for specific questions and keywords pertaining to teachers' explanations of difficulty. One coding decision we faced was determining how to unitize (Campbell, Quincy, Osserman, & Pedersen, 2013), or chunk the text into meaningful, codable parts. In semistructured interviews, it is often the case that relevant ideas (e.g., an explanation regarding a source of difficulty) unfold over the course of multiple turns of talk (Campbell et al., 2013). Therefore, in an effort to ensure coders were coding the same unit of text, we aimed to code at the unit of a turn of talk. However, within NVIVO, we also captured whatever relevant text we used to make sense of a particular talk turn.

For each relevant passage, coders assigned a code of "productive" (explaining students' difficulty as a relation between student(s) and instructional and/or schooling opportunities); "unproductive" (attributing students' difficulty to an inherent property of students and/or as produced in relation to something other than instructional opportunities); or "mixed" (wavering between the two previous kinds of explanations). (See App. Table A1 for an abbreviated coding scheme with examples of coded explanations of difficulty.) If all coded passages were categorized as unproductive, the interview was coded as unproductive. If all coded passages were considered productive, the interview coded as productive. The interview was coded as mixed if either all coded passages were categorized as mixed, or if there was a combination of productive and unproductive passages.

Coders were doctoral students and were required to reach at least 80% exact agreement at the interview level with previously scored transcripts before coding new transcripts. The instrument developer (the third author) double coded 20% of the interview transcripts to check for ongoing reliability, and the overall exact agreement rate was 71.7% (kappa 0.506). Some teachers in the larger MIST study are missing a code for explanations of difficulty, mostly due to the fact that interviewers did not probe regarding teachers' explanations of difficulty to sufficient depth to make a coding judgment.[1] Because teachers' explanations of sources of students' difficulty are central to our analysis, we limited our sample to the teacher observations with coded explanations of difficulty. Of the teacher observations with explanations of difficulty scores, 33% were scored as unproductive, 40% were scored as mixed, and 27% were scored as productive.

Mathematical knowledge for teaching. Mathematical knowledge for teaching (MKT) is mathematics content knowledge specific to the work of teaching (Ball, Thames, & Phelps, 2008). There is evidence that MKT influences teachers' instructional practice (Charalambous, 2010; Copur-Gencturk, 2015; Hill et al., 2008). In March of each year of the larger study, we assessed all participating teachers' MKT by using a pencil-and-paper instrument developed by the Learning Mathematics for Teaching Project (Hill et al., 2004). For our analysis, we used a combined average of developer-provided item response theory scale scores from two subtests (number concepts and operations; and patterns, functions, and algebra) to form a single MKT score for each participating teacher in each study year.

Vision of high-quality mathematics instruction. A teacher's instructional vision is a dynamic conception and articulation of (future) practice. It can be viewed as an indication of the extent to which teachers have appropriated conceptual tools for teaching (Grossman, Smagorinsky, & Valencia, 1999) such as those introduced in professional development settings, and has been shown to relate to aspects of teachers' practice (Munter, 2015; Wilhelm, 2014). We assessed the sophistication of teachers' visions of high-quality mathematics instruction (VHQMI; Munter, 2014) in the January interviews by asking a series of interview prompts and coding the responses with leveled rubrics. Specifically, participants were asked what they would look for when observing a mathematics teacher's instruction to determine whether the instruction was of high quality. Depending on the breadth of their responses, participants were then asked a series of follow-up probes specific to our coding dimensions, which included the role of the teacher, discourse, and mathematical tasks (see Munter [2014] for more details). The number of score levels vary by rubric, but, in general, scores should be considered as ranging from 0 to 4, with directionality roughly mirroring that of the IQA. All interviews were transcribed and coded by research team members, with weekly reliability checks performed by the first two authors (maintaining an overall rate of exact agreement of 80% across all years combined). To estimate participants' VHQMI, we calculated the mean across all dimensions on which their transcripts were scored.

Years of experience teaching mathematics. Previous studies suggest that teachers' mathematics teaching experience influences their instructional practices

(Charalambous, 2010; Escudero & Sánchez, 2007; Remillard & Bryans, 2004). Therefore, to control for the influence of teaching experience on students' participation in quality mathematical discourse, we included teachers' number of years of experience teaching mathematics.

Student-level characteristics. It is reasonable to assume that the relation between teachers' explanations of sources of students' difficulty in mathematics and students' opportunities to learn in a given classroom might vary based on whether there are indeed students who have previously had some difficulty with mathematics. One proxy for students' past difficulties—about which teachers may have knowledge—is the class mean score on the previous school year's state standardized test (mean student achievement [SA]).

