ENSEMBLE LEARNING AND KNOWING

Developing a Walking Scale Geometry Dilation Strategy

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While typical formal schooling is generally more concerned with the learning of individuals, much of human activity takes place in groups, and little research focuses on ensemble learning, where activity and learning necessarily occurs in groups and is treated as a property of groups. For this reason, an analysis of the knowledge developed in the course of ensemble learning should focus, at least in part, on the learning of the group. A common theoretical assumption of Interaction Analysis is that “knowledge and action are fundamentally social in origin, organization, and use, and are situated in particular social and material ecologies” (Jordan & Henderson, 1995, p. 41); an Interaction Analysis takes as given that knowledge under development in a setting is inextricably tied to the exchanges between members of the group, as well as to the social, cultural, and material resources available to and recruited by them. In the spirit of exploring the possibilities of treating knowledge and learning as an ensemble property, I offer the analysis of the learning of a group of students solving a geometry task designed to be completed by multiple individuals together. As an extreme example of ensemble learning, one where the task was designed to be difficult or impossible to complete individually, I hope to provide a useful comparative case as fodder for discussion regarding the knowledge and learning of multiple individuals in interaction. I present a multimodal Interaction Analysis of an episode of a group’s problem-solving activity. In their insightful commentary to this chapter, Conlin and Hammer extend the exploration by viewing the case through the lens of Knowledge Analysis.

The analysis of the learning of the ensemble activity presented in this chapter follows the work of others who have very deliberately expanded their units of analysis from individuals to groups and other resources in the setting. Specifically, I draw from Cobb, Stephan, McClain, and Gravemeijer (2001) in their thinking...
about emergent classroom mathematical practices, and Hutchins’ (1995a) distributed cognition approach to more explicitly account for representations and material resources.

Cobb and his group (2001) felt that, given their design orientations for classroom mathematics learning, it was important to be able to characterize the collective learning of a classroom community in order to make design conjectures or claims about individual student learning, necessarily situated in the activity of the classroom community. Rather than focus on the historically developed ways of joining or being in an established community or discipline, they adapted the sociocultural idea of “cultural practice” to account for the local, emergent activity of teachers and students in a particular classroom. They analyzed the collective learning of a classroom community by attending to “normative taken-as-shared ways of talking and reasoning” (p. 119) at three levels: classroom norms (participation structure; they cite Erickson, 1986), socio-mathematical norms (related to mathematics in particular), and classroom mathematical practices (related to a specific topic of mathematics).

Hutchins’ (e.g., 1995a, 2010) distributed cognition approach is a different, though related, treatment of a social analysis of knowledge. Cobb and his colleagues (2001) acknowledged the contribution of theories of distributed cognition to their work. However, Hutchins (1995a) resisted attributing particular forms of knowledge, memory, or cognition to individuals based on observations of activity. Instead, he argued that the whole setting (for example, the cockpit of a commercial airplane or the bridge of an aircraft carrier) should be taken as the cognitive system. Analysis, in alignment with the cognitive science view of cognition as computation, followed “the creation, transformation, and propagation of representational states” (p. 49). Hutchins (2010) argued that “cognitive science made a fundamental category error when it mistook the properties of a person in interaction with a social and material world for the cognitive properties of whatever is inside the person” (p. 91). Larger systems may have cognitive properties that are irreducible to individuals within the system. Additionally, an advantage of this expanded unit of analysis is that many of the representations and actions of the cognitive process are directly observable in activity.

Following Cobb and colleagues, I track the emergent mathematical practices of the group as they solve problems together. However, like Hutchins, I make claims only about the cognition and learning of the group as a whole, rather than as individuals. This is not to claim that there is not individual knowledge or mental activity in the heads of individuals. Instead, the focus on the activity of the group highlights the problem-solving accomplishments and learning of the three students in interaction; the developing representational tools; and the contributions of the material resources, including students’ bodies, available in the setting. Over the course of the analysis, it is inevitable that the contributions of particular individuals and materials are highlighted. However, overall, the analysis provides
insight only into the ensemble as a unit of analysis. Additionally, I argue that the accomplishments and learning of the ensemble would not be possible given solely the knowledge of any given individual (or the properties of any material resources) in the episode. The problem-solving strategy carried out in the episode below is a collective accomplishment. The analysis will show how it came about.

I focus on questions of whether or not the group is learning, what they are learning, and how. Individuals within the group see, know, and learn very different things, but by the end of the task, the group has the capacity to perform the dilation of a geometric figure, which it did not have at the start. I take a sociocultural, interactional, and embodied stance on learning and knowledge. From a sociocultural and interactional perspective, learning is a trajectory of participation in activities within communities of practice, and knowing is the ability to perform tasks and solve problems in ways valued by that community, aligned with the goals of the group, in coordination with its tools, technologies, and representational systems, and in ways distributed across people and practices (Greeno & Middle School Mathematics through Applications Project Group, 1998; Hall, 1996; Lave, 1988). Following the traditions of conversation and Interacion Analysis, the analysis foregrounds how participants are making sense of ongoing activity (and therefore their learning) in moment-to-moment, unfolding interactions (Goffman, 1964; Sacks, Schegloff, & Jefferson, 1974; Schegloff, 1991; Stevens, 2010). I pay deliberate attention not just to talk in interaction but to bodies, materials, and spatial relations as well, in an attempt to represent a richly multimodal theory of interaction, learning, and knowledge. From an embodied stance, whole bodies are actors in and resources for learning and knowing, not just for doing (Hall, Stevens, & Torralba, 2002; Nemirovsky & Ferrara, 2009; Núñez, Edwards, & Filipe Matos, 1999). The analysis in this chapter combines these views to present an example of multimodal interaction analysis that takes seriously the role of multiple actors, whole bodies, and other material resources as constituent parts of learning and knowledge.

Setting

The focal episode of the chapter follows a group of three high-school students completing a large-scale geometry task outdoors on a university campus lawn using everyday materials such as ropes, lawn flags, and flagging tape. I call this type of task Walking Scale Geometry (WSG), to emphasize the engagement of students’ whole bodies in motion during problem solving, using the ground as a “drawing” surface. Participants are not allowed to engage in problem solving using paper and pencil; instead, they use their bodies and other everyday materials as representational tools.

I draw from part of a design experiment (Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) that introduced a geometry task setting for secondary
students that deliberately disrupted targeted aspects of typical classroom activity (Ma, 2012, 2014). The geometry task setting was designed to drastically alter the setting of problem solving so as to support more diverse opportunities to learn by inviting students to engage in geometry activity in novel, sensible (to them) ways.

