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Journal Title: Learning technologies and the body: integration and implementation in formal and informal learning environments / edited by Victor R. Lee.

Volume: Issue: 2014 Pages: 112-131

Article Author: Hall, R

Article Title: Rescaling Bodies in/as Representational Instruments in GPS Drawing


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6 Rescaling Bodies in/as Representational Instruments in GPS Drawing

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INTRODUCTION

Geometry as a topic in school is typically understood as a generative relation between the symbolic or graphical description of ideal objects (i.e., what can be drawn, written, or proved through derivation, typically on paper or in a computer interface) and physical realization of models of those objects at a grasable scale (i.e., physical construction and manipulation of 2-D and 3-D objects). In this chapter, we describe work to disrupt the scale and modality of school geometry by using location-aware technologies (i.e., wearable global positioning system [GPS] devices and mapping software). We present three cases in which students in different learning settings were asked to imagine, draw, and ‘walk through’ typical geometric constructions or problems while wearing GPS devices. These GPS units recorded their locations in space as they walked in their planned paths. Their tracks were then uploaded into mapping software so that they could be viewed superimposed on maps and satellite imagery of the areas in which they had walked, in effect producing/collecting drawings on the landscape that could later be analyzed as mapped figures. We call this activity GPS drawing, following artist and cartographer Jeremy Wood (www.gpsdrawing.com; O’Rourke, 2013), whom many credit with the development and popularization of using global positioning devices in this way.

Making and transforming familiar geometric objects at walking scale was part of a series of design experiments (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) in which we deliberately disrupted familiar construction techniques at paper scale (e.g., using compass, straightedge, and pencil), by introducing a new form of drawing (collecting track points with a handheld GPS device) and asking study participants to develop effective drawing/construction techniques as a group activity. Our chapter presents three cases as empirical material for making sense of how changes in the scale and modality of GPS drawing supported novel coordinations of time, perspective, and semiotic and material resources. We do not treat the cases as samples for making inferences about a population of similar groups, working on similar tasks. That might be valuable but would require a
different study design. More specifically, the GPS drawing tasks were part of a design experiment in which making and transforming geometric figures at walking scale disrupted conventional practices of paper-scale geometric construction and demonstration (Hall, Stevens, & Torralba, 2002; Ma, 2012). Our disruption changed the scale of the activity in space (from paper to field) and social participation (from solo to group construction), removed familiar paper-based construction tools (e.g., straightedge, compass, and pencil), and required use of a new drawing technology (GPS device and online mapping engine). We expected participants would struggle with walking geometry, and in their use of the new tool of GPS drawing, we would be able to analyze how they developed a new understanding of how geometric figures could be constructed at walking scale, how their transformation could be accomplished with enough precision that others could recognize the figure, and how these new practices preserved mathematical properties of geometric figures. This deliberate disruption allowed our analysis to follow the development of a new activity (walking-scale geometry using GPS drawing) and explore how students understood this new activity in mathematical terms.

For example, in our second case, a group of students in a mathematics course for pre-service mathematics teachers, who were working on transformations of geometric objects, created a treble clef (a figure in musical notation) in an open field. They used rope, place markers, and carefully choreographed movements while walking/drawing on the field with a handheld GPS device. These students ‘drew’ geometric transformations of the original clef by changing location, resetting place markers, and walking through copies of the clef that had been either rotated, expanded (a dilation), or displaced into another area of the field (translation). Later, while explaining how they were able to make spatially precise copies of this walking-scale clef to illustrate geometric transformations, one member of the group reported that she had used (and taught other members of her group) choreographing strategies familiar from her participation in a marching band. Thus, changing the scale and modality of a standard school mathematics topic (transformation of geometric figures) both created new learning opportunities for these pre-service teachers and provided evidence for interplay between distinctly different spatial practices (geometric construction, marching choreography).

In this chapter, we focus on changes in the scale and modality of mathematical activity as a historical disruption to relatively stable, cultural understandings of geometric shapes and their properties. We analyze multi-party responses to these disruptions as a process of learning and cultural invention that re-places the body in genuinely novel, mathematical activities. If concepts are bundles of social and technical practice that develop over historical time (Hall & Greeneo, 2008), learners’ bodies are positioned or placed in these practices just as are technologies. Changing the scale and modality of these conceptual practices invites learners to create and stabilize new forms of embodied mathematical activity. GPS drawing re-places the body
because a new division of labor is required in working with a concept and
new demands are placed on talk in coordination with the activity and the
participation structure of the activity itself (e.g., different bodies were used
differently to mark places, to make traces over the surface of the earth, and
to perform/monitor changing parameters of construction). Bodies are also
located in new ways, creating mobile and diverse perspectives on geometric
objects, as well as new capacities to feel and touch aspects of mathematics.

