Examining Relations between Teachers' Diagnoses of Sources of Students' Difficulty in Mathematics and Students' Opportunities to Learn

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We present a quantitative analysis of the relation between a large sample of middle-grades mathematics teachers' views of students' mathematical capabilities and the quality of learning opportunities they provide for their students across large, urban U.S. districts pursuing reform. Specifically, we examine the relation between teachers' diagnoses of sources of students' difficulty in mathematics and the distribution and quality of students' mathematical discourse. Our findings suggest that the relation is stronger in classrooms with higher percentages of students of color or students identified as limited in English proficiency.

Fostering rich classroom mathematical activity and discourse requires fundamentally different practices on the part of both teachers and students than are typical in mathematics classrooms. For teachers, in addition to developing the mathematical knowledge for teaching and teaching skills to enact such forms of practice, it likely requires the development of particular stances regarding what students are able to do (Lampert, 2001). For example, if teachers view their students as incapable of explicating their mathematical reasoning, it is unlikely that they will press or support students to explain their reasoning in classroom instruction. Several small-scale qualitative studies support this supposition (e.g., Diamond, Randolph, & Spillane, 2004; Horn, 2007; Jackson, 2009), but we know of no study that has investigated on a large scale how teachers' views of their students' mathematical capabilities might relate to the quality of learning opportunities they provide for their students.

In this paper, we present a quantitative analysis of the relation between a large sample of middle-grades mathematics teachers' views of their students' mathematical capabilities (specifically, their diagnoses of sources of students' difficulty) and the quality of learning opportunities they provide for their students across large, urban districts in the United States pursuing reform. Of particular interest was the extent to which that relation varies, depending on the classroom composition of students they teach.

Framing Ideas

Teachers' Views of Their Students' Mathematical Capabilities

In recent years, with our colleagues on the Middle-School Mathematics and the Institutional Setting of Teaching (MIST) project (Cobb & Jackson, 2011), we have worked to understand what it takes to improve the quality of middle-grades mathematics teaching at the scale of large, urban districts. One focus has been teachers' views of their students' mathematical capabilities. In our investigation, we were particularly influenced by empirical work that suggested that how teachers framed (Goffman, 1974) the issue of student difficulty in mathematics would be a potentially useful way to get at what they viewed their students as capable of. In general, framing refers to how a particular certain situation is understood or interpreted (e.g., Benford & Snow, 2000; Goffman, 1974). It offers a particular representation of a problem, and as such, suggests particular solutions as possible (and occludes others). Sociologists attend to at least two "framing tasks" when analyzing framing processes—diagnostic framing and prognostic framing (Benford & Snow, 2000). The former involves articulating sources of the problem, whereas the latter involves identifying potential solutions, but with the two tasks being "intertwined, in that prognostic framing often rests implicitly on the problem definition and attribution that is part of diagnostic framing" (Coburn, 2006, p. 357).

In the MIST project, we developed an interview-based assessment to elicit information regarding these two framing dimensions, which as Jackson, Gibbons, and Dunlap (under review) argued, provide considerable insight into teachers' views of their students' mathematical capabilities. In what follows, we describe in more detail what we mean by teachers' diagnostic framing regarding sources of students' difficulty in mathematics, as that is the focus of this particular paper.

Teachers' Diagnoses of Sources of Students' Difficulty in Mathematics

Building on the work of Horn (2007), we make the following distinction when assessing how teachers diagnose students' difficulty in mathematics: does the teacher describe student difficulty in terms of inherent traits of the child or due to factors outside of instruction (e.g., families, community), or does the teacher frame a problem of student difficulty in relation to the nature of instruction, or learning opportunities? We have termed the former unproductive diagnoses of student difficulty and the latter productive diagnoses to signal that if teachers frame sources of student difficulty as outside instruction, it is unlikely that they will act to examine or alter current instruction to support students facing difficulty, but if teachers frame sources of student difficulty as in relation to what happens instructionally, we conjecture that they would be amenable to examining, and even altering instruction.