The WSG setting was designed to support conceptual agency. Because of the scale and available materials of WSG, students needed to adopt new tools for drawing, representing, and completing mathematical procedures. They no longer had paper, pencils, rulers, protractors, or compasses, and could no longer engage by filling out solutions to problems on, for example, worksheets. Instead, students were asked to reason with and about everyday material objects and their bodies, and find ways to mathematize them. Additionally, individual students’ perspectives on the same geometric objects were radically different. Not only were students much smaller in comparison to the objects, they had to view them from within, rather than from a birds-eye view as they did when working at paper-and-pencil scale. As a consequence, judging whether constructed lines were straight or parallel, or whether angles had familiar degree measures (e.g., 90 degrees) became problematic, inviting or requiring new techniques for making or judging these mathematical properties.

WSG was also designed to support access to participation for all students. Students were no longer able to draw and manipulate the geometric representations individually because of the scale of the figures and the students’ perspectives. The large size of the figures made the materials difficult to manipulate and fix as inscriptions. Students standing within or alongside the figures did not generally have the same view as others. This meant a figure looked significantly different to each individual, and they needed to coordinate and assemble varying perspectives in order to reach agreement.

At walking scale, the division of labor of geometry problem solving was distributed so that individual solutions to tasks were difficult, if not impossible. Students had to work together and communicate effectively to develop, negotiate, and accomplish their goals. WSG was designed to support ensemble learning (Hall & Ma, 2011), where learning must be done together. Students were held accountable to each other to participate and to coordinate access to participation for others in their group in order to complete tasks.

The episode below took place in a summer enrichment course (SEC) held on a university campus, centered on the theme of spatial reasoning and analysis. The 12 students in the course had only known each other for one day before engaging in this task, and had spent just a half-hour prior working on similar tasks together. While general norms of school mathematics were clearly in play, the SEC setting of the episode produced a complex dynamic of informal learning and exploration, untethered to the regimes of discipline and assessments typically found in school. The students, coming from different schools and grade levels, actively negotiated participation norms in ways apparent to the research team, who served as instructors for the course.
The three students in the episode, Lauryn, Natalie, and Tahir (pictured in Figure 11.1), all had successful histories of participation in school mathematics, and demonstrated positive mathematics identities. Natalie was a rising 10th-grader and had already completed a year of high-school-level geometry before SEC. Lauryn and Tahir were rising 9th-graders, and had not. Over the course of the episode, various researchers dropped by to check on the group’s progress. Nate carried the video camera and was present throughout. Other researchers who appear in the transcript include Rogers, Jasmine, and Jillian.

Dilating a WSG Quadrilateral

Before the start of the episode, the group had made a large quadrilateral out of flagging tape (Figure 11.1). The three had wanted to make a trapezoid, but Natalie pointed out that it was only “trapezoid-like” and “trapezoid-ish” because, as Lauryn explained, “we didn’t measure it exactly.” Now they were at the beginning of solving a problem involving scaling their quadrilateral 1.5 times. Natalie quickly proposed a dilation strategy that she vaguely remembered learning in school. It involved choosing, at random, a point to be the center of dilation, then stretching pieces of tape from that point out through each vertex of the quadrilateral. The length of each of these “spokes” (Tahir’s term, later when describing their inscription) would be 1.5 times the distance from the center of dilation to the relevant vertex. The endpoints of the spokes would be the vertices of the new quadrilateral (Figure 11.2).

Natalie’s explanations of her dilation strategy were not sufficient to convince Lauryn and Tahir, even though she attempted to describe her idea many times.
Lauryn and Tahir wanted to scale each side of the quadrilateral 1.5 times, and the group initially began this process by producing one new side. Natalie was dissatisfied with this strategy, since simply scaling each side of the quadrilateral would not guide the group in arranging them so as to produce angles congruent to those in the original. Lauryn and Tahir had not considered this aspect of their plan, and Natalie had some trouble explaining her concern to the other two. After some disagreement, and the prompting of a researcher (Nate), Lauryn and Tahir agreed to try Natalie’s strategy, although they were still reluctant. Lauryn and Tahir not only did not know how their mathematical goal (scaling their quadrilateral 1.5 times) and envisioned plan (making each side longer by a factor of 1.5) related to what Natalie wanted to do, but they also did not have a sense of Natalie’s plan with respect to material manipulations. It is worth noting that Natalie had not fully formulated her plan yet either. She was explicit that she was not sure how much to dilate the distance from the center of dilation to the original vertex in order to find the new vertices. As the group worked, it also became apparent that the details of implementing the dilation of each spoke were still under development.

The episode is divided into three excerpts below. Each excerpt begins with a brief summary of what to expect, then the transcript. Following the transcript, I describe the progress of the group, focusing on talk, bodily engagements, and material manipulations, along with the meanings they take on in the unfolding interaction. This will provide a moment-to-moment accounting for how this part of the dilation strategy was accomplished. Following the three excerpts, I make some general comments about the contributions of components of the system to the group’s learning.

**Excerpt 1**

As the episode opens, the group was beginning to try Natalie’s dilation strategy. They already had a strip of green tape that had been measured to be 1.5 times
the length of one side of the quadrilateral. Natalie had already placed a flag inside the quadrilateral as their center of dilation, but not near what might be considered to be the center of the figure. In this first excerpt, Lauryn and Tahir tried to engage in implementing the strategy and make sense of it at the same time. Natalie alternated between responding to their questions and statements that did not align with her plan, and trying to direct their actions to carry out her plan.