Elaborating on his classic experimental studies on the sense of touch,
Katz (1989) comments, "... for the sense of touch there are no such
things as a magnifying glass or microscope" (p. 60). As opposed to vision,
which enables us to be inside small things by magnification, or outside
large things by contraction, touch and kinesthesia pose an inherent 'body
scale' required for tactile availability. This further motivates our interest
in walking-scale geometry, since activity at paper scale makes weak
provision for experience inside of a geometric figure. GPS drawing makes
it possible to experiment bodily with mathematical entities by including
inside perspectives, different from the 'bird's-eye view' that dominates
paper-scale diagramming. What can be seen or felt from within opens up
new possibilities for interaction between participants who take different
roles and stances in making and transforming figures. Walking-scale
geometry (WSG) with GPS drawing repositions the knower of geometry
within the shape in a constitutive way. The coordinated interaction of
whole (and many) bodies, novel tools, and talk invite a different lived
experience of the geometry of shapes, their properties, and relations. As
we will show, these new experiences do not replace but are interleaved
with paper-scale experiences of geometry.

METHODS FOR MAKING AND STUDYING WALKING-SCALE
GEOMETRY WITH GPS DRAWING

In each case, we organized WSG with GPS drawing as a form of experi-
mental teaching that contrasted with paper-scale, school geometry. We typi-
cally started with a presentation of the idea that GPS devices use satellite
signals to locate places on the earth with variable precision. We then shared
examples of how these devices could be used to track moving objects (dogs,
cars, or people) and, by treating tracks as a way of marking the surface
of the earth, to create large-scale drawings or writing. After demonstrat-
ing how to operate the GPS devices and load track data into a mapping
game engine, we posed a walking-scale drawing or geometric task. As our designs
for experimental teaching were refined, we also provided bags with rope,
stakes, pieces of wood, cardboard, and tape. We did not tell study partici-
pants how to use these things, but rather thought of them as materials that
might be used to invent new tools for walking-scale geometry. In this sense,
our experimental teaching was organized as a form of double stimulation
GPS Drawing

(Engestrom, 2007; Vygotsky, 1978) in which a geometric task at walking scale was the first stimulus (a deliberate disruption to paper-scale, school geometry), the tools and materials were a second stimulus (materials for invention), and the techniques and tools created by study participants were a genuinely novel cultural activity.

The cases presented in this chapter were treated as iterations in an ongoing design experiment. We describe the types of data collected and our methods of analysis briefly. Since we hoped to follow the development of novel cultural activity, we gathered several types of data over time. These included written field notes during design and teaching activities, digital records of constructive activity (e.g., track data sets from GPS devices, digital photographs of constructions in progress, and drawings rendered as maps in Internet mapping engines), video and audio recordings of activity in classroom spaces and on the field, and video records of semi-structured interviews during and after instructional activities. Recorded materials were indexed together in time, with coarse-grained content logging of the contents of records of talk and activity. As our analysis of these materials developed, we selected for closer analysis what appeared to us as critical moments in the developing activity of walking-scale geometry with GPS drawing. For example, we transcribed and analyzed closely moments in which ways of making the 'same' or 'similar' shapes were invented, using materials we provided to study participants (e.g., knotting a rope to scale or dilate a shape). Our analysis was inductive, using methods of video-based interaction analysis (Derry et al., 2010) to follow and make inferences about how study participants solved the problems presented by our tasks and understood the techniques they invented in mathematical terms. A full presentation of these design studies is beyond the scope of this chapter (see Ma, 2012, for a detailed presentation of one design study).

In the following we present cases of GPS drawing from three different study sites. The first case, "Walking Awesome," describes a group of seventh graders planning and implementing a GPS drawing. We demonstrate the coordination of bodies and various perspectives on designing and making a figure over time. In the second case, "Walking a Clef," students reenacted their GPS drawing for the research team, then explained their walking-scale strategy using paper and pencil. Our analysis of the case provides insight into how the disruptions of GPS drawing may invite aspects of spatial practices from different settings as resources for developing new strategies for drawing, and how the walking-scale experiences are brought back into the classroom and incorporated into paper-scale drawing. The third case, "Animated Drawings at Walking Scale," investigates how a group of high school students drew multiple instances of a circle across a long stretch of lawn to be assembled later to create a bouncing ball animation. The case highlights students’ multiple perspectives on the figures as they drew each one, but also how they kept track of the (invisible to them during construction) relations between past and future iterations of the shapes used in the animation, through coordinated forms of bodily and social engagements with the materials and each other.
These cases were chosen because they highlight different aspects of the disruption in scale and modality, how bodies together with location-aware technologies coordinate over space and time as newly developing representational systems, and students’ novel mathematical engagements. The goals and theoretical orientation motivating us to explore and disrupt how bodies engage with representational systems overlap with other work described in this volume (e.g., Enyedy & Danish, this volume).

Case 1: Walking Awesome

Our first case presents how a group of students coordinated their joint activity in planning and implementing the movement of a human stylus to create a drawing on the school soccer field. The case took place in a seventh-grade mathematics class in a struggling (by state testing standards) public middle school in the Southeast. The GPS drawing task was part of a larger study of disruptions to the scale and tools of typical classroom mathematics where these students engaged in a variety of geometry tasks outside at large scale. Before the events of this case, the students had already constructed and transformed geometric figures at walking scale using their bodies and everyday materials (Ma, 2012; see Chapter 5), so they had some experience making geometric figures at large scale. The episode presented here was from the class’s first attempt at GPS drawing. After being introduced to GPS drawing, the students were asked to draw a figure of their choosing on the school’s soccer field. The focal students in this case, Ben, Dean, Eddy, and Harry, decided to write the word ‘Awesome.’ This was a nickname given to Eddy, one of the school’s star soccer players.