Related Constructs

There exist several related, but distinct, ideas that aim to account for teachers' views of their own capability or responsibility in supporting all students' learning. Measures of *self-efficacy* generally refer to a teacher's belief that s/he "can influence learning" (Sosa & Gomez, 2012, p. 879), whereas measures of *responsibility* "[focus] more on the teachers' willingness to take responsibility for helping all students learn rather than on teachers' beliefs about their professional effectiveness" (Halvorsen et al., 2009, p. 183). However, very few studies have examined relations between teachers' self-efficacy or responsibility and the quality of mathematics classroom instruction. Those that have investigated such relations have generally been qualitative, small-scale studies (e.g., Diamond et al., 2004; Sosa & Gomez, 2012). In general, studies of teachers' self-efficacy and responsibility have found that "achievement gains are significantly higher in schools where teachers take responsibility for students' academic success or failure rather than blaming students for their own failure" (Lee & Smith, 1996, p. 103), but that such responsibility tends to be higher in schools serving greater percentages of White and economically advantaged students, as compared to schools serving more racial minority students and students from lower-income backgrounds (Diamond et al., 2004; Lee & Smith, 1996).

We located no large-scale studies focused on the relations between teachers' self-efficacy or responsibility and teachers' instructional practice. However, based on their observations of classroom instruction, Diamond et al. (2004) found that teachers they described as having low responsibility also tended to enact instruction that communicated low expectations for students. For example, in a school characterized by teachers' low sense of responsibility, they observed that a fifth grade mathematics teacher assigned tasks that focused solely on developing procedural facility, and "spoon[fed the students] correct answers," and that teachers in schools characterized by low responsibility were, in general, "reluctant to try 'new things' because they feared that students would not be able to handle more innovative practices" (p. 88).

Equity in Opportunities to Learn

The literature referenced above suggests a notable pattern: there is evidence that teachers view students from historically under-served populations in less productive terms than students from historically advantaged populations. Further, there is at least small-scale evidence that such views matter for the quality of learning opportunities that teachers then provide for their students. This evidence indicates that a reason why teachers might not enact high-quality instructional practices in classrooms is because they do not view their students as capable of engaging in such activity.

These findings map onto current discourses about students' performance in mathematics more generally. As scholars such as Flores (2007) and Martin (2009) have argued, the dominant way of understanding disparity in performance on mathematics assessments between historically under-served populations and historically advantaged populations is through a lens of the "achievement gap." This way of understanding student performance often ends up, explicitly and implicitly, as casting historically under-served populations as the problem in need of fixing (Martin, 2009); further, rarely are the root causes of the "achievement gap" discussed (Flores, 2007). An alternative way of understanding such disparity in performance is as an "opportunity gap" (Flores, 2007). An opportunity gap perspective highlights that current disparities in achievement are the product of long-standing structural inequities, such as access to highly qualified teachers, resources, and so forth (Darling-Hammond, 2007; Flores, 2007). In other words, from this perspective, the "problem" does not rest with the individual students or the communities they come from, but with the opportunities that have (or have not) been provided to students.

In this analysis, we are concerned with both the quality of the learning opportunities teachers provide to students and the distribution of those opportunities. In light of the existing literature, we conjectured that teachers' diagnoses of the problem of student difficulty in mathematics (as due to individual traits of students or their communities, or as in relation to the opportunities provided to learn in the classroom) may differentially relate to the quality and distribution of learning opportunities provided to their students, depending on the composition of students in the classroom.

