Introduction to the Special Issue: Modalities of Body Engagement in Mathematical Activity and Learning

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Introduction to the Special Issue: Modalities of Body Engagement in Mathematical Activity and Learning

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How is the active human body—through gesture production, manipulation of tools, mobility in the local environment, and interaction with others—included in mathematical thinking and learning? This special issue of the Journal of the Learning Sciences presents three articles, each taking a different approach to this question. Two commentaries by accomplished scholars in the cognitive and learning sciences provide a critical perspective on the articles and pose far-reaching questions about studies of embodiment in human thinking and learning. These articles and commentaries appear amid a diverse set of theoretical proposals for how cognition is necessarily embodied, supported by a growing collection of empirical findings regarding the body, concepts, and cognition that come from a range of scientific disciplines. Not surprisingly, these articles and the larger interdisciplinary field reflect different commitments to the nature of the mind, to modes of scientific investigation in the learning sciences, and to what should count as an adequate or a productive explanation in our field.

In this introduction, we start with a brief review of proposals for embodied cognition and what they can tell us about mathematical activity in particular. What is the nature of mathematical knowledge, what is the role of the body in a domain with such seemingly abstract or imaginary entities, and how is mathematics (or how could it be) taught and learned in ways that exploit phenomena of embodiment? We next introduce a synthetic proposal for research on embodied mathematical cognition that encourages close analysis of modal engagements with
physical, cultural, and social settings for mathematical activity. We end by introducing the articles as examples of new research within a productively diverse set of approaches to understanding embodied mathematical cognition. We conclude with an invitation for new research in this area.

RECOVERING THE BODY FOR MATHEMATICAL THINKING AND LEARNING

Until quite recently, most of us have been encouraged (even schooled) to understand mathematical thinking as made up of individual intellectual processes. Our bodies have been peripheral under this received view, both in scientific accounts of mathematical cognition and in folk theories of how mathematics is done or should be taught, learned, and assessed. If relevant at all, our bodies have served to carry around a mind that processed mental content, typically expressed in propositional forms that were free of modalities engaged by the body in physical or social settings (e.g., the feel of acceleration, the difference between being inside a geometric figure and looking down at it on paper, or the sense that an interlocutor “gets it” when giving an explanation). It has been hard to avoid thinking of the working content of mathematics as anything other than the stuff of mind. At best bodies and modalities of perception were peripheral; at worst their concrete involvements contaminated or limited mathematics as an intellectual accomplishment. This view of mathematical (or any human) thinking has strong historical roots.

Close to three centuries ago, John Locke posed the following thought experiment:

Imagine a case of body-switch: suppose that a prince and a cobbler both go to sleep one night and that in the following morning a person wakes up with the cobbler’s body but all the thoughts, memories, and feelings of the prince, and another person wakes up with the prince’s body but all the thoughts, feelings, and memories of the cobbler. Which person, Locke asks, is identical with the prince, and which with the cobbler? His answer is that the person with the prince’s thoughts and memories and the cobbler’s body is the prince, and the person with the cobbler’s thoughts and memories and the prince’s body is the cobbler. (Dicker, 2004, p. 140)

Locke’s unhesitant response to this thought experiment has been prevalent in Western philosophy. Having a body was viewed as akin to dressing up in a certain costume; although wearing an unusual costume alters one’s appearance, it does not change who one is.

Over the past century a major stream of work in continental philosophy has begun to challenge assumptions of this kind and to develop alternative frameworks...
for what being a body amounts to. Overcoming the traditional dualism of body and mind is not a matter of arguing that they are very closely related but of questioning the split between them in the first place. This entails rethinking what a living body is and how it participates in the use of language, incorporates tools, and engages in interpersonal transactions.