During the planning phase in the classroom, the group was given a generic outline of a soccer field on which to sketch their drawing and plan their walk. They gathered around Dean’s desk, all leaning in toward Eddy’s writing. After he wrote the word ‘Awesome’ once on the planning sheet (Figure 6.1a), he began to trace over it, the pencil emulating his walking path, narrating as he traced (“Like that, then up, like that . . .”). As he did so, Dean placed his finger over the pencil tip, tracing along with him (Figure 6.1b). Occasionally Eddy would stop, and Dean would continue, suggesting a path for the next few curves. In anticipation of the GPS drawing activity, this writing/drawing event had already begun to take on new forms, including a new division of labor. The students were all intently focused on Eddy’s retracing of his drawing, attending to the development of the stroke rather than the shape of the figure itself. Two different student hands took the position of stylus, one using a pencil tip and the other his finger tip to trace the inscription. The group was anticipating the work of creating the figure and walking a continuous path that traced the word ‘Awesome’ (Figure 6.1g shows a trace of Eddy’s walk, along with images of Eddy at various moments during the walk).

After the teacher, Ms. N, looked at their plan and approved it, they practiced it in front of the classroom, a smaller space than the soccer field. Eddy walked in small shuffling steps. Dean followed closely behind, with his hands
on Eddy's shoulders, guiding and turning his body (Figure 6.1c). At the 'w,' Eddy stopped and demonstrated, turning his whole body back and forth, that he had to stop and make sharp turns. As Eddy continued on with the first 'e' without Dean, Harry took Eddy's shoulders and turned him, then released him and walked the 's' alongside him. Eddy then finished walking the word with the other three looking on. When he finished, Dean walked the word himself (Figure 6.1d), although he would not be walking it out on the field. As he walked, he used his left index finger to point ahead of him in anticipation of where he would go next.

In this practice run Dean and Harry both physically directed Eddy and walked at least a part of the word. They engaged in the drawing both by guiding Eddy (as one would guide a person-sized pencil) and by standing in

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**Figure 6.1** Ben, Dean, and Eddy plan, practice, and draw 'Awesome.' The group drew the word on the soccer field outline (a), then rehearsed the tracing of the word at paper scale (b). Eddy (c) and then Dean (d) practiced walking at the front of the classroom. Out on the field Eddy walked the word while Harry and Ben watched, followed, and directed from below on the field (e). Dean watched from above (f). Figure 6.1g is a composite image of Eddy at various points throughout his walk, and his entire path is traced to be made visible.
for him. The students all participated in the drawing, with Eddy as stylus and the others coordinating their bodies and gazes to direct and experience the ongoing representation.

Out on the field, Dean told the group and Ms. N that he would watch from the hill adjacent to the field and give directions (Figures 6.1e–f show the view from down below on the soccer field, and then from Dean’s perspective). The rest of the group headed down, and positioned themselves approximately where the beginning of their plan indicated. Harry carried the plan. Eddy, carrying a backpack and the GPS device, walked, while the others, including Ms. N, looked on and made occasional comments. The following excerpts provide samples of how the students accomplished walking ‘Awesome’ on the field.1

1. ((Eddy starts. He walks part of the length of the first leg of the ‘A,’ and then looks up at Dean, while continuing to walk. 3s.))
2. Dean: Start to curve! ((Eddy turns his head back to watch his walking path, beginning a wide curve.))
3. Harry: Turn now!
4. ((Eddy turns and walks down the second side of the ‘A.’ 11s.))

Dean, from his bird’s-eye perspective, had a better sense of what Eddy’s developing and already-drawn invisible trace looked like. For Eddy there was little in the visual field to help keep track of parts of the trace. He was inside the figure, and there was little to help him differentiate the parts of his drawing surface. Eddy’s glance up the hill (Line 1) positioned Dean to provide feedback on his walking path. He walked, continuing to look at Dean, until he was instructed to “Start to curve!” (Line 2). As soon as Dean said this, Eddy returned his gaze directly in front of him, indicating that he had heard what he was waiting for. Harry also reiterated the direction to “Turn now” (Line 3), as Eddy began to turn his body. The three students were beginning to negotiate their roles in the representational system.

5. Harry: ((As Eddy gets just past halfway down the second side of the ‘A.’)) Don’t forget about the - [cross in the ‘A.’]
6. Eddy: [I am, I'm [starting it at the bottom.
7. Dean: [Alright (0.5s), right there’s good.
8. ((Eddy stops just as Dean says “right,” and turns so that he will walk diagonally up toward the first side of the ‘A.’))

Harry showed he was monitoring Eddy’s progress here, although his reminder about the “cross in the ‘A’” (Line 5) demonstrated his lack of familiarity with the planned path, where Eddy was to turn up from the bottom of the second side of the ‘A’ to make the horizontal stroke. Dean looked on, calling out to Eddy when the length of the second side of the ‘A’ was the same as the first (Line 7).
9. Eddy: *(in a singsong voice)* (I'm gonna walk a little 'A.')
10. Dean: *(Eddy returns to where the first side of the 'A' would be. 8s.)*
    Right there.
11. *(Eddy stops and turns to make the horizontal segment of the 'A.)*
12. Dean: Go forward.
13. *(Eddy walks forward. 5s.)*
14. Dean: That's where the 'A' ends.
15. *(Eddy takes a few more steps, and then stops and turns back. 3s.)*
16. Eddy: Right here, right?
18. Dean: Yeah.