Further, when teachers describe subgroups of students who struggle in mathematics class, they often include students who live in poverty, students for whom English is their second language, and/or students of color (e.g., African American students, Latino/a students). Therefore, along with students' prior mathematics achievement, we included variables pertaining to the percentage of students in a school eligible for free or reduced-price lunch (% FRL),[2] the percentage of students in a class classified as limited English proficient (% LEP), and the percentage of students of color in the class (% SoC).

We used standardized versions of each in order to allow for interpretation of interaction effects. However, in the case of % SoC, because of the left-skewed distribution, we divided by a half (rather than a full) standard deviation, which means that a one-unit increase corresponds to a half of a standard deviation increase in the percentage of students of color in the class.

Analyses

To answer our first research question, we employed a series of linear regression models to investigate how teachers' explanations of difficulty are related to both of the students' participation in quality mathematical discourse outcomes described previously: students' participation in providing and students' participation in linking. In each model, we controlled for other factors that could explain the relation between teachers' explanations of difficulty and opportunities to learn, including teachers' MKT, VHQMI, and years of experience teaching mathematics. Additionally, because teachers taught in four different school districts, we included dummy variables to account for district membership. We constructed multilevel models in order to account for the nested nature of our data: observations nested within teachers, nested within schools. In other words, there were multiple observations for some teachers over the years of the study, and multiple teachers per school.

To answer our second research question, we added information about student-level characteristics of the classroom, and investigated statistical interactions between those characteristics and teachers' explanations of difficulty. Prior to estimating the models, we calculated correlations between the different student-level characteristics to identify any interdependencies between the measures. Next, to test whether student-level characteristics moderate the relation between explanations of difficulty and students' participation in quality mathematical discourse, for each of the two outcomes, we estimated four models including interactions between different student-level characteristics and teachers' explanations of difficulty. We first tested the models with just the addition of the class mean score on the previous year's state standardized test, along with statistical interactions between this mean score and explanations of difficulty. If the class mean is lower than that of other classes, it may be that a greater percentage of the students have had (and perhaps are having) difficulty, in which case the relation between their teacher's explanations of difficulty and their opportunities to learn in that teacher's class might be more tightly linked.

With the last few models we tested whether the relation between teachers' explanations of difficulty and students' participation in quality mathematical discourse varies with respect to the three other student-level characteristics: percentage of students in the school eligible for free or reduced-price lunch, percentage of students in the class classified as limited English proficient, and percentage of students of color in the class. A statistically significant interaction between student characteristics and teachers' explanations of difficulty would indicate that the nature of the relation between teachers' explanations of difficulty and students' participation in quality mathematical discourse is related to the composition of students in the classroom.

Results

In this section, we report the results from models addressing our research questions along with relevant correlations between variables. Our initial focus is on the relation between teachers' explanations of sources of students' difficulty in mathematics and students' participation in quality mathematical discourse. We then examine variation in that relation with respect to student-level characteristics in teachers' classes.

Teachers' Explanations of Difficulty and Students' Participation in Quality Mathematical Discourse

Results from the two models of the relation between teachers' explanations of difficulty and students' participation in quality mathematical discourse are given in Table 2. First, across the two models, results suggest that on average, for this sample of teachers, teachers' explanations of difficulty are significantly related to students' participation in providing ($b = .315$, $p < .05$) but not to students' participation in linking ($b = 0.083$, $p = 0.58$). In particular, student providing of reasoning was about a third of a standard deviation higher in classrooms of teachers who articulated productive explanations of difficulty than in classrooms of teachers who articulated unproductive explanations of difficulty. Of the control variables included, only the cognitive demand of the classroom activity was significantly related to either outcome: teachers who chose and implemented tasks with higher cognitive demand tended to have better student participation in quality mathematical discourse. Teachers' MKT and VHQMI were not significantly related to students' participation in quality mathematical discourse, the meaning and significance of which we elaborate on in the Discussion section.