Study Context

The data we analyzed were collected in the first four years of the MIST project. The research team collaborated with the leaders of four large, urban school districts located in three states. The four collaborating school districts were typical of large, urban districts in the U.S. in that they had limited resources, large numbers of historically under-served populations of students, high teacher turnover, and disparities among subgroups of students in their performance on state standardized tests (Darling-Hammond, 2007). The districts were atypical, however, in their response to high-stakes accountability pressures. They responded by focusing primarily on improving the quality of instruction rather than on focusing exclusively on raising student test scores. Namely, each district was attempting to achieve a vision of mathematics instruction in which all students would have regular opportunities to collaboratively make sense of and solve challenging mathematical tasks, and, in discussing their solutions, develop robust understandings of key mathematical ideas. Three of the four districts (which we will call Districts A, B, and D) adopted a popular U.S. reform textbook as their primary textbook resource, and the fourth (District C) encouraged teachers to at least supplement their more typical series with that same reform text. Additionally, each district had initiated comprehensive professional development plans intended to support teachers in improving their instruction (e.g., curriculum frameworks, coaching, regularly scheduled time to collaborate with colleagues on issues of instruction, and/or professional development for instructional leaders.)

In each of the four districts, the research team and district leaders selected 6 to 10 middle-grades schools that reflected variation in student performance and in capacity for improvement in the quality of instruction across the district. Within each school, up to 5 mathematics teachers were selected to participate in the study, for a total of approximately 30 teachers per district. The schools remained constant throughout the study, but, as is typical, some of the teachers changed schools or roles during study. In each case, we recruited replacements in order to maintain a representative and consistently sized sample.

The collaborating districts provided settings in which we could investigate our relations of interest within the context of four, distinct, at-scale reform efforts. Specifically, our research questions were: 1) How are teachers' diagnoses of sources of students' difficulty related to the distribution and quality of students' mathematical discourse? and 2) Does the relation between teachers' diagnoses of sources of students' difficulty and classroom discourse vary depending on student-level characteristics of the classroom?

Methods

Sample and Primary Measures

Our primary analytic sample included 165 middle-school mathematics teachers pooled over the four years of the study with multiple years of data for some of the teachers, resulting in a total of 275 (statistical) observations (9 teachers with 4 years of data, 20 teachers with 3 years, 43 teachers with 2 years, and 93 teachers with 1 year of data). Because we do not have information about student-level characteristics for some of those teachers, we use a reduced sample of 156 teachers with 238 observations to answer the second research question. In each of the four years of the study (2007-2011), we collected several types of data to test and refine a set of hypotheses and conjectures about district and school organizational arrangements, social relations, and material resources that might support mathematics teachers' development of high-quality instructional practices at scale. We drew on a number of these data sources in this analysis, including video-recordings of teachers' classroom instruction; interviews with teachers; and a written assessment of teachers' mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004). Additionally, our analyses included student demographic information collected and provided by the districts. The number of students served in the four districts ranged from approximately 35,000 to 160,000 students. On average, 29% of the students were White, 33% of the students were Black, and 36% of the students were Hispanic. Approximately 20% of the students were classified as limited English proficient, and 68% of the students were eligible for free or reduced price lunch.

The two outcome variables in our analyses are measures of the distribution and quality of students' mathematical discourse. In each year of the project, teachers' instruction in two consecutive lessons with the same class was video-recorded mid-year. Each lesson was scored using the Instructional Quality Assessment (IQA; Boston, 2012), which, with its 8 rubrics, aims to assess the "academic rigor" and the quality and distribution of Accountable Talk® in instruction. For our outcome variables, we employed the three Accountable Talk® indices that focus on students' contributions to and participation in classroom discourse: *student providing* accounts of his/her reasoning, *student linking* to and building on each other's ideas, and the percentage of student *participation* in class discussion. With respect to the first two, a score of 0 represents no whole-class discussion, a score of 1 represents no student providing or linking, a score of 2 represents providing procedural accounts of reasoning (e.g., describing steps taken or calculations performed) or superficial linking, a score of 3 represents a few strong efforts to provide conceptual accounts of reasoning or link ideas, and a score of 4 represents consistent strong efforts to provide conceptual reasoning or link ideas. For participation, a score of 0 represents no whole class discussion, 1 represents at most 25% of the students participating, 2 represents 26-50% of students participating, 3 represents 51-75% of students participating, and 4 represents 76-100% of students participating. Because we were interested in the distribution and quality of students' mathematical discourse, we used the student providing and student linking scores as two indicators of quality and weighted each by the participation score, which accounted for the distribution of students' discourse. We divided each product by 4 to match the usual IQA scale.