As Eddy walked back to the first side of the 'A,' Dean let him know where his current path intersected with that past, invisible stroke (Line 10). As Eddy walked, Dean let him know "where the 'A' ends" (Line 14). Eddy continued to walk, as dictated by the plan, until he got to a place where he thought the 'w' should begin (Line 15). He turned up to Dean and asked, "Right here, right?" (Line 16), and both Harry and Dean confirmed.

19. *(Eddy turns to make the 'w.' They are quiet, until he starts making the 's.' 40s.)*
20. Harry: You're on the 's!' *(They all laugh)* Just making sure he knows where he is *(to no one in particular).*
21. *(Eddy continues to walk. Harry and Ben start to walk toward him. He starts the first vertical of the 'm.' 19s.)*
22. Dean: Alright! *(Eddy turns to double back to the top of the 'm.)*
23. Harry: Are you on the 'e' now?
24. Eddy: No, (I'm doing the 'm. ')
25. Harry: ['m']?
26. Ms. N: *(Eddy, when you get done just stop and I'll turn it off *(referring to the GPS device)).
23. *(Eddy continues walking. He gets to the end of the second 'e.' 26s.)*
25. Eddy: Can I do an exclamation point?

For the next few letters, the group was quiet as Eddy walked. The steps he took were small and the turns tight (Line 19). It is possible that these intricate moves were difficult to follow, or that they felt that their guidance was unnecessary since the scale of these letters was so small. When Eddy started to make the 's,' however, Harry spoke up again, informing him "You're on the 's!'" (Line 20). The group laughed, and Harry perhaps realized the strangeness of this statement, and joked that he was "Just making sure he knows where he is." Harry's participation in the group's GPS drawing here did not seem to make any direct contributions to the technical
development of the representational system. However, he did demonstrate some capabilities to participate by echoing or anticipating Dean’s feedback, and by actively monitoring Eddy’s progress, although he was not always successful in doing so. Finally, Ms. N directed Eddy to stop when he reached the end of the last ‘e’ (Line 24). Eddy wanted to continue, and draw an exclamation point (Line 25), but Ms. N held him accountable to the plan, admonishing him to “Just leave it” (Line 26).

The group had developed some capacity for coordinating on- and above-field perspectives in directing a moving, human stylus. Dean, in a stationary position above the field, monitored and directed Eddy’s current progress in relation to the path already walked, now no longer visible except through memory. Harry and Ms. N, walking in the direction of the progression of the ‘Awesome,’ had an extrinsic, but up-close, changing view. They had more success in monitoring Eddy’s ongoing progress (Lines 5, 20, 26). In the next case students make use of bodies and materials on the field in a very different way, combining talk, spatial relations, and physical properties of their resources to develop a routine for walking, dilating, and translating a treble clef.

Case 2: Walking a Clef

The analytical focus of this “Walking a Clef” case is the return to paper-based diagramming after a group of undergraduate, teacher preparation students created a GPS drawing of a treble clef on a campus field. We show that their past experience with a large-scale drawing became fully incorporated into their drawing/talking on paper as they spontaneously came to use typical conventions of paper-drawn geometric diagrams to recount their activity. We suggest that the disruption created by large-scale drawing, far from being excluded in subsequent work at paper scale, enriched their work on paper and enabled them to develop complex temporalities and a symbiosis of notational conventions.

This activity took place in a required mathematics class at a public university, in which most of the students were pre-service high school mathematics teachers. The activity was designed to practice geometric transformations, these being a major aspect of the general subject of the course (complex functions). The students, divided in teams that would create and present GPS drawings for further discussion, were asked to generate three drawings on campus. The first one was any drawing they chose; the second and third drawings were to be the same shape transformed by combinations of scaling, rotation, or translation. Teams presented their drawings, and the class was asked to identify types and parameters of transformation (e.g., find the center of dilation).

Unknown to us at the outset, the team we focus on for this chapter included a student, Tracy, who was music major and trainer of marching bands. Tracy proposed to draw a musical clef and brought to class a long
rope of the kind she used for marching band practices. She showed her team-}
mates how to use the rope to measure and create elements of the musical
clef, and she explained to us in a debriefing interview that ropes were
a common tool in teaching and practice for competition marching bands.
Figure 6.2a shows the first clef. Figure 6.2b overlaps the second clef, rotated
and scaled up. Figure 6.2c includes the third clef scaled down and trans-
lated. Tracy stood at a fixed position for each curved section releasing or
pulling rope at variable rates, while Karen walked at the other end of the
rope and held the GPS device.