These philosophical projects had a deep impact on the cognitive and learning sciences during the late 1980s and early 1990s. These came through a series of critical appraisals of computational and representational theories of mind, typically understood as processes of symbolic information processing. Drawing from phenomenology, Winograd and Flores (1986) argued that human intelligence depended intimately on having a body that was active in the world. Varela, Thompson, and Rosch (1991) made similar arguments but drew from different traditions of Buddhist thought and practices of mindfulness to bring the body forward in basic research in cognitive and developmental psychology. Drawing from ethnomethodology (also with phenomenological roots) and conversation analysis, Suchman (1987) argued that sense-making practices in human conversation were qualitatively different from extant proposals for human/machine interaction based on symbolic computation. Two other critical appraisals were more thoroughly historical and cultural. Engeström (1987) extended a cultural and historical theory of activity based in Vygotsky’s writing (e.g., Vygotsky, 1986) to outline a theory of “learning by expanding” that was concerned with transforming both individuals and the activity systems in which they lived and worked. Drawing as well from practice theories in sociology and anthropological studies of apprenticeship, Lave and Wenger (1991) proposed a widely influential theory of situated learning as changes in participation within the structured activities of communities of practice.

These critical reappraisals of cognition have brought the human body, acting in cultural settings, out of the periphery and onto center stage for research in the learning sciences. However, it would be misleading to say that a unified theory of embodied cognition is available for studies of mathematical activity and learning. Our field is at a turning point regarding how mind, body, and the broader social and cultural world fit together as people engage in and learn mathematics (or other cultural practices represented in school instruction). We next review a variety of arguments for embodied cognition before turning to research that is specific to mathematics.

Not surprisingly, questions of how the mind is embodied have been intensely debated in cognitive science, because many of the criticisms we have reviewed were directed toward core commitments of this field. Wilson’s (2002) synthetic review provides an excellent starting point. Though she is cautious or even dismissive of phenomena beyond the level of individual thinking (more on this in a moment), Wilson explores two claims that are particularly relevant for our
purposes. The first is that people “off-load” cognitive work onto the proximal environment (Wilson, 2002, pp. 628–629)—as in making lists, arranging materials and tools in an order that supports assembling something, or marking events on a calendar to remind ourselves of things that need to be done in the future. These apparently self-reflective, epistemic actions (Kirsh, 1996; Kirsh & Maglio, 1994) are particularly useful in advance of activity when one’s attention might be occupied with other matters. The second claim Wilson endorses is related and considers how sensorimotor cognitive processes that were originally involved in doing something are repurposed “off-line” on later occasions (i.e., to imagine doing that thing or something like it). As put most directly by Barsalou (2008, 2010; Barsalou & Wiemer-Hastings, 2005), this view of embodied cognition argues that concepts and conceptual understanding depend upon covert forms of mental simulation that reenact modality-specific traces of perceptual and motor experience acquired earlier while learning or observing others. If this is the case, mechanisms of modality-specific simulation bypass entirely the need (in theory) for propositional (amodal) mental content in having and using concepts. Just as thinking about (or recognizing) a cup might involve sensorimotor preparation for grasping or drinking, thinking about an equilateral triangle might involve covertly reenacting modality-specific experiences of physical measurement or the construction of geometric objects (e.g., forming a triangle with rigid, same-length segments).

These claims—that the embodied mind is extended by changing the environment, that conceptual understanding is based on modality-specific mental simulation—are now widely accepted as working conjectures in our field. This is in part a response to intriguing evidence from research in cognitive neuroscience that our perception of space is dynamically shaped by tool use (the embodied mind is extended) and that understanding by covert simulation also provides forms of shared intentionality when we are observing or interacting with others (understanding concepts through modality-specific simulation). It is important to recognize that these claims draw a boundary around cognition—at the skull or the skin of the individual—that sidesteps or dismisses critical questions about human interaction and cultural practice that we reviewed earlier. For example, if embodied cognition involves creating and using an environment layered with things that extend the acting body, then how does the body, either alone or acting with other bodies, engage with or produce these things? Similarly, if concepts are grounded intimately in actual or vicarious experiences with physical things, then why not also expect interactions with other people to be central, modal resources for having or understanding a concept?