The key independent variable of interest in our analyses was interview-based assessments of teachers' diagnoses of sources of students' difficulty in mathematics (hereafter "diagnoses"). During annual interviews, we asked questions like, "When your students don't learn as expected, what do you find are typically the reasons?" We also asked teachers to describe the challenges they face, which often provided insight into how they framed student difficulty. All interviews were transcribed and coded at the unit of a turn of talk and any relevant text we used to make sense of that particular talk turn. For each relevant passage, coders assigned a code of "productive," "unproductive," or "mixed" (wavering between the two previous kinds of explanations). In order to use these categorizations in our quantitative analyses, we assigned numerical values to each interview based on our coding. If all coded passages were categorized as unproductive, the interview was assigned a value of 0. If all coded passages were considered productive, the interview was assigned a 2. A score of 1 was assigned if either all coded passages were categorized as mixed, or if there was a combination of productive and unproductive passages.

In our models, we also included student demographics and prior achievement, and measures of (a) *cognitive demand of classroom activity* (average of IQA scores for the potential and implementation of mathematical tasks, based on the mathematical task framework of Stein, Grover and Henningsen, 1996); and teachers' (b) *mathematical knowledge for teaching* (scores from a pencil-and-paper instrument, Hill et al., 2004); (c) *visions of high-quality mathematics instruction* (an interview-based assessment of the sophistication of teachers' instructional vision across key classroom dimensions: role of the teacher, mathematical tasks, student engagement in classroom activity, and discourse, with scores generally ranging from 0 to 4, and directionality roughly mirroring that of the IQA; Munter, 2014); and (d) *years of experience teaching*.

Analyses

To answer our first research question, we employed a series of linear regression models to investigate how teachers' diagnoses are related to each of the classroom discourse outcomes described previously (student providing and student linking), each weighted by participation. In each model, we controlled for other factors that could explain the relation between teachers' diagnoses and opportunities to learn, including teachers' mathematical knowledge for teaching; instructional vision; years of experience; and district membership. We constructed multi-level models to account for the nested nature of our data: observations within teachers, within schools.

To answer our second research question, we added information about student-level characteristics of the classroom, and investigated statistical interactions between those characteristics and teachers' diagnoses. For each of the two outcomes, we estimated four different models including interactions between four different student-level characteristics and teachers' diagnoses: 1) students' prior mathematics achievement; 2) the percentage of students in a school eligible for free or reduced price lunch; 3) the percentage of students in a class classified as limited English proficient; and 4) the percentage of students of color in the class.

Results

Across the two models of the relation between teachers' diagnoses and distribution and quality of students' discourse, results suggest that on average, for this sample of teachers, teachers' diagnoses are significantly related to the distribution and quality of student providing (b=.315, p<.05) but not to student linking (b=0.083, p=0.58). In particular, student providing of reasoning was about a third of a standard deviation higher in classrooms of teachers who articulated productive diagnoses than in classrooms of teachers who articulated unproductive diagnoses. Of the control variables included, only the cognitive demand of the classroom activity was significantly related to either outcome. Teachers who chose and implemented tasks with higher cognitive demand tended to have better distribution and quality of students' mathematical discourse.

The parallel series of models testing for statistical interactions yield a number of interesting results. For student providing, we found a significant interaction between mean prior student mathematics achievement in the class and teachers' diagnoses (b=-.2999, p<.05), suggesting that the relation between teachers' diagnoses of sources of students' difficulty in mathematics and student providing varies based on the prior achievement of students in the class. In general, teachers' diagnoses are not related to differences in the distribution and quality of student providing when they teach in classrooms in which most students have previously been successful on standardized mathematics assessments. However, we did not find a similar interaction with respect to student linking (b=-.211, p=.142).