Two months later the team volunteered to be interviewed about their
GPS drawing experience. After the interview they went outdoors to dem-
onstrate the sequence of actions they had originally performed to draw
the clef. The intent of this demonstration was to complement the con-
versation, allowing for a fuller understanding on the part of the inter-
viewer. Figure 6.2d is a recreation of the path they walked during this

*Figure 6.2* Tracy and her team make GPS drawings of a musical clef: (a, b, c)
Google Map view of their original clef and two transformations; (d) team recreation
of the smallest clef during a later interview (Tracy at center of rotation, holding
rope), clef path recreated; (e) drawing and annotation of the clef at paper scale.
demonstration; the figure was obtained by composing the video sequence of images. The ensuing analysis focuses exclusively on the interview prior to the demonstration outdoors (Figure 6.2e).

The team and the interviewer went into a classroom and talked, as they drew and pointed on a piece of paper, about how they had created the clef as a GPS drawing at walking scale.

1. Tracy: So what we did was, we had two people, we only had two people, so I’m going to be called A (writes A) because I was holding the rope in the center. And then Kate is B (writes B next to A). We’re actually at the same spot.

Tracy started in past tense (“So, what we did was . . . ”) in reference to what they had done in the field, but immediately she shifted into future continuous time (“so I’m going to be called A . . . ”) on account of the fact that she was going to write the letter ‘A’ on the paper. This utterance merged the past activities in the field with the upcoming writing on the paper. In “and then Kate is B, we’re actually at the same spot,” Tracy developed a present ‘now,’ encompassing at once her writing of the letter ‘B’ and their being “at the same spot” in the field (for a related analysis of tense shifting and hybrid predication in the work of experimental physicists, see Ochs, Gonzales & Jacoby, 1996).

Then Tracy described the process they had followed to draw the first loop (Figure 6.2e), with her hands changing the gesturing space three times:

2. Tracy: ((pencil above point B)) She started moving in a clockwise direction, and as far away from me as she could get with the length of rope that I let out.

3. Tracy: ((hand gesturing above the table)) We marked the rope so that there were two equal sections, so that we could double so the dilation later. But I knew that when I got to the first mark, that was as far as she could go.

4. Tracy: ((pencil comes back to the point B)) So I watched it and only let her go a certain amount.

Line 2 was a description of what Kate “had started” (i.e., moving clockwise, going as far as Tracy let her by means of rope release). While holding the pencil right above ‘B,’ Tracy made slight movements on the tip of the pencil as if holding Kate’s “start,” giving herself time to describe the actions Kate was about to start. In Line 3 Tracy interrupted Kate’s start altogether to go further back in time, allowing herself to elaborate on what she had been planning to do (i.e., use the second knot for a subsequent tracing of the dilated clef, whereas for this initial clef she expected to release just enough rope to get to the first knot). The piece of paper that Tracy was using had become for her a site for the detailed reconstruction of their past
activities on the field. Since the preparations she now described occurred even before they went to the field, it makes sense that Tracy removed her hands from the paper and gestured out into the space proximal to her upper body. It was as if the space strictly 'on paper' and the one in front of her fulfilled different roles, the latter for digressions away from the actual tracing on the field. Line 4 was a return to the field, signaled by the pencil going back to 'B.'

5. Tracy: So she started walking away from me; she's walking basically north, I guess, and clockwise. So she's going to start getting farther and farther away from me.

Line 5 started in past tense ("So she started . . ."), and shifted to present tense ("she's walking away . . .") and then to future continuous ("she's going to start . . ."). Tracy appeared to constitute a 'now' right after the start of the curve drawing, a time and space that merged past action (GPS drawing of a curve) with a demonstration upcoming (i.e., after Kate started to walk but before the loop got on its way of being drawn). Although the act of drawing put Tracy in a 'bird's-eye view' perspective with respect to the sheet of paper, she described the emerging curve in terms of how Kate walked away from her. The tip of the pencil was Kate walking with the GPS (while holding the rope tight and so forth) as Tracy was imaginatively seeing Kate walking away from her. It was not only that the temporality of Tracy's clef drawing on paper became embedded in the temporality of her holding and releasing the rope in the field, but also that its spatiality overcame her 'view-from-above' toward seeing Kate getting farther and farther from her.

6. Rogers: And you're ((to Kate)) paying out line continuously, as best you can?
7. Kate: Right, and I'm pulling it tight.

Rogers' and Kate's words were uttered while Tracy completed drawing the first loop: Tracy's act of drawing constituted a present shared by the three of them, in which Tracy is releasing the rope and Kate is pulling it tight. On reaching a point where her curve crossed the center of the clef, Tracy stopped drawing the curve and traced a centerline segment between the Start and the Crossing points.

8. Tracy: So whatever the distance was that we ended up with, just call that X ((draws segment labeled 'X' between A and the endpoint of the first loop; see Figure 6.2e)). She would stand here at B prime ((labels the point B'; see Figure 6.2e)), and I would run all the way around to the other side on this line, X distance again ((moves her pencil, without drawing, a distance X, from B' and away from A)).
Tracy adopted the usual convention for an unknown (i.e., labeled X), which was a distance set by the length of rope paid out to Kate. Of course, the smooth curve was obtained through many contingencies related to the release of the rope over time. Once the rope distance, X, was used up in Kate’s rotating movement (i.e., the segment marked between A and B’), Tracy reused this rope segment (distance X) by running “all the way around” (Line 8) and establishing a new center for tracing a curve on the opposite side of B’. Kate waited with the GPS device until Tracy was set in the new center, and then Tracy controlled the rope as Kate completed a second curve, drawing the top loop of the musical clef (i.e., reaching the ‘top point’ in Figure 6.2d). This innovative use of a rope to measure and trace smooth curves, taught by Tracy to her teammates in the field, was merged with conventional ways of labeling paper-scale geometric constructions during the interview. This involved complex forms of tense shifting (e.g., creating the field and a ‘now’ of coordinated action on the table surface) and multiple points of view on the emerging geometric figure (e.g., Tracy measures and pays out rope, while Kate pulls tight and walks deliberately to create a GPS trace for the emerging clef).