In our view (Hall & Nemirovsky, 2010), there is nothing about the extended body or covert sensorimotor simulation for conceptual understanding that rules out interactive, cultural, or historical aspects of embodiment. We think of concepts (in mathematics but also in other domains) as forms of modal engagement in which bodies incorporate and express culture. As more research accumulates
in this area of interdisciplinary study, this more inclusive view is gaining traction (Farnell, 1996; Gallagher, 2005, 2008; Gallese, 2007, 2009; Wilson, 2008). In studies of mathematical activity specifically, we are now able to ask how people and their bodies (plural) are engaged in using drawing surfaces persuasively, or in building and manipulating physical models that are understood and appreciated as explanations. But there are challenges ahead. For example, as we move beyond the skin to consider tool use in multiparty activity, how can we choose units and levels of analysis that are productive for studies of mathematical activity and learning?

Within the diverse set of proposals reviewed here, three common themes may help us approach embodied mathematical cognition. These are the central role of specific modalities of experience for understanding and using concepts, the constitutive role of acting bodies in mathematical thinking, and possibilities for extending the body (and modalities of experience) associated with making and using representational tools. These themes are increasingly addressed in research on embodied mathematical cognition that has appeared in the past decade, and they are explored further in each of the articles and commentaries in this special issue.

ARTICLES AND COMMENTARIES IN THIS SPECIAL ISSUE

Perhaps the most widely influential, recent contribution to studies of embodied mathematical cognition is Lakoff and Núñez’s (2000) analysis of how mathematical concepts (e.g., number, arithmetic operations, infinity) can be understood as metaphorical extensions and combinations of image schemas that are commonplace in language production and grounded in actions of the body (see also Núñez, Edwards, & Matos, 2006). Williams’s article (2012/this issue) works within the framework of cognitive semantics to show how elementary school children learn to tell time using an analog clock face. According to Williams’s analysis, errors documented as children grow older and have more experience with telling time can be explained by the need to coordinate different image schemas that support correctly reading time at different scales. This paper makes two important contributions. First, Williams explores yet another domain of mathematics instruction in which the conceptual metaphor framework has explanatory power, in this case the mundane but widely distributed capacity to use aspects of modular arithmetic involved in reading an analog clock face. Second, his article demonstrates using a close analysis of how talk and gesture are coordinated with instructional devices to understand conceptual change on the part of learners, with potentially useful implications for how teachers might systematically anticipate student thinking when using these devices.

Of similarly wide interest is the role of gesture in mathematical thinking and problem solving, which has been studied with increasing intensity over the past
decade. Drawing primarily from psychological studies of how gesture is coordinated with talk conducted both in experimental settings (McNeill, 1992, 2005) and in classrooms (Goldin-Meadow, 2003), researchers have increasingly seen gesture as an activity of the body that reflects (or is co-produced with) thought in ways that are consistent with the claim that concepts are based on image schemas (mentioned previously). Gestures also appear to be designed by speakers in ways that are finely tuned to listeners. Hostetter and Alibali (2008) provided a comprehensive review of research in this area, including studies of learning and teaching mathematics. They argued that gesture not only facilitates problem solving and communication in mathematics (and other domains), but at a more basic level gestures show visible embodiment of processes of sensorimotor simulation that are involved in having and understanding concepts. They called this the “gesture-as-simulated-action (GSA) framework” (p. 502), and this treatment of gesture as evidence for (and constitutive of) embodied mathematical cognition is further updated in the article by Alibali and Nathan in this special issue. Here, evidence from a range of studies is used to argue that pointing (deictic) gestures ground talk about mathematical entities in the proximal environment and that some types of metaphoric gestures (e.g., touching points along a horizontal path while talking about the passage of time) show direct evidence of using the types of conceptual metaphors described by Lakoff and Núñez (2000).