[254] Natalie: Ok. Let's just try it. Ok(h)ay.
[255] Lauryn: [Are we gonna use the same length? ((holding the green tape that was just measured to be 1.5 times the length of the side, and handing one end to Natalie))]
[256] Natalie: [Guess we can use this to measure.
[257] Lauryn: Cause this is [the one and a half,=]
[258] Natalie: [Well,
[259] Tahir: =That's not the center point though.
[260] Natalie: It doesn’t need to be. You can dilate from any point. Just as long as you go out the same distance from ca- ((kneels down at the flag at the center of dilation with her end of the green tape in her hand)) So like make this line go out from ((points to the vertex between her and Lauryn)) that point?
[261] Lauryn: What?
[262] Natalie: Like(h),
[263] Tahir: Cross this ((points to green tape)) over here ((points to flag marking the vertex)) (on) the flag- [(front) flag
[264] Natalie: [Yeah, Okay.=
[265] Lauryn: =Like that?=
[266] Natalie: =Put it to the ground? And hold it there? And then- (addressing Tahir)) could you like, ((points to vertex)) hold it- [˚there?
[267] Tahir: [(Yeah Like here?) ((Figure 11.3, Panel A> kneels down and holds green tape down at vertex with right hand))
[268] Natalie: Yes. ((stands up with her end of the tape and walks toward vertex)) Okay. Okay ((backs up and kneels down again near center of dilation)) sorry. ((microphone transmitter falls off )) Yikes! Okay. And then like, here. Loosen up just a tad? Just so it can like, pull out here ((Tahir lifts his fingers a little))? Okay.Yah. And then like, ((walks with her end of the tape to the vertex, puts it down by Tahir's hand)) find whatever half of this, is? Like hold that tight ((Tahir puts his hand on the tape)) there too? ((steps back, pulling the doubled tape through her hand until it is taut <Figure 11.3, Panel B>)) Okay. ((puts down green tape)) So we’re probably gonna have to keep ((picks up yellow rope that’s nearby on the ground, and takes the end of it to the center of dilation flag)), like this just to make sure we’re on the same line. So put- ((points at yellow rope)) the- yellow rope ((Tahir picks up yellow rope with left hand)) like ((points at the vertex)) over there ((Tahir starts moving the yellow rope toward the vertex)) too? But like make sure it can still reach- ((pulls back on the yellow rope)) eh(eh(hhh)) it’s mostly,
[269] Tahir: Yeah, the rope is kinda tied [(together)
[270] Natalie: [It's okay, jus:- I think it's better now ((Tahir stands up and walks over to where the rest of the yellow rope lies in a tangled ball)), cause-]
As Lauryn handed Natalie the end of the green tape that they had just measured to be 1.5 times the length of the side, the two spoke at the same time, at cross-purposes. Lauryn, handing Natalie one end of the green tape, asked, “Are we gonna use the same length? Cause this is the one and a half” [255, 257]. The green tape was the new side that they had produced earlier in the first, aborted, strategy. To Lauryn, it was still the side of the new quadrilateral, measured to be 1.5 times the length of the corresponding side on the original. She was asking if they would use it, since it had already been measured. Meanwhile, Natalie no longer thought of it as a part of their new quadrilateral, but just a long piece of tape that they could “use … to measure” [256] where a new vertex of the quadrilateral would go.
Tahir, still unsure that this plan would work, pointed out that the flag (their center of dilation) was not at the center of the quadrilateral. Natalie claimed that it did not have to be, but did not explain why, mathematically. She only answered him in terms of the rules of the procedure she remembered, and did not even complete her thought [260]. She knelt down with her end of the green tape at the flag, interrupting herself to tell Lauryn to pull it taut so it would stretch through a nearby vertex (“make this line go out from that point?” [260]). Lauryn asked, “What?” leading Tahir to translate, telling her to “Cross this over here (on) the flag- (front) flag” [263], pointing to the green tape (“this”), then to the vertex of the quadrilateral that was between Lauryn and Natalie (“over here on the flag”). Once Lauryn did this, Natalie asked her to put the tape to the ground and hold it there, and asked Tahir to hold the tape at the front flag. They had now produced a line segment between the center of dilation and the vertex. In order to find the new vertex, they needed to extend this line segment by half its own length. Another way to say this is that they had traced, with the green tape, the distance from the center of dilation to the first vertex. The new vertex would be half this distance beyond the old vertex, in the same direction.

In her instructions to Lauryn, Natalie used some mathematical language to refer to parts of the materials they were using (“So like make this line go out from that point?” [260]). Lauryn did not know what Natalie wanted her to do, and Tahir revoiced the instruction for her. Tahir’s version of the instructions substituted gestures and language referring to the materials (“Cross this over here (on) the flag” [263]) for the mathematical entities that Natalie referred to (“this line,” and “that point”). Natalie approved of Tahir’s revision of her direction, and Lauryn completed the action.

This is not to say that Lauryn does not know what a point or line is, or that she was not familiar with the group’s now established practice of representing points with flags and lines with tape. However, it was not clear to Lauryn which tape-as-line and which flag-as-point Natalie was talking about. Additionally, with her still-developing understanding of Natalie’s strategy, and her lingering interpretation of the green tape as one of the sides of the new quadrilateral, it would not make sense to place it where Natalie was asking her to place it. However, Tahir’s translation allowed Lauryn to engage with everyday language and objects in order to execute the direction, even if she had not yet made mathematical sense of it. In this way, both Lauryn and Tahir had access to participating in the task even as they were still making sense of the solution strategy. The everyday materials and manipulations became resources for the two to engage, and therefore to have access to the mathematics that Natalie was proposing. At the same time, Natalie was able to obtain Lauryn and Tahir’s help to try out her strategy, even if she did not yet have a clear plan for how to carry it out.

Natalie began to find half the distance from the center of dilation to the vertex by bringing her end of the green tape over to Tahir and asking him to hold it. She then backed up a few steps, holding the now doubled-over green tape. In backing
up she was able to pull this until it was taut. Then she placed it down on the
ground. At this point Natalie decided that they should use the yellow rope that
was lying on the ground “just to make sure we’re on the same line” [268]. While
Natalie had a general plan about both how they would dilate the quadrilateral
and how they would find the new corresponding vertices, the exact actions they
would execute were still under development for her. In particular, she was still
figuring out how to find 1.5 times the distance from the center of dilation to the
original quadrilateral vertex. She wanted to use the yellow rope as a visible trace
of this distance, and they continued to use it throughout the task, even though it
was mathematically unnecessary. The yellow rope was mostly in a tangled ball, and
Tahir stood up to untangle it.

Excerpt 2

As Tahir abandoned his post at the vertex to deal with the yellow rope, Natalie
took the opportunity to check in with Nate for reassurance. Lauryn and Tahir
took this opportunity to express their confusion and resistance to this strategy fur-
ther. In this excerpt, Natalie repeatedly asked Nate if her idea was “stupid,” while
Lauryn and Tahir struggled to make sense of Natalie’s plan. Some researchers also
join the group to check in on their progress.