Case 3: Animated Drawings at Walking Scale

In this section, we consider a case in which rising Grade 9 and 10 students in an interdisciplinary summer course on spatial thinking created an animated GPS drawing of a large ball (over 100 feet in diameter) bouncing ‘up and down’ over a college campus lawn, visible as a satellite map (plan view) in Google Maps. Like the middle school students described in our first case study, these students had already spent some time constructing geometric figures at walking scale. High school students in this case produced considerably more sophisticated GPS drawings (as well as figures in walking-scale geometry). Their efforts, as well as the clef drawing by pre-service secondary teachers in Case 2, suggest the range of activities and mathematical understandings possible in making GPS drawings.

Scenes from the stop-motion video created by a team of secondary students—Lauryn, Natalie, and Casey—can be seen at the bottom of Figure 6.3d. Three simultaneous camera views of a single moment of their work can be seen at the top of Figure 6.3. For the video recording, two students wore GoPro head-mounted cameras (Figures 6.3a, b), while an instructor carried a third video camera (Figure 6.3c). Their drawing used a single GPS device (Garmin Foretrex 201), computer mapping and digital movie editing software, about 100 feet of rope and flags that could be staked in the ground, and the landscaped lawn of the surrounding campus (i.e., an environment built for purposes other than large-scale drawing). We first describe how their idea of creating a stop-motion video with GPS drawing arose in the activities of the course. We then describe challenges that arose in their efforts to make a series of coordinated images of circles that could be seen as ‘bouncing’ in a satellite view of the college campus. Finally
Figure 6.3 Animated drawings at walking scale: (a, b, c) multiple perspectives on drawing a circle; (d) images from a stop-motion video of a ball ‘bouncing’ on the campus lawn (ball enhanced).

we explore how working through the challenges of multi-person drawing at large scale involved multiple perspectives on figures in the drawing, the tactile feel of producing large circles, and recurring forms of social interaction that supported making and making sense of the animated GPS drawing.

The idea of making an animated GPS drawing first arose in talk between Casey and Natalie when instructors in the summer course showed images of Jeremy Wood’s drawings. There were questions about making drawings in modes other than walking, and examples of video showing interactive trails of dogs and their human companions were explored. In this context, Casey wondered if it would be possible to make stop-motion videos with GPS drawings.
When the class was shown a clef drawing created by pre-service teachers (the second case in this chapter), Casey and his teammates began talking about how to make and animate multiple circles with a rope. Casey suggested they could turn the GPS device on and off to eliminate unwanted motion tracks, and Natalie proposed spacing the balls so they appeared to “fall” down the campus lawn. They decided the ball should fill the width of the campus lawn, so they chose a rope that gave a radius of suitable size. But they worried that making ovals to show the ball “squish” at the bottom of its fall “down” the lawn was probably going to be hard. Without knowing quite how to put these ideas together into action, but delighted with their plan (indicated by high fives all around), the group selected a rope, grabbed a GPS device, and headed outside. Instructors (our research team) followed with camera gear.

In their attempt to draw a shape (a “ball”) that could be animated and deformed consistently, these students experienced several challenges. Their response to these challenges put their bodies and talk to work in ways that were productively different from more familiar, paper-based drawing practices. These new forms of embodied engagement (Hall & Nemirovsky, 2012) with tools and each other rescaled their understanding of how to maintain the identity of a figure through time, to move a figure through space uniformly, and to create a reversible change of one figure into another (i.e., from a circle into an oval and back). The students also managed to make a relatively mundane object come ‘to life’ in a way that drew the applause of their peers during a later viewing of GPS drawings made in the class.

Their initial challenge was to make a circle that would ‘read’ both on their GPS device (a small screen, difficult to see in bright sun) and on a satellite image of the campus. As shown at the top of Figure 6.3, they used the rope as a fixed radius, which Lauryn held at the center of the lawn (Figure 6.3a), while Natalie and Casey walked with the GPS stylus along the circumference formed at the end of their taut rope (Figure 6.3b). Keeping the rope taut was more difficult than they expected—Lauryn was pulled off center by the walkers, and she eventually requested more rope to create a handle that eliminated her “wiggling” at the center.

Their first attempt produced only a few usable track points, so with help from instructors, the team reset the sampling rate on the GPS device and decided to walk in a “regular” way, several times around the circumference of each circle. Returning to the lawn, their revised drawing routine produced a cleanly readable circle, which elicited cheers⁴ by the drawing team and an announcement by Natalie that this was “the highlight of my life.” Whereas their talk was playful, it was also clear they felt they had accomplished something new.