Table 2. Teachers' Explanations of Difficulty and Students' Participation in Quality Mathematical Discourse

	Providing		Linking	
	Coef.	*SE*	Coef.	*SE*
Mixed exp. of difficulty	.257^{+}	.135	.116	.134
Prod. exp. of difficulty	.315*	.151	.083	.149
Task	.481***	.110	.427***	.110
VHQMI	−.080	.089	.063	.087
MKT	.038	.085	.073	.082
Teaching experience	.002	.008	.007	.007
District B	.184	.202	.078	.232
District C	−.010	.218	−.035	.249
District D	−.032	.204	.070	.235
Constant	−1.336**	.420	−1.451**	.428

$^{+}$ $p < .10$.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

Variation in the Relation between Teachers' Explanations of Difficulty and Students' Participation in Quality Mathematical Discourse, by Student-Level Characteristics

Before describing results from the models of statistical interactions between student characteristics and teachers' explanations of difficulty, we examined correlations between the different student-level characteristics in our sample. We found that mean student achievement was negatively correlated with % FRL, % LEP, and % SoC ($r = -.24$, $r = -.35$, and $r = -.35$, respectively). Relatedly, % FRL, % LEP, and % SoC are all positively correlated. For the most part, the magnitudes of the correlations are not large enough to raise concerns about multicollinearity. The one exception is the correlation between the percentage of students in the class who were classified as limited English proficient and the percentage of students of color in the class ($r = .41$). Therefore, we did not include those two variables together in any of the models.

Tables 3 and 4 display the results of our models examining relations between teachers' explanations of difficulty and students' participation in providing and linking, respectively. In each case, we first list results from a model including a statistical interaction between teachers' explanations of difficulty and the mean student achievement in the class (1), followed by three models that each test an interaction between teachers' explanations of sources of students' difficulty in mathematics and a particular student-level characteristic: poverty status (2), language proficiency (3), and racial/ethnic minority status (4).

Beginning with model 1 in Table 3, the significant interaction suggests that the relation between teachers' explanations of difficulty and students' participation in providing does vary based on the mean prior student mathematics achievement in the class ($b = -.2999$, $p < .05$). This relation is depicted in Figure 2. In particular, the solid line represents teachers with unproductive explanations of difficulty and the dotted line represents teachers with productive explanations of difficulty. In comparing the differences in students' participation in providing for classrooms

Table 3. Variation in How Teachers' Explanations of Difficulty Relate to Students' Participation in Providing, by Student-Level Characteristic

	(1) Prior Achievement		(2) % FRL		(3) % LEP		(4) % SoC	
	Coef.	*SE*	Coef.	*SE*	Coef.	*SE*	Coef.	*SE*
Mixed exp. of diff.	.248+	.145	.247+	.145	.245	.143	.226	.144
Prod. exp. of diff.	.331*	.162	.340*	.162	.237	.163	.266	.162
Mean SA	.089	.098	.090	.099	.064	.107	.061	.105
Mixed × mean SA	−.089	.150	−.096	.151	−.072	.157	−.064	.155
Prod. × mean SA	−.299*	.150	−.346*	.164	−.148	.160	−.136	.162
S. char.			−.004	.116	−.051	.109	−.027	.161
Mixed × S. char.			−.009	.150	.051	.155	.046	.149
Prod. × S. char.			−.101	.176	.360*	.149	.214*	.181
Task	.434***	.120	.430***	.122	.424***	.120	.438***	.119
VHQMI	−.014	.094	−.022	.097	−.004	.093	.004	.093
MKT	.034	.091	.020	.093	.038	.091	.058	.092
Teaching experience	.003	.008	.003	.008	.003	.008	.004	.008
District B	.346	.220	.341	.220	.439	.234	.309	.235
District C	.192	.240	.204	.245	.219	.240	.147	.264
District D	.152	.224	.157	.226	.233	.244	.254	.282
Constant	−1.539**	.457	−1.508**	.462	−1.583***	.452	−1.589***	.453

+ $p < .10$.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

Note.—Exp. of diff. = explanations of difficulty; S. char. = student characteristic and is % FRL in column 2, % LEP in column 3, and % SoC in column 4.

Table 4. Variation in How Teachers' Explanations of Student Difficulty Relate to Students' Participation in Linking, by Student-Level Characteristic