((Natalie looks back at Nate.))
Natalie: I feel like I’m like, doing something really stupid.
Tahir: I don’t think either of us really understand what you’re doing.
Lauryn: Yeah I’m not really,
Natalie: You know like dilating on a, grid? Like when you get like a triangle
and you pick a point in the center and you go out with your ruler? To
like make it bigger? Like
((Lauryn shrugs.))
Natalie: Like I don’t know how to like describe it if, ((looks back at Nate again.))
Nate: Just keep trying it, see what happens.
Natalie: Ok.
Tahir: I thought we were gonna like add onto this ((sweeps arm along the side
of the quadrilateral that they had originally already scaled)) and make it a
bigger trapezoid.
Natalie: Well it’s like easier cause now we don’t have to like mess with, angles.
So just like, ((laughs)).
((Rogers walks over with a microphone, connected to the roof cam, for Lauryn.))
Natalie: I feel like I’m, being stupid. Ok. So just like,
Tahir: (This is a) very tangled rope.
Natalie: (hh)Yeah(hh). I don’t even know if this is like, half. ((Picks up the green
and unfolds it, pulling it back to the center flag.)) Could you loosen up on
it a little, like, step closer just a tiny bit. Ok there. That’s good.
Tahir: (I don’t see how to untangle this rope.)
Natalie: Yeah(h)}
Tahir: I don’t even think it’s tangled I think it’s just really like, (()).
Lauryn: So how is this gonna, (1s) like,
Natalie: ((sighs, looks back at Nate)) Well I don’t know yet.
Lauryn: Yeah.
Natalie: Well I mean like, I don’t know what like the, [growth factor is supposed to be.
Lauryn: (Wow, from this angle, our sides are really off.
Tahir: (yeah)
Natalie: Yeah they are.
Lauryn: I didn’t notice that before.
Natalie: ((Looks back at Nate again.)) Am I just,
Nate: They’re not o- They’re not off, right? Because you just were supposed to make a quadrilateral.
Natalie: Yeah, yeah.
Lauryn: Yeah that’s true but
Natalie: Am I just doing something really stupid?
Nate: Just try it, it’s fine! There’s no right answer, just give it a shot, see what happens.
((Jasmine comes over asking Lauryn why she thinks it looks off. This cuts in and out (mostly out) on the other audio stream.))
Lauryn: So, we’re gonna go <Figure 11.4, Panel A>, this way? ((She uses her forearm to trace a quadrilateral rotated from the original, treating the green length as a side.))
Natalie: Like, we’re going a<Figure 11.4, Panel B>round<Figure 11.4, Panel C> it<Figure 11.4, Panel D>. Like the <Figure 11.4, Panel E> same growth factor.
Lauryn: O::::::OH!
Natalie: Yeah.
Lauryn: Yeah.
Jillan: So what is that stake in the middle (doing)?
Natalie: Well this is just like the point of dilation. Like where we’re taking it out from.
Jasmine: Is that a specific point or is that- you just picked it randomly?
Natalie: Just wherever.
Jasmine: So you’re dilating from that point, one point five times.
Natalie: Yeah.
Jasmine: Ok.
Natalie: Is that gonna work? Ca-
Tahir: I think it might need to be the center.
Jasmine: I don’t know. I think it’s a cool idea.
Natalie: I doesn’t- It shouldn’t need to be a center, it’ll just be li-
Lauryn: I think we should have (did) a square.
((Lauryn and Jasmine laugh.))
Jasmine: So- how did you decide on this green thing?
Natalie: Well, we’re just waiting-
Tahir: Well the green thing is actually one point five times the length of that.
Lauryn: Yeah.
Natalie: Yeah.
Jasmine: Ok.

Natalie: So we were gonna do something else but then we realized like the angles would be hard to like, copy, so, now we’re just getting this untangled, so we can li-

Jasmine: Oh, so you’re avoiding copying angles. I see.

Natalie, knowing that the others were not in agreement with her about the strategy, looked at Nate and said that she felt like she was “doing something really stupid” [272]. Tahir responded, “I don’t think either of us really understand what you’re doing” [273] and Lauryn agreed. Natalie tried to explain again, this time appealing directly to the paper-and-pencil version of the procedure: “You know, like dilating on a grid? Like when you get like a triangle and you pick a point in

FIGURE 11.4  Still frames from Excerpt 2. Panel A: Lauryn traces the first side of her imagined new quadrilateral with her arm, starting with the green tape as a new side. Panels B–E: Natalie traces her imagined new quadrilateral with both of her arms, with her body inside of it.
the center and you go out with your ruler? To like make it bigger?” [275]. Lauryn just shrugged in response. She and Tahir did not know about this, because they had not yet learned it in school. Natalie’s description was of a procedure rather than the mathematical relationships they needed to manipulate in order to produce a scaled figure. Talk alone was not enough to communicate the plan to her group mates.

Although Nate encouraged them to “Just keep trying it, see what happens,” [278] Tahir and Lauryn remained unhappy with the plan. Tahir reminded them of their original plan by saying “I thought we were gonna…” [280], implying that their current course of action was a surprise to him, not what he thought they had agreed to do. Soon after, Lauryn asked, “So how is this gonna, like” [289]. She did not finish her question, but her quick affirmative response to Natalie’s “Well I don’t know yet” [290] revealed her lingering skepticism. If even Natalie did not know what was going to happen, there was clearly a problem. However, Natalie was not willing to give in, clarifying that what she did not know was “what like, the growth factor is supposed to be” [292].

At this point, Lauryn interrupted Natalie to notice that, “from this angle, our sides are really off” [293]. Lauryn, standing in her position from outside and some distance from the quadrilateral, was attending to their representation as a whole and assessing it. Nate reminded her that they had agreed they made a quadrilateral, not specifically a trapezoid [298]. After I walked over to ask Lauryn what she meant by saying that their sides were off [303], two seconds passed, and Lauryn, still looking out over the whole quadrilateral, asked, “So, we’re gonna go <Figure 11.4, Panel A>, this way?” [305]. Using her forearm to represent sides of the quadrilateral, Lauryn traced a quadrilateral in a way that began with the green tape lying on the ground as one of the new sides. Natalie responded, “Like, we’re going a<Figure 11.4, Panel B>round <Figure 11.4, Panel C> it <Figure 11.4, Panel D>. Like the <Figure 11.4, Panel E> same growth factor” [306]. Kneeling inside the original quadrilateral, at the center of dilation, Natalie used both arms to trace a quadrilateral around her. In response to this, Lauryn exclaimed, “O:::::OH! Okay I get it now” [307, 309]. It is unclear what sense Lauryn made of Natalie’s response, but the contrast highlighted in this exchange at least served to specify the planned orientation of the new quadrilateral and its spatial relationship to the original one.