In preparing to make the next circle, they decided to continue in this configuration with Lauryn in the center, since they were becoming “experts” at their drawing roles. However, they faced a second challenge over how to align a series of identical circles so these “dropped” uniformly down the lawn.
Tentatively at first, they used their fixed radius (the rope, held by Lauryn and Natalie at each end) to extend what they called an “increment” of movement. This increment allowed Lauryn to walk to the ending GPS position of the just completed circle, and then they used that position as a new center, while Natalie and Casey (carrying the GPS stylus) walked down the lawn to a point on the circumference of their next circle. This fairly complex, multi-person movement allowed them to translate the original circle into a series of overlapping copies that would serve as the ‘same’ circle up until the final step in their animation. These copies appear to fall down the lawn before ‘squishing’ at the bottom in Figure 6.3.

But coordinating bodies and tools to make GPS drawings of the ‘same’ circle was initially difficult. Figures 6.3a–c show a moment of trouble in which Lauryn shouts and gestures, “Y’all are gonna have to come this way, a little bit.” From her view (Figure 6.3a), the taut rope clearly misses the center of the lawn (a judgment probably aided by intersecting sidewalks). Natalie’s point of view is shown in Figure 6.3b, looking up while Lauryn shouts, and probably noticing that the rope is well off the center of the lawn. In this same image, Casey can be seen looking down at the GPS device, probably preparing it for the next circle. The play of multiple gazes, tools held to form shapes in the drawing space, and shouted directives are all new forms of work for the body.

A third and more difficult challenge arose when the team reached the bottom of the lawn and wanted to make two “ovals” that flattened out as if the ball were bouncing off the bottom of the lawn (rightmost image in sequence Figure 6.3d, taken from their completed animation). Though Lauryn departed for study hall, Casey and Natalie created knots that shortened the radius, then walked ovals by paying out and reeling in the rope between these knots and the length of the original radius (i.e., the knotted radius corresponded to the flattest part of the oval). Despite running out of time and losing a team member, they made GPS drawings of two ovals, gathered their materials and returned to the classroom, where an instructor later arranged their drawings into a stop-motion animation, using iMovie video editing software.

DISCUSSION

These three cases present the planning, rehearsal, and implementation of a GPS writing (“Walking Awesome”), the reenactment through talk and paper-scale representation of a GPS drawing that used rope as a tool to strategically place and move bodies (“Walking a Clef”), and the implementation of multiples of the ‘same’ ball, translated and then ‘squished’ for later assembly into an animation (“Animated Drawings at Walking Scale”). In all cases, novel forms of mathematical activity emerged from and were reflected in the multiple perspectives of participants. Creating and making
sense of shapes required that participants coordinate these different perspectives using talk, body placement, and physical materials in new ways (e.g., fitting a word, musical clef, or ball into a grassy field). In accomplishing these things, students developed tools and interactive routines that involved new uses of touch, lines of sight, and dynamic placement of bodies. We see these accomplishments as cultural inventions, as new ways of doing and understanding an old conceptual practice of mathematics (e.g., explorations in transformation geometry).

Our design studies of walking-scale geometry using GPS drawing have helped us to see how the doing and knowing of mathematics is contingent on representational infrastructure and embodied routines that have become invisible to us over historical time. For example, in paper-scale practices of geometry, figures are made and felt at the scale of the fingers, hand, and jointed motion of the arm. Learners are thought to know that these figures (shapes and their motility) could be of any size. This can be seen not just as an assumption but also as a conceit of schooling, in the sense that those unaware could be caught out as being unfit to learn or as having the wrong (or misconceived) prior knowledge. Our disruption of paper scale for geometry and drawing deliberately challenges this conceit, not just as a knowledge structure or misconception but also as a materially embodied practice with a history. In walking-scale geometry using GPS drawing, the body is put to new work at a different scale, and in ways that give learners and knowers new experiences with fundamental mathematical ideas (e.g., what about a figure is the ‘same’ through time and over space).

Our design studies located students in and around the geometric figures, and so provided them with unfamiliar and partial views of these figures. Since students took different positions in the unfolding figures, their multiple perspectives—both inside and outside the emerging shapes—needed to be coordinated during assembly. As multi-party activity in open space, this placed new demands on talk, the organization of gaze and lines of sight, regimented movement of bodies, and the manipulation of physical media (e.g., knotting, paying out, and reeling in rope). When making the initial loop of the clef, Tracy “watched it and only let her [Kate] go a certain amount” (Case 2, Line 4). In the Animation group, in order to make sure the balls were all aligned as the group progressed down the quad, Lauryn, Casey, and Natalie had to continuously check their relative positions, and there were frequent efforts to repair their configuration to produce adequately precise marks/tracks across the surface of the lawn. Of course, what is visible in this coordinated activity is not simple. Completing a usable drawing required moving between inside and outside perspectives on shape. The group went back inside to upload their GPS tracks after they completed their first circle, to see what they looked like on the computer. They also continuously checked the (less accurate) trace on the GPS screen. The “Awesome” group had a member standing up on an adjacent hill, where he could see the entire drawing surface at once, approximating a plan view.
We also note that multiple perspectives in time were important as students moved between the field and the classroom. In all our cases, students managed a recurrent tension between doing and seeing (something that is unified in paper-scale drawing and geometry). At walking scale, movements were whole-bodied rather than at the scale of the wrist or hand. However, no trace was left behind in the field. Students could not see the figure until they returned to the classroom and loaded their track data into a mapping engine for display. As they planned and enacted walking-scale drawings, students needed to anticipate and manage this relation between doing (in the field), marking (within the GPS device), and seeing (in later computer projection). The “Awesome” group practiced their drawing through various modalities, tracing with fingers, pencils, and bodies at different scales and over different surfaces. We found similar evidence of anticipation and imagination among students in the “Clef” case, as they described their walking-scale GPS drawing strategies indoors, in a classroom, and at paper scale. Coordination in time as well as space was most visible in the “Animation” team, who carefully ordered their GPS drawings to show the ‘same’ shape moving through space over time.