Building and using representational tools has long been an active area of research for mathematical activity and learning (Cobb, Yackel, & McClain, 2000; Hall, Stevens, & Torralba, 2002; Kaput, 2009; Noble, DiMattia, Nemirovsky, & Barros, 2006). Attending to how the body is deployed in this activity and how tools might extend the embodied mind in creating and working with mathematical entities is a more recent development. Nemirovsky, Rasmussen, Sweeney, and Wawro (2012/this issue) examine the deployment of bodies, gaze, and representational materials as groups of pre-service, secondary mathematics teachers worked on a tile floor—treated as the complex plane—to explore arithmetic over these types of numbers. Their analysis follows a phenomenological tradition, asking about the horizon of possible meanings as people take action and talk to each other using different representational means (e.g., algebraic derivation on paper, moving points on the floor, and depicting both in the speaker’s gestural stage). Their analysis provides further descriptive evidence that understanding and using mathematical concepts is intimately related to how the body inhabits and moves in space, particularly space that is already layered with systems of representation that have particular mathematical meanings. One of their more provocative claims is that the horizon of possible meanings is not determined by task or environment but enacted in ways that can shift radically with even small changes in what people take as relevant or worth attending to in their activity (e.g., exploiting the color of floor tiles for enacting geometric operations, producing gestures with feet and body that are fitted to the scale of these tiles).
The commentaries on these articles are generally appreciative, but they raise critical questions for the studies reported here and for new research on embodied mathematical cognition. Rafael Núñez, coming to mathematics from cognitive science and a primary developer of a conceptual metaphor approach to mathematical knowledge, draws critical distinctions between studies that operate in a confirmatory scientific mode, posing questions that can be falsified with empirical evidence from planned comparisons, and more descriptive approaches. Although Núñez (2012/this issue) characterizes descriptive or exploratory analyses as subject to bias and untestable beliefs, he notes that much early work in this field was descriptive or based on philosophical analysis. We would add that science proceeds as a necessary interplay between two modes, one of discovery (often descriptive, exploratory, and naturalistic) and the other of confirmation (planned comparisons and hypothesis testing). Both are required to advance scientific understanding, particularly for a field that has only seen about a decade of focused research, as is the case with studies of embodied mathematical cognition.

Reed Stevens, our other commentator, approaches the articles as a learning scientist and mathematics educator, working from a stance consistent with ethnomethodology and social practice theory. Stevens (2012/this issue) contrasts cognitivist and interactionist aspects of each paper in ways that index the critical reappraisal of relations between mind and body that we reviewed earlier. Within a cognitivist framework (Alibali & Nathan, 2012/this issue, and Williams, 2012/this issue, by Stevens’s reading), claims about cognitive mechanisms (image schemas, schematic blending, gesture as sensorimotor simulation) rest on a set of assumptions about how mental processes create the visible regularities of social life. Stevens argues (and most proponents of multimodal interaction analysis would agree) that embodied mathematical activity might be examined directly in social interaction (including activity with representational media). But Stevens is also critical of the phenomenological approach of Nemirovsky et al. (2012/this issue), arguing that mathematical concepts and understanding are not exclusively the province of bodies but should be studied in ways that include whatever participants within the activity take to be relevant for their own sense making. Sometimes the body has primacy, but at other times shared expectations about how representational systems are supposed to work are more relevant as resources for understanding.

We close by inviting readers to take up the diverse issues and framing proposals in this special issue in the hopes of moving research on embodied mathematical cognition forward. Tensions remain over where and how to draw boundaries around embodied cognition, though there seems to be growing agreement that human thinking and learning is intimately tied not only to the body but to a body that interacts with others and is active in social and cultural settings already rich with mediating artifacts that afford particular kinds of joint activity. Our hope is that future studies in this area will build upon accounts of phenomena and
findings presented in these articles—teaching and learning as the coordination of conventional schemas for using shared technology (Williams, 2012/this issue), the generative and communicative functions of gesture in learning and explaining mathematics concepts (Alibali & Nathan, 2012/this issue), and the open horizon for mathematical imagination associated with taking up diverse representational media in learning environments that are designed to position bodies in new ways (Nemirovsky et al., 2012/this issue). We also encourage careful attention to learners’ histories of modal engagement, both in descriptive analyses of processes of learning and teaching and as a way to think about how to design learning environments that anticipate and leverage new understandings of embodied mathematical cognition.

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