At this point, Jillian and I (Jasmine) began asking the group some clarifying questions about the center of dilation and the green length of tape [310–330]. This gave Natalie more opportunities to describe her strategy, and given the specificity of the questions, she was positioned to articulate, in interaction with her group mates, the materials, and us, some of the particulars of the process. Natalie named the flag inside the quadrilateral (“the point of dilation” [311]), reiterated that it was set “Just wherever” [313], and I summarized the plan (“So you’re dilating from that point, one point five times”), articulating it as a procedure that was familiar to me [314]. Natalie again requested confirmation that it would “work” [317], and her uncertainty provided Tahir another opportunity
to voice his concern about the center of dilation being in the center of the quadrilateral [318].

When I asked the group about how they “decide[d] on this green thing,” [323] Natalie began to explain that it hadn’t been “decide[d]” on yet, but Tahir cut her off, letting me know that it had “actually” been measured to be “one point five times the length of that” [325], the side of the quadrilateral. Both Lauryn and Natalie agreed with this; the green tape did not yet have a new mathematical identity, although Natalie had new plans for it.

**Excerpt 3**

Once Tahir finished untangling the yellow rope, the group resumed creating the first spoke of their dilation. He brings the yellow rope over so that one end is at the center of dilation where Natalie stands, and lays it out through the vertex of the quadrilateral. Tahir and Lauryn continue to participate both physically and in talk, at times trying to anticipate Natalie’s unfolding plan, but also following her directions and asking questions. As Natalie folds the green piece of tape into three pieces, each half the distance between the center of dilation and the vertex, both Tahir and Lauryn continue to demonstrate confusion and some resistance.

[331] Tahir: Okay I think this is [probably (about)]
[332] Natalie: (Okay) Okay. Do you wanna just like, pull it over to that corner? Just so we have like a straight [line as comparison
[333] Lauryn: [Okay I get it now=/
[335] Lauryn: =I-I just didn’t really understand what you were saying for a minute, ‘til we actually started doin’ it.
[336] Natalie: Okay. (To Tahir) Yah. And just like leave it there?
[337] Lauryn: And is he going to this one? (points at vertex to her left)
[338] Natalie: Uh, no. Not yet. ((She folds the portion of green tape inside the quadrilateral, bringing the end to the vertex, to represent half the distance between the center of dilation and the vertex. To Tahir) And then just, could you hold that there ((the end of the green tape at the vertex))? With that. Okay. ((She holds the now folded end of the green tape to the ground. Then, to Lauryn)) And then, fold? ((pointing down the length of the green tape)) Hm, the green, the other green back over?
[339] Lauryn: [This one?]
[340] Natalie: [Just so it’s the same length at this ((points to the folded segment she and Tahir are holding))? (To Tahir) But keep– holding that there ((points to vertex)). So we like measure off, ((Lauryn brings her end of the tape over to the vertex and bends down)) NO like,=
[341] Lauryn: =Okay now I’m confused.
[342] Natalie: Like bring this (hh) over (hh)here. ((Lauryn hands Natalie the tape end)) [And then like, ((puts Lauryn’s end at the end of her folded half))]
Natalie asked Tahir to bring over the yellow rope to act as a straight line, for reference. Lauryn declared again that “I get it now,” that “I just didn’t really understand what you were saying for a minute, ’til we actually started doin’ it” [333,335]. But then she tried to anticipate Tahir’s next move, “And is he going to this one?” [337] pointing to the vertex to her left. That vertex would not come into play until after this spoke was finished; Lauryn’s anticipated next move was not aligned with Natalie’s strategy.

Natalie walked her end of the green tape over to the vertex and asked Tahir, squatting there, to hold it down. She backed up to the folded halfway point, telling Lauryn to “fold? Hmm, the green, the other green back over? Just so it’s the same length at this?” [338, 340]. She wanted Lauryn to bring her end of the green tape to her so they could fold that part of the tape at Tahir’s vertex and lay it over the already folded “half” between Natalie and Tahir. Lauryn’s part of the green tape would determine where they tore Lauryn’s section of tape to produce a length 1.5 times the distance from the center of dilation to the vertex. Lauryn walked to the vertex, and began to put the end of tape down. In the green tape’s previous role as a side of the new quadrilateral, both of the ends had meaning as the endpoints of that particular measured line segment. However, for the dilation strategy, the procedure that they were in the midst of implementing would determine the new endpoint of that green tape (which would also tell them where the new vertex should be). As a length of tape with an as yet undetermined length, the end of the
tape that Lauryn was holding did not have any mathematical meaning. Lauryn attributed some meaning to it by bringing it to the vertex.

Natalie stopped her (“NO like”), prompting Lauryn to quickly respond, “Okay now I’m confused” [341]. Lauryn’s visible, whole-body motion made it clear that they still did not share an understanding of the plan. Natalie then gestured for Lauryn to bring the green tape end over, and Lauryn handed it to her. Natalie herself then struggled to complete the measurement. Her action was similar to Lauryn’s, in that she momentarily gave mathematical meaning to the end of the green tape, putting it at the fold that she was holding (which demarcated the half-way point between the vertex and the center of dilation). She then hesitated and stumbled in her talk ([342]). Finally Natalie pulled the green tape end back past her body (and also the center of dilation) until it was folded at the vertex that Tahir held fixed.

Here, Lauryn’s manipulation of the green tape was not what Natalie expected or wanted, and Natalie took over and did it herself. Unlike earlier, when Tahir revoiced Natalie’s instruction for Lauryn [263], the spatial configuration of this sub-task (finding the final “half” to create 1.5 times the distance between the center of dilation and the vertex) was such that it was not difficult for Natalie to take it over once Lauryn handed her the tape end. Although Natalie denied the opportunity for Lauryn to engage in the task, by doing so she provided feedback to Lauryn about her current understandings. She was able to do so because of the visual availability of Lauryn’s actions. The large-scale physical space, whole-body engagements, and distributed perspective of WSG gave students additional resources for assessing each other in the midst of problem solving, and the division of labor made it necessary to give each other feedback in order for the activity to progress.