Coordinating different perspectives in space and time was only a part of the new division of labor created in response to changing the scale and modality of making and transforming geometric shapes. Familiar tools at paper scale (e.g., straight edge and compass) had to be recreated at walking scale. For example, both the “Clef” and “Animation” teams used rope and anchored centers to create a tool very similar to a compass, in which two or more people walk through (and inscribe, using the GPS device as a stylus) circles by submitting their bodies to the discipline of a taut rope, felt and made as a rigid constraint on motion. This discipline of the straightness and apparent (felt) rigidity of the radius simultaneously was created by and provided constraints on the coordination of their bodies. This coordination was built, edited, and stabilized with the support of social interaction (talk, gesture, demonstrative tool use), leading to new forms of talk, gaze management, and body mobility in an emerging practice of GPS drawing.

We end by drawing attention to how emerging aspects of walking-scale geometry with GPS drawing were a genuinely creative accomplishment of each team, developed in response to our disruption of more familiar, paper-scale drawing and geometric construction. We were learning every bit as much as they were learning. As we iteratively refined our design studies, we brought along creative ideas that were developed in early studies into our later efforts. For example, we drew from Jeremy Wood's artistic creations at the outset of our work. We also learned about and incorporated ropes and stakes as tools from Tracy’s explanation of her prior history with competitive marching bands. When we showed the series of completed GPS drawings from “Walking a Clef” (Case 2, Figures 6.2a–c) to high school students in “Animated Drawings at Walking Scale” (Case 3), Casey and his teammates read these multiple versions of a treble clef in an unexpected
(for us) way—they saw and developed the idea of making an animated GPS drawing. These lines of development, consistent with the idea that disruptions lead to new concepts, as well as new meanings for ‘old’ concepts (Hall, Wieckert & Wright, 2010) is important. Changes in scale and modality of paper-scale geometry, undertaken across our series of design experiments, led not only to new understandings of how geometric figures could be the same or similar (i.e., basic properties of the geometry of shape and transformations of shape) but also to genuinely new forms of mathematical activity. This is perhaps a mundane point, but to the best of our knowledge, Casey and his team invented the world’s very first animated GPS drawing.

Our cases provide a close look at multi-party, material, and embodied engagements of inventing routines for walking-scale geometry using GPS drawing. These inventions involve new tool use (ropes and flags as drawing instruments, individuals’ moving bodies plus GPS device and its interface as a stylus), repurposing the built environment as a drawing surface, fitting shapes to the built constraints and affordances of this repurposed surface, negotiating doing and seeing as separable parts of drawing, and disciplining one’s body to participate in these routines. These adaptive, embodied actions can be seen as making up a newly emerging practice that rescales bodies and technologies in the mathematics of space and motion. Of course, our efforts are only a start toward shifting the scale and modality of these venerated practices, and we invite interested readers to join us in what we think will continue to be a rich design space.

NOTES

1. Transcript conventions used in this chapter include the following: Line numbers denote turns at talk. Actions temporally separate from talk are numbered as turns. (Descriptions of actions) appear in italicized double parentheses. (Transcriber uncertainty) appears in single parentheses. Pauses in speech are indicated by (Xs), where X is the duration of the pause. The onset of overlapping talk is shown with left brackets.

2. Based on what we learned from Tracy and her team, Hall and Ma (2011) subsequently studied design and ensemble learning in a high school marching band, following instructors and students over a 9-month competition season.

3. How and why speakers take action in the gestural stage is a topic beyond the scope of this chapter. We refer readers to an excellent, synthetic account by Sterken (2009) of the difference between gestures with which speakers appear to create conceptual structures while talking (ceiving) and those used to depict structures for listeners. Gero & Kofsky (2010) presents an illuminating analysis of the role of differently scaled gesture and whole-body movement in the learning of mathematics.

4. The editors noticed these students seemed very cheerful while using GPS to draw animated geometric figures. We have not studied this systematically, but by comparison with practices of paper-based school geometry that we did observe (e.g., in Cases 1 and 2), participants were excited to try geometric constructions at walking scale and use satellites to draw on the surface of the earth. As cultural inventions go, GPS drawing has been both novel and enjoyable.
5. This might be a controversial claim. Jeremy Wood’s GPS drawing website does contain ‘animations’ (see www.gpsdrawing.com/animations/wingfest360.htm), but these are gif animations either of a moving object or a ‘fly through’ of a completed set of tracks. Casey and his team created a form of stop-motion filmmaking, more like a Claymation movie than a visualization of spatial data.

REFERENCES