	(1) Student Prior Achievement		(2) % FRL		(3) % LEP		(4) % SoC	
	Coef.	*SE*	Coef.	*SE*	Coef.	*SE*	Coef.	*SE*
Mixed exp. of diff.	.050	.140	.048	.140	.058	.137	.043	.138
Prod. exp. of diff.	.139	.155	.142	.156	.045	.155	.097	.155
Mean SA	−.026	.094	−.021	.095	−.050	.103	−.081	.100
Mixed × mean SA	.006	.144	−.004	.145	.049	.151	.053	.149
Prod. × mean SA	−.211	.143	−.251	.157	−.046	.151	−.081	.156
S. char.			.012	.114	−.041	.107	−.091	.161
Mixed × S. char.			−.025	.147	.123	.149	.056	.143
Prod. × S. char.			−.095	.168	.390**	.143	.179*	.173
Task	.421***	.116	.420***	.118	.412***	.115	.430***	.116
VHQMI	.129	.090	.125	.093	.136	.089	.138	.090
MKT	.102	.086	.093	.088	.110	.085	.112	.087
Teaching experience	.006	.008	.005	.008	.005	.008	.007	.008
District B	.239	.254	.236	.254	.369	.264	.284	.272
District C	.195	.274	.201	.279	.223	.272	.263	.303
District D	.319	.258	.322	.261	.452	.275	.257	.314
Constant	−1.751***	.458	−1.732***	.462	−1.817***	.451	−1.804***	.457

+ $p < .10$.
* $p < .05$.
** $p < .01$.
*** $p < .001$.

Note.—Exp. of diff. = explanations of difficulty; S. char. = student characteristic and is % FRL in column 2, % LEP in column 3, and % SoC in column 4.

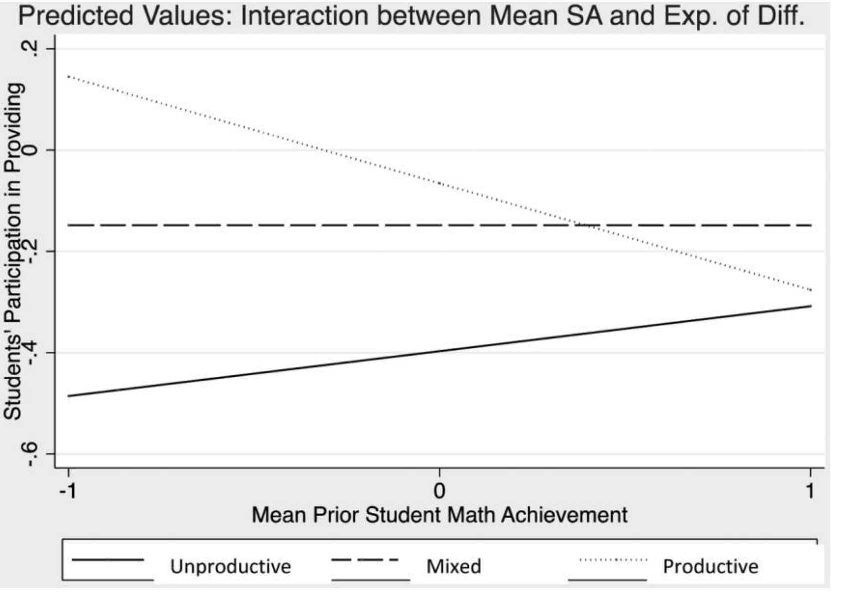

Figure 2. Mean student achievement moderates the relation between students' participation in providing and teachers' explanations of difficulty.

with students whose prior achievement is 1 standard deviation below the mean, classrooms of teachers who articulated productive explanations of difficulty had significantly better participation in providing than those of teachers who articulated unproductive explanations of difficulty. That difference diminishes as the mean prior student achievement increases (as indicated by the negative coefficient for the interaction variable and the negative slope of the dotted line). In other words, teachers' explanations of difficulty are not related to differences in students' participation in providing when they teach in classrooms in which most students have previously been successful on standardized mathematics assessments. This suggests that the relation between teachers' explanations of sources of students' and students' participation in providing varies based on the prior achievement of students in the class.

The next three columns in Table 3 display the results of our models testing for relations between students' participation in providing and whether teachers' explanations of difficulty vary depending on students' poverty status (2), students' language proficiency (3), and classroom racial/ethnic composition (4). Results suggest that there is not a significant interaction between teachers' explanations of difficulty and the percentage of students eligible for free or reduced-price lunch, but that there is variation in the relation between teachers' explanations of difficulty and students' participation in providing based on the percentage of students classified as limited English proficient and the percentage of students of color. We offer possible interpretations of these results in the Discussion section.