At this point I came over to see what they were doing. Both Tahir and Lauryn were quick to say that they did not know. Natalie reported, “Now we’re going-out the- point- ([Tears the excess off the green tape.]) five. So we already have the one that’s within?” [347, 349]. As soon as Natalie tore the green tape at the point it overlapped with her halfway fold, in conjunction with her utterance that this is “point- five,” Tahir exclaimed that he understood how the newly ripped tape is 1.5 [350, 353], even though two turns at talk before he had just told me that he did not know what was happening. Lauryn agreed that she understood as well [352]. Tearing off the excess destroyed the previous mathematical measure (and corresponding physical properties) of the green tape, allowing it to take on the properties of the new scaled measure of the distance between vertex and center of dilation.

I responded by stating my understanding of the 1.5 (“[So that’s one ([Pointing at the center of dilation and vertex]), and then that’s point five ([Pointing at vertex and other end of green tape])? [And so that’s how you know your dilation?” [358]) and Natalie agreed. She then narrated the end of the procedure, “and then we need, this is our vertex corresponding to that one” [360]. This was the first time that
anyone had stated explicitly that what they were doing was finding a new vertex (rather than a new side, or something else entirely). Up until now, Natalie had pointed outward past vertices and referred to the vertices (old and new) as “that” and “over there.”

As the students completed the rest of the quadrilateral dilation, while the process was not completely smooth, it was clear that their understandings of what they were going to do were aligned. Rather than challenging and questioning each other, they successfully completed each other’s sentences or left sentences unfinished rather than overlapping and interrupting. They anticipated each other’s actions and material manipulations. They swapped roles (e.g., for the next spoke, Lauryn stood in the middle, Natalie held the tape down at the original vertex, and Tahir handled the end of the tape that would become the new vertex). By the time they were working on the fourth vertex, much of their talk was topically unrelated to coordinating their actions, including chitchat with the researchers about their school geometry course experiences, and the dilation activity was completed with little explicit negotiation. Lauryn and Tahir took the lead, and in the end it was they who initiated and completed producing the sides of the new quadrilateral, excited to find out if the group’s strategy worked.

Discussion

The multimodal interactional analysis above demonstrated how talk, bodies, materials, and other representational resources combined to constitute the group’s learning and knowledge of this dilation strategy. There were three primary obstacles to, or goals for, the successful implementation of the quadrilateral dilation strategy. First, Lauryn and Tahir had to be recruited to participate. Natalie could not implement her idea on her own. Second, they needed a way to participate, to know what to do. Natalie was having trouble articulating, through talk, what the general plan was and the specific actions she wanted them to take. Finally, new vertices needed to be placed for the new quadrilateral, based on the original. For the purposes of this chapter, the episode focused on just the first new vertex, a sub-goal of the solution in its entirety.

Because of the large-scale, distributed division of labor of the WSG setting, it was crucial for all students to participate in order for the strategy to develop. The dynamic of this group was also such that they always tried to come to agreement and work together. Therefore, Lauryn and Tahir’s ongoing skepticism and reluctance was a visible source of trouble for the group and specifically for Natalie, who repeatedly made appeals to the researchers for validation that the strategy would work. Lauryn and Tahir consistently made bids for further explanation or feedback while the group worked together. They did this by anticipating next moves (“Are we gonna use the same length?” [255]) or by challenging Natalie (“That’s not the center point though” [259]; “I don’t think either of us really..."
understand what you’re doing” [273]). Lauryn and Tahir’s physical actions and engagements with the materials also served as openings for, and at times requests for, further explanation or feedback from Natalie (e.g., [340–342]). While Lauryn and Tahir’s reluctance and confusion may be taken as obstacles to the development of the strategy, these exchanges provided a resource in the form of opportunities for the group to articulate, rehearse, and revise the strategy, and to assess their ongoing work together. As Lauryn and Tahir became convinced the strategy was a good idea, the mood of the group shifted (as they produced the final three vertices, talk mainly turned to banter and small talk), and the two moved around more (and more quickly), taking ownership of different aspects of the dilation.

Additionally, the many researchers who came and went over the course of the episode played a role in the recruitment of Lauryn and Tahir (and in Natalie’s persistence). Knowing that this was a solution no other group had tried in our work with WSG tasks, we were all eager to see how it might play out. Nate had let us know earlier that Natalie had proposed something interesting, and Jillian and I were spending a lot of time observing the group. Rogers had also got wind of the dilation effort, and brought over the extra microphone to try to capture as much of their activity as possible [282]. These were all additional resources available for the group to implement this strategy. Natalie actively recruited Nate’s reassurance and validation in the face of her classmates’ skepticism. He voiced strong support, and helped her idea maintain its status as something to try. Jillian and I tried not to “give away” that the strategy would work, but we helped to name some parts of Natalie’s plan (e.g., “you’re dilating from that point” [314]). In the end, I also validated Natalie’s concern about copying angles by explicitly understanding (“I see” [330]) why they switched from scaling the sides to this new strategy.

In the end, Lauryn and Tahir’s conversion to enthusiastic dilators (Lauryn later announced, “This is a really good idea now”) was the demonstration of a successful placement of a new vertex. I call it a demonstration because Lauryn and Tahir could see and experience the actions necessary for the vertex to be located, and the 1.5 relationship was also made visible to them through the manipulations of the green tape, and through talk and gesture, and they played a crucial role in the production of this demonstration.

The three students, with differing initial (and final) understandings of the mathematics, all participated in solving the task, and as they did so, they had more opportunities to participate more centrally in mathematically significant ways. Lauryn attended intently to what Natalie was saying and doing, and followed Natalie’s instructions carefully. In the beginning, Natalie gave her instructions using mathematical language to refer to parts of the materials they were using (“So like make this line go out from that point?” [260]). Tahir’s revoicing used gestures and terms related to the materials to help Lauryn comply with Natalie’s request. As they progressed, the group continued to use gestures and language rooted in the everyday uses of the materials rather than the mathematical referents.
while negotiating and coordinating their actions. The three group members all had access to participation due to the everyday materials and whole-bodied, readily visible actions required to manipulate them. This led to increased and more mathematically engaged levels of participation as the activity progressed.

As for the placement of that first new vertex, the negotiation of meanings for and mathematical relationships between the WSG “drawing” materials was crucial. The everyday materials of WSG were constantly mathematical tools-in-the-making, as determined by the activity of the group. As the students shared a local history of engaging in the tasks together, the materials’ mathematical meanings or uses became more stabilized and tacit. This at times served as an impediment to developing new understandings (e.g., the end of the green tape), and at times allowed the continuing co-construction of the mathematical strategy (e.g., a shared routine of doubling tape over, one person holding the ends together and another holding the fold, developed in the construction of the green tape length the first time). However, as the students negotiated meanings and uses for the materials, they all could – and because of the division of labor, had to – still manipulate their bodies and the materials in the context of the mathematical activity.