With respect to both student providing and student linking, results suggest that there is not a significant interaction between teachers' diagnoses and the percentage of students eligible for free or reduced price lunch, but that there is variation in the relation between teachers' diagnoses and outcomes based on the percentage of students classified as limited English proficient and the percentage of students of color. With respect to the first outcome, this suggests that perhaps what manifested as variation in the relation between teachers' diagnoses and student providing based on prior student achievement may have actually been attributable to these other student characteristics. For both of these characteristics, as the percentage of students in the subgroup increases, so too does the relation between teachers' diagnoses and student providing (%LEP: b=.360, p<.05; %SoC: b=.214, p<.05) and between teachers' diagnoses and student linking (%LEP: b=.390, p<.05; %SoC: b=.179, p<.05).

For example, at the mean percentage of limited English proficient students in the class (12.7%), teachers' diagnoses are not significantly related to student providing, but at 1 standard deviation above the mean (27.5%) diagnoses are significantly related to student providing. Specifically, the results suggest that, on average, in classrooms with 27.5% of their students classified as limited English proficient, student providing is just over a third of a standard deviation better for teachers who articulated productive diagnoses than for teachers who articulated unproductive diagnoses. Also, at the mean percentage of students of color in the class (78.4%), teachers' diagnoses are not significantly related to student providing, but at half a standard deviation above the mean (90.7%), teachers' diagnoses are significantly related to student providing. This finding suggests that, on average, in classrooms with 90.7% of students of color, student providing is nearly a fifth of a standard deviation better for teachers who articulated productive diagnoses than for teachers who articulated unproductive diagnoses. More generally, we find that the relation between teachers' diagnoses and both student providing and linking is stronger in classrooms with higher percentages of historically under-served students.

Discussion

Our findings suggest that whether there are more or less equitable opportunities to engage in rich discourse in mathematics classrooms is related to teachers' diagnoses of sources of students' difficulty in mathematics. On average, students in our sample were more likely to participate in discussions in which they and their peers provided their reasoning and made connections between strategies if their teacher diagnosed sources of student difficulty in mathematics as related to the nature of instruction or learning opportunities, rather than in terms of inherent traits of the students or due to factors outside of instruction (e.g., families, community). We view this finding as providing an important insight into the factors associated with teachers' enactment of practices that afford important learning opportunities to all of their students, and as large scale confirmation of what previous, smaller scale studies have asserted that teachers' views of their students' mathematical capabilities are related to the quality of learning opportunities they afford their students. It also provides potential insights into how teachers act on their sense of responsibility (Halvorsen et al., 2009) in terms of the nature of discourse that they foster through instruction.

But more than this, the results of our statistical interaction analyses suggest that the strength of this relation depends on the composition of students in the classroom with respect to race, ethnicity and/or language status. For example, students in classes composed (almost) entirely of students of color were more likely to have opportunities to participate in discussions in which students provided reasoning for their solutions if their teacher articulated productive diagnoses of sources of their difficulty. In other words, without White students in the room, teachers' views of their students' mathematical capabilities—whether productive or unproductive—were, on average, more likely to be reflected in their instructional practice.

Given that the interactions that we observed between teachers' diagnoses and both percentage of students with limited English proficiency and percentage of students of color did not hold for percentage of students who were eligible for free or reduced-price lunches, these findings imply that teachers' views of students' mathematical capabilities may be more influenced by students' race or cultural background than by their economic status or even past achievement. To the extent that this is the case, it may be that the more "visible" characteristics of race and limited English proficiency, as compared to social class and prior achievement, are more likely to trigger the kind of low responsibility/low expectations-oriented instruction identified by Diamond et al. (2004).

Our findings suggest that there is clearly much work to be done in shifting how teachers frame the problem of students' difficulty in mathematics, particularly when serving historically disadvantaged groups of students. How can teachers be supported in developing more productive views of students' mathematical capabilities and translating new diagnostic frames into productive prognostic approaches in instructional practice? Our sense is that answering those questions will require further investigation of teachers' cultural and racial competence (Milner, 2003) in discipline-specific contexts, and a better understanding of and support for teachers in building relationships with students that extend beyond knowing them from a cognitive standpoint, potentially including integrating current models of instructional practice with the affective, relational, and emotional aspects of classroom interactions.

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