In both of the latter two models, the significant interaction between teachers' explanations of difficulty and prior student achievement (detected with model 1) disappears and is replaced by a significant interaction between teachers' explanations of difficulty and either the percentage of students classified as limited English proficient (model 3), or the percentage of students of color in the class (model 4). This suggests that perhaps what manifested as variation in the relation between teachers' explanations of difficulty and students' participation in providing based on prior student achievement may have actually been attributable to these other student characteristics. For both of these student characteristics, as the percentage of students in the subgroup increases, the relation between teachers' explanations of difficulty and participation in providing increases (% LEP: $b = .360$, $p < .05$; % SoC: $b = .214$, $p < .05$). For example, at the mean percentage of limited English proficient students in the class (12.7%), teachers' explanations of difficulty are not significantly related to participation in providing, but at 1 standard deviation above the mean (27.5%), explanations of difficulty are significantly related to participation in providing. Specifically, the results suggest that, on average, in classrooms with 27.5% of their students classified as limited English proficient, students' participation in providing is just over a third of a standard deviation better for teachers who articulated productive explanations of difficulty than for teachers who articulated unproductive explanations of difficulty. Also, at the mean percentage of students of color in the class (78.4%), teachers' explanations of difficulty are not significantly related to students' participation in providing, but at half a standard deviation above the mean (90.7%), teachers' explanations of difficulty are significantly related to participation in providing. This finding suggests that, on average, in classrooms with at least 90.7% of students of color, students' participation in providing is nearly a fifth of a standard deviation better for teachers who articulated productive explanations of difficulty than for teachers who articulated unproductive explanations of difficulty. More generally, we find that the relation between teachers' explanations of difficulty and students' participation in providing is stronger in classrooms with higher percentages of historically underserved students.

In Table 4, we report the results from a parallel set of analyses that we conducted for students' participation in linking. Recall that in the original model for students' participation in linking, we did not find a significant relation between teachers' explanations of difficulty and participation in linking. First, as shown in column 1 of Table 4, we also did not find that the relation between teachers' explanations of difficulty and participation in linking varied significantly by prior student achievement ($b = -.211$, $p = .142$). As with participation in providing, the relation also did not vary for the percentage of students eligible for free or reduced-price lunch. However, we found significant interactions for both the percentage of students in the classroom classified as limited English proficient and the percentage of students of color (% LEP: $b = .390$, $p < .01$; % SoC: $b = .179$, $p < .05$). This suggests that for the average teacher in our sample, teachers' explanations of difficulty were not significantly related to students' participation in linking, but when we consider the composition of the students in the classroom, teachers' explanations of difficulty were significantly related to students' participation in linking in classrooms with larger percentages of students from historically underserved populations.

Discussion

For years, a key factor of interest in linking teaching and learning in mathematics education research has been "opportunity to learn" (Hiebert & Grouws, 2007). Although the measures of students' contributions we used in our analyses—communicating one's reasoning ("students' participation in providing") and drawing connections among strategies and representations ("students' participation in linking")—do not capture all that high-quality mathematics instruction entails, they do represent two such opportunities that, at least since 1989 and the promotion of its "equity principle," the NCTM (1989) has argued that all students should have access to. The findings we have reported in this article suggest that whether there are more or less equitable opportunities to engage in such forms of mathematical reasoning is related to teachers' explanations of sources of students' difficulty in mathematics. On average, students in our sample were more likely to participate in discussions in which they and their peers provided reasoning and made connections between strategies if their teacher explained sources of student difficulty in mathematics as related to the nature of instruction or learning opportunities, rather than in terms of inherent traits of the students or due to factors outside of instruction (e.g., families, community). We view this finding as providing an important insight into the factors associated with teachers' enactment of practices that afford important learning opportunities to all of their students.

Before discussing our findings further, we acknowledge a number of limitations of our study. First, whereas we identified a relation between teachers' explanations of students' difficulty and students' participation in quality mathematical discourse, we do not mean to imply any directionality in the relation. That is, we are not suggesting that teachers' explanations caused them to enact a kind of instruction (or that changes in the former would lead to changes in the latter). Indeed, the relation could very well be in the opposite direction, as the introduction of new instructional practices has been shown to be revelatory to teachers with respect to students' capabilities (see Franke et al., 2001), and such new insights could lead to reexaminations of the implications of one's prior practice. For example, through engaging in whole-class discussion with students, teachers might learn that the instructional decisions they make contribute to students' difficulty (or success) in mathematics, which could change how they explain students' difficulty. Future research might investigate how changes in instruction relate to change in teachers' explanations of students' difficulty, capitalizing on longitudinal data and qualitative data to understand the directionality of the relation.