The large-scale spatial configurations, interconnected whole-body and material manipulations, and distributed perspective in this setting allowed the students visual and physical access to each other’s actions, making possible ongoing assessment and providing opportunities for feedback. When these actions were counter to the mathematical meaning or relationship one student attributed to materials, or prevented a student from completing his or her own action, there was a clear indicator of divergent understandings. Natalie could see and physically feel Lauryn and Tahir’s actions, easily identifying when they were misaligned with her dilation plan. Because of the need for multiple bodies to complete the WSG tasks, it was in the best interest of the students to give each other feedback and continue to negotiate their understandings.

This analysis provided an example of a form of multimodal interaction analysis that accounts for a larger set of resources for meaning making and the accomplishment of a mathematics task. The analysis attended to whole bodies, available material resources, developing representational infrastructure, and spatial relations, as well as talk and gestures. The analysis took a sociocultural perspective on learning and knowledge, inviting questions of what counts as evidence of learning and knowledge, taking all of these resources into account. The development of the strategy emerged though the interaction of the group, and the capacity for dilating quadrilaterals was a property of the ensemble, not of any one individual.
Commentary

FROM THE INDIVIDUAL TO THE ENSEMBLE AND BACK AGAIN

Luke D. Conlin and David Hammer

Jasmine Ma presents an analysis of three students’ interactions during an episode of a mathematical activity, Walking Scale Geometry, which she designed to support “ensemble learning.” In this case, the challenge was to dilate a quadrilateral, laid out on the lawn with flagging tape, by a factor of 1.5. Her analysis of the students’ interactions with each other and the physical materials shows that it was the ensemble – Natalie, Tahir, and Lauryn as a group – that learned to complete the task. Thus Ma has offered a rich and compelling example of Interaction Analysis (IA).

The aim of KAIA and this volume being to bridge IA with KA, Knowledge Analysis, we see ourselves charged with adding a KA take on the data. Our interest more broadly is to offer a view on coordinating IA and KA, which we believe is not only possible but necessary for progress. Like Cobb (1994), we argue for coordinating IA and KA based on the evidence at hand.

Most of the literature in both IA and KA makes assumptions about the unit of analysis based on a priori theoretical commitments and interests. Ma is explicit about that here. Following Hutchins (1995a, 1995b), Ma attributes cognition and learning to the ensemble, and resists making claims about individuals, although “this is not to claim that there is not individual knowledge or mental activity.” To be sure, she makes numerous individual attributions in the course of her analysis, nested within her compelling case for ensemble learning. From our perspective, this is not only appropriate but necessary: The dynamics of cognition and learning take place at a range of scales, in Ma’s data as in general, from within individual minds to across multiple minds and materials. Understanding ensemble learning involves attention to its members. Of course, we argue the reverse as well: Any sensible account of individual knowledge and mental activity must attend to the larger situation. Ideally, an approach that bridges KA and IA would include strategies to guide researchers’ selection of the unit of analysis, perhaps analogous in some ways to strategies researchers use to guide their development of theory from qualitative data (Glaser & Strauss, 1967) or synthesizing coding categories (Chi, 1997).

Elsewhere (Conlin, Gupta, & Hammer, 2010a, 2010b), we have posited empirical heuristics for keeping track of evidence of the scale of cognitive dynamics,
allowing for decisions regarding the unit of analysis based on the data. In what follows, we review that previous work, present the heuristics, and apply them to Ma’s data. Our analysis agrees with Ma’s central claim that the ensemble learns a strategy for how to dilate the quadrilateral. However, by allowing our attention to shift to individuals, guided by our heuristics, we can say more about how those individuals’ roles may contribute to ensemble learning.

Heuristics for Tracking the Unit of Framing

Our focus in earlier work was on students’ epistemological framing, a construct Redish (2004) proposed, connecting resource-based accounts of intuitive epistemologies (Hammer & Elby, 2002; Rosenberg, Hammer, & Phelan, 2006) with accounts of framing in cognitive science (Minsky, 1988) as well as sociology and anthropology (Bateson, 1955; Goffman, 1974; Tannen, 1993). The former consider learners’ intellectual resources for understanding knowledge and knowledge-related activities. The latter consider people’s sense of “what is it that’s going on here?” (Goffman, 1974, p. 8). “Epistemological framing” considers learners’ sense of what is going on with respect to knowledge. Most examples of epistemological framing in the literature focus on individuals (Bing & Redish, 2009; Lising & Elby, 2005), but a number attribute framing to a group or class (Louca, Elby, Hammer, & Kagey, 2004; Scherr & Hammer, 2009). Scherr & Hammer (2009) argued:

The question of which system to consider, or what scale of system to treat as the cognitive unit, should not be decided a priori, and it does not have a general answer. Rather, it should be decided by the case at hand, by the evidence of what resources (cognitive, physical, material, representational, etc.) participate in the dynamics of reasoning.

(p. 173)

Our group at the University of Maryland conducted a variety of studies of collaborative groups working on tutorials in introductory physics (Elby et al., 2007). As a collection, these studies reveal a range of dynamics, at various scales. We drew on this work to extract a set of heuristics to identify the relevant unit of analysis, specifically with respect to epistemological framing (Conlin et al., 2010a, 2010b). Scherr and Hammer (2009), for example, presented evidence of framing for a group as a whole. Scherr had noticed collective shifts in behaviors at the group level. For instance, groups transitioned from what Scherr labeled the “blue” cluster – students hunching over the table, eyes on their papers, and speaking in soft tones with hands gesturing discreetly – to what Scherr labeled “green” – sitting up, making eye contact, speaking in full voice and gesturing conspicuously. Analysis of the discourse in these distinct clusters of behavior revealed that they corresponded to different epistemological frames, “completing the worksheet” vs.
“having a discussion.” The locus of stability of these frames tends to reside with the group, as evidenced in moments in which a group would be completing the worksheet and one student would pop up in a “bid” to have a discussion, only to hunch back over and return to their worksheet when nobody else followed suit.