A second limitation concerns our proxy for students' opportunity to learn—students' participation in discussion in which they provide reasoning for their solutions and make connections between and to classmates' ideas. Although these are important components of rich, mathematical activity and discourse, this operationalization likely fails to capture other important opportunities for learning. For example, it would have been informative if we had included some indication of whether, perhaps in small-group work, students had opportunities for sense-making (Putnam, Lampert, & Peterson, 1990), building on current understanding (Carpenter & Lehrer, 1999), or generating and testing conjectures (Lampert, 2001)—and whether those opportunities were similarly related to teachers' explanations of difficulty. For this particular analysis, we were limited in our choice of a proxy for students' opportunity to learn because we explored relations within a larger research project that had a number of data-collection priorities. However, we used theory to choose our set of outcomes—students' participation in quality mathematical discourse in the whole-class discussion—from the available set within the larger research project.

A related, third limitation concerns the level at which we addressed aspects of an opportunity gap. Our analysis focused on individual teachers' explanations of student difficulty and interactions between teachers and students within classrooms; it did not extend to ways that differences in opportunity are produced systemically or structurally (although we view these two levels as being inextricably linked). Moreover, although our study highlights important differences in learning opportunities related to *race* (and language and culture), we do not claim to provide insights into how *racism*—whether individual or structural—produces the differences.

Last, it is important to acknowledge the limits of the generalizations that can reasonably be made based on our study's sample. As previously explained, we collected our data in schools that, typical of urban districts, were serving large numbers of historically underserved populations of students, with limited resources and high teacher turnover (Darling-Hammond, 2007), but, atypically, were pursuing a vision of mathematics instruction in which all students would have regular opportunities to collaboratively make sense of, solve, and discuss challenging mathematical tasks. The relations we have identified may vary in other settings, depending on the student populations being served or the commitments to particular forms of mathematics learning and instruction, as represented in curriculum adoption, professional development support, teaching evaluation processes, and other policies or reform efforts.

Teachers' Explanations of Students' Difficulty and (Equitable) Opportunities to Learn

Our study's main finding regarding the relation between teachers' explanations of sources of students' difficulty and students' participation in quality mathematical discourse—at least with respect to students explaining their reasoning—provides a large-scale confirmation of what previous, smaller-scale studies have suggested: that teachers' explanations of sources of students' difficulty in mathematics are related to the quality of learning opportunities they afford students. But more than this, the results of our interaction analyses suggest that, in the context of this study, the strength of this relation depends on the composition of students in the classroom with respect to race, ethnicity, and/or language status. To expand this point, we further emphasize two key findings.

First, our results provide further support for abandoning an "achievement gap" lens and focusing instead on inequities in opportunity as they relate to race and cultural background (Flores, 2007; Milner, 2011). By itself, the first interaction we tested in model 1, between teachers' explanations of difficulty and students' prior achievement, yielded interesting insights: the significant interaction term suggests that, on average, whether previously "low-achieving" students have opportunities to participate in discussions in which students provide reasoning for their solutions is related to whether their teacher explains students' difficulty in mathematics productively. If not, a lack of prior achievement may be associated with a lack of access to particular opportunities to learn. But, the results of our analyses suggest that the relation between teachers' explanations of difficulty and students' participation in providing depends more strongly on other student-level characteristics—classroom proportions of students of color or students identified as limited English proficient. For example, the significant interaction term in model 4 in Table 3 suggests that students in classes composed (almost) entirely of students of color were more likely to have opportunities to participate in discussions in which students provided reasoning for their solutions if their teacher articulated productive explanations of sources of their difficulty. However, in classrooms with smaller percentages of students of color, the difference in instructional practice between teachers with productive and unproductive explanations of difficulty was not as dramatic. In other words, without White students in the room, teachers' explanations of students' difficulty in mathematics—whether productive or unproductive—were, on average, more likely to be reflected in (or reflective of) their instructional practice.

A second, related finding that we wish to highlight is that the statistical interactions that we observed between teachers' explanations of difficulty and both % LEP and % SoC did not hold for % FRL; for this sample, the relation between teachers' explanations of difficulty and students' participation in quality mathematical discourse did not depend on the proportion of students in poverty, at least as measured by the percentage of students who qualify for free or reduced-price lunch. This suggests that teachers who articulated unproductive explanations of difficulty were no less likely to foster rich mathematical discussion in classrooms with high proportions of economically disadvantaged students than were teachers who articulated productive explanations of difficulty. That said, this finding could also be an artifact of the inadequate use of % FRL as a proxy for poverty (Bryk et al., 2010).

Taken together, these findings imply that teachers' explanations of sources of students' difficulty in mathematics may be more influenced by students' race or cultural background than by their economic status or even past achievement. To the extent that this is the case, it may be that the more "visible" characteristics of race and limited English proficiency, as compared to social class and prior achievement, are more likely to trigger the kind of low responsibility/low expectations–oriented instruction identified by Diamond et al. (2004).

These findings raise an important question: *why* was students' participation in quality mathematical discourse better in classrooms of teachers who articulated productive explanations of difficulty? Mathematics education research has identified the level of skill or support it takes to orchestrate whole-class discussions in which students provide conceptual evidence for their reasoning and connect their ideas to those of their peers and the discipline (Ball, 1993; Schoenfeld, 2011; Stein et al., 2008; Stylianides & Stylianides, 2014). However, organizing discussions that elicit and make use of student thinking to build toward a mathematical agenda likely requires that a teacher views students as capable of articulating ideas that are, indeed, worth building upon (Lampert, 2001). In other words, it could be the case teachers who articulated productive explanations of difficulty more generally viewed their students' thinking as worth understanding and building upon. Another conjecture is that those teachers who articulated productive explanations of difficulty are more likely to enact pedagogical practices that are culturally relevant (Ladson-Billings, 1995), responsive (Gay, 2000), or sustaining (Paris, 2012), and, in turn, those practices give rise to greater participation in richer mathematical discourse (Ottmar, Rimm-Kaufman, Larsen, & Berry, 2015). Or, it could be that how teachers explain students' difficulty in mathematics is indicative of a more general disposition with which they approach the relationships they develop with students, thereby fostering a level of trust necessary for achieving such discourse.

Connections to Prior Research

In addition to identifying how teachers' explanations of the sources of students' difficulty relate to students' opportunities to participate in quality discourse, our analysis confirmed and, in some cases, raised questions about existing research on relations between teacher knowledge, conceptions, and practice. First, we consistently found that students' participation in quality mathematical discourse was, on average, higher in lessons in which teachers posed cognitively demanding tasks and maintained that demand in their implementation. That engagement with such tasks—itself an important opportunity to learn—is associated with richer discussion confirms previous findings (see Stein et al., 1996), contributing further evidence for the important role of complex mathematical tasks.

Second, unlike some previous studies (see Charalambous, 2010; Hill et al., 2008), we did not find a relation between teachers' mathematical knowledge for teaching and instructional outcomes. However, we do not view this as a contradictory finding. We investigated the relation between MKT and specific aspects of a particular kind of instruction—opportunities for students to provide reasoning for their ideas or link their solution methods to others' contributions. Our findings suggest that, in the context of this study, MKT is not directly related to the extent to which students are afforded such opportunities, but our findings do not preclude the possibility that MKT is related to other aspects of instructional practice or student outcomes defined in other ways.

Similarly, we did not find that teachers' visions of high-quality mathematics instruction were related to students' participation in quality mathematical discourse. Previous research has demonstrated connections between teachers' beliefs about or conceptions of mathematics and instruction (see Lloyd, 1999; Remillard & Bryans, 2004; Stipek et al., 2001). However, our findings regarding teachers' instructional vision are not surprising, given that vision has been shown to be related more strongly to other aspects of instruction (Munter, 2015; Wilhelm, 2014) and, in particular, future *change* in instruction (Munter & Correnti, 2017). Still, our finding suggests that teachers' articulations of commitments to rich student discourse are not necessarily reflected in their current practice.

Future Directions

On the whole, our findings point to a need for shifting how teachers explain the problem of students' difficulty in mathematics, particularly when serving historically underserved groups of students. Figuring out how to support such a shift will likely require further investigation of teachers' cultural and racial competence (Milner, 2003) in discipline-specific contexts, and a better understanding of and support for teachers in "building relationships for doing and learning mathematics" beyond knowing students from a cognitive standpoint (Franke et al., 2007, p. 242). Regarding the latter, it could be worthwhile to integrate into current mathematics instructional models the affective, relational, and emotional aspects of classroom interactions. It is possible that there is more to be leveraged from the very kinds of instructional practice that we have privileged in our analyses, which have been described as characteristic of "relationship-enhancing pedagogy" (Wallace & Chhuon, 2014, p. 958).

It is also important to keep in mind that relations such as those examined in this article play out in and are likely influenced by aspects of the organizational contexts in which teachers work. For example, Jackson (2009) attributed a charter school network's choice to focus its curriculum on memorization of mnemonic devices and procedures to the network leaders' assumption that students did not have the requisite skills and knowledge to engage in conceptually oriented mathematics. Horn (2007) illustrated how teachers *jointly* construct ways to make sense of and respond to students' difficulties in mathematics in the context of regularly scheduled time to collaborate (e.g., common planning time, department meetings). And, Diamond et al. (2004) identified the role of school leaders as having the potential to shape teachers' sense of responsibility for their students' learning. Together, these findings suggest that teachers' opportunities to collaborate and school leaders' instructional expectations are two aspects of the organizational context that, along with professional development, curriculum, and other policies, have potential to moderate the influence of (and ultimately shift) teachers' explanations of sources of difficulty in mathematics.

Conclusion

Even given the limitations acknowledged above, we view the results reported in this article as providing important insights regarding the challenge of affording equitable opportunities to students to engage in rich mathematical work and talk. Our findings suggest that teachers' explanations of sources of students' difficulty in mathematics are related to students' participation in quality discourse, and that this relation is stronger with respect to the proportions of students of color and students of limited English proficiency in classrooms. These results may be, sadly, not entirely surprising. But even if they confirm what many suspected was the case, to our knowledge, this report is the first to provide large-scale empirical evidence of such relations in mathematics classrooms and may therefore serve as a catalyst for increased and more focused attention to understanding and resolving inequities in opportunities to learn mathematics.

Appendix

Table A1. Abbreviated Version of Coding Scheme to Assess the Nature of Participants' Explanations Regarding the Source(s) of Students' Difficulties in Mathematics

Code and Definition	Example
Productive Student performance (e.g., failure, success, engagement, interest) is described as a relationship between student(s) and instructional and/or schooling opportunities.	Interviewer: So, in your own classroom when students don't learn as expected, what do you usually find are the reasons? Teacher: Why a kid didn't learn? Because I didn't make him. Interviewer: How do you make a kid learn? Teacher: I don't know. That's always the problem, isn't it? I, I do, and also again I, I might be different on that, but I, I really feel like if a kid's not learning in a classroom, it's my fault. That it's something that I'm not doing. There has to be a reason. I mean, I, you know, especially in the eighth grade, I mean, they can learn something. There is, there's something they can be doing. There's some way they can be doing it. And so, I mean, if a kid's just flat out not learning then there's something that I need to do better to make him learn and I don't always know what that is, but I mean, I do put most of the emphasis back on me.
Mixed Participant wavers between explaining student performance (e.g., failure, success, engagement, interest) (1) as a relationship between student(s) and instructional and/or schooling opportunities and (2) as due to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional opportunities (e.g., parents don't value education, therefore students don't).	Interviewer: In your classrooms, when the students do not learn as expected, what do you find are the typical reasons? Teacher: Probably me . . . I don't put blame on the students. I mean, I think it's a combination. They have to do their part, and I have to do mine, so if they're not getting it, it may, and this, this may not be the best way, but I'll be honest, I look to the students that are consistently successful, and if they don't understand something, I know I'm doing something wrong, so I need to go back, and I need to think it through again or come up with a different strategy or a way of showing them to do the problem. You know, if it's a kid that is consistently off task and playing around or something, then I might just kind of think that, "Well, they're not paying attention," so, it's just kind of like what the majority of the class is doing, and I kind of judge off that.
Unproductive Student performance (e.g., failure, success, engagement, interest) is attributed to an inherent property of students (e.g., students are lazy) and/or as produced in relation to something other than instructional and/or schooling opportunities (e.g., parents don't value education, therefore students don't). Explanation presents students' mathematical capabilities as relatively stable (i.e., they are not likely to change).	Interviewer: So what are some of the major challenges . . . of teaching mathematics in this school? Teacher: The kids already don't want to learn math. They have this notion of not caring for it and usually it's instilled by their parents 'cause their parents didn't get it, so they think its okay that they didn't get it.

Notes

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1. One hundred eighty-five of 460 teacher observations are missing a code for explanations of difficulty (which results in a primary sample of 275 teacher observations). The instructional quality of the subsample with a code for explanations of difficulty is not significantly different from the instructional quality of the full sample.

2. We recognize that the percentage of students in a school who are eligible for free or reduced-price lunch is not an ideal proxy for poverty (Bryk, Sebring, Allensworth, Luppesco, & Easton, 2010), but it is the best proxy made available to us by the school districts.

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