In another analysis of students in physics tutorials (Conlin et al., 2010a, 2010b), we presented evidence of two students’ (Veronica & Jan) contrasting frames, drawing on analyses from Lising and Elby (2005). Working on a tutorial in optics, Veronica and Jan clashed over how to describe the path taken by light from a bulb to a screen. Veronica gave an intuitive answer of how the light moves, while Jan was trying to make things more “physics-oriented” by using official vocabulary (“rays” and “vectors” that are “polarized”). Veronica resisted Jan’s use of these phrases, saying that she was “making it too complicated.” A disagreement over which way the light moves revealed that Jan and Veronica had different stable framings of what it means to be “physics-oriented.”

Frank (2009) showed multiple scales of dynamics in his analyses of tutorial groups. Across various groups, he showed evidence of ensemble thinking, the group as a whole making a particular inference about the speed of an object and shifting, as a group, to a different inference. Interestingly, groups’ shifting in conceptualization co-occurred with their shifting in epistemology, from reporting something they found obvious to reasoning carefully based on mechanism. Looking moment to moment at one group, however, Frank showed evidence of contrasting dynamics for one student, who was doing something different from the others. Her thinking, at first distinct from the group’s both conceptually and epistemology, led the group to shift as a whole. This analysis demonstrates the dynamics of how the locus of stability of a framing can shift levels, from the individual to the group (and back again).

Looking across this collection of analyses, we posited a set of four heuristics that highlight distinct forms of evidence for deciding the relevant unit of cognition. While they are influenced by a complex-systems view of cognition, the heuristics are largely consistent with forms of evidence detailed in Interaction Analysis (McDermott, Gospodinoff, & Aron, 1978; Schegloff, 1991). We describe them here, and then we apply them to Ma’s data.

**Clustering**

Clustering refers to the patterns of co-occurrence in multiple aspects of what researchers observe, such as Scherr (Scherr & Hammer, 2009) recognized in students’ posture, gestures, and speech. McDermott et al. (1978) have also noted that mutually interacting participants tend to organize their postures in coherent patterns that signal their mutual sense of what is going on. We take these behavioral clusters as evidence of the unit of framing which resides across whatever actors and materials are involved in the pattern. We have described how distinct
clusters of speech and behaviors can at times signal a shared epistemological frame (Conlin et al., 2010a, 2010b; Scherr & Hammer, 2009). At other times, the clustering is at the level of the individual, as was the case for Jan and Veronica (Lising & Elby, 2005), who each had a distinct way of framing what it means to be “physics-oriented.”

**Transitions**

We see evidence of the unit of framing in transitions between clusters of speech and behavior across the set of actors and materials that participate in the transition. This is consistent with the observation in McDermott et al. (1978) that transitions between group members’ positionings can reveal shared aspects of their sense of what they are doing together. Scherr and Hammer (2009) describe how tutorial groups would often transition from one group activity (having a discussion) to another (completing their worksheets) abruptly and without any explicit bid to shift activities. This transition thereby provides evidence that they shared a sense of these alternative ways of framing their activity together.

Transitions can reveal individual differences in framing as well, as was the case when a new tutorial group member asked “Can we discuss our answers now?”, thereby challenging and ultimately shifting the established norms of the group, which had been dismissive of discussions up to that point (Conlin, 2012). Frank’s (2009) analysis of tutorial groups also detailed a case in which one student’s thinking led to a shift in the group’s thinking. These transitions provide evidence of both the individual-level framing before the shift and the new group-level framing after the shift.

**Persistence**

Persistence refers simply to duration in a set of observables. Again, we take it as evidence of the unit of cognition, i.e., the set of actors or materials participating in a pattern over time. Regarding the tutorial groups, it was typical to observe the group in a particular behavioral cluster, e.g., hunched over in the blue-worksheet cluster, for minutes at a time. For most of the groups, their time in tutorial could be generally described as long periods of shared behavioral clusters, punctuated by brief transitions between clusters. But at times, and for some groups more than others, we found evidence of individual students’ distinct epistemological frames that persisted over time, as was the case for Jan and Veronica.

**Resistance**

Resistance refers to persistence in the face of a challenge. In the physics tutorials, the group-level patterns of behavior and reasoning would often show
resistance to “bids” from individual students for a shift in activity, say, from completing the worksheet to having a discussion. These bids could be implicit, as when a group is hunched over their worksheet and one student sits up for a moment, puts her pen down and looks to talk, but finds the group unresponsive and so returns her attention to the worksheet. In other instances, the student would sit up and stay up, start speaking and trying to engage others in the group more directly, as in, “Can we discuss our answers now?” (Conlin, 2012). Persistence of the group in the face of individuals’ bids for a shift in activity would be evidence that the locus of stability of the framing of their activity resides with the group.

Further examples of resistance can be evident in repairs of understanding (Schegloff, 1991) and explicit disagreements. Conversational repairs offer resistance against whatever is being repaired, be it a conceptual understanding of a phenomenon or an epistemological understanding of what’s going on in a situation. Veronica’s resistance to Jan’s use of physics vocabulary signaled that each of them had their own framing of what it means to be “physics-oriented.”

Of course, there are always many scales and layers of framing. A group framing their activity as “having a discussion” is typically comprised of individuals framing their participation in different but coordinated ways. While one member momentarily frames their participation as articulating an idea, the others may be framing it as trying to understand the idea. The group’s framing may be stable even if individuals momentarily diverge: the stability in one respect does not imply stability in another. For example, consider a group that remains stable in having a discussion even after realizing at some point that individuals have been making different assumptions – “I thought we were talking about dinner tonight, and you’ve been thinking of sometime next week.” Thus the heuristics apply to an aspect of what is taking place (“having a discussion” vs. understanding of the details of that discussion), and they guide analysis of the scale involved in that aspect.

We have suggested these heuristics can support researchers in selecting the unit of analysis with respect to framing, and perhaps regarding other aspects of cognition as well – such as a “concept” or an affective state. Importantly, they allow for the possibility of identifying shifts in the evident unit of cognition, moment to moment. These shifts can proceed from the individual to the group, and vice versa, as Frank’s (2009) analysis showed. We believe this is evident in Ma’s data as well.

We turn to her data now, using these heuristics. We agree with Ma that the ensemble learns a strategy to dilate a quadrilateral that they did not know before; her analysis allows us to see the moments that constitute ensemble learning. However, we argue that when we apply these heuristics, the unit of understanding the strategy evidently begins with the individual and shifts to the ensemble. And, we hope, perhaps it shifts back again to the individuals.
Tracking the Unit of Learning

In tracking the unit of analysis, it is important to be clear on just what aspect of cognition we are referring to when we say cognition is shared, situated, and/or embodied. Hutchins (1995b) referred specifically to the act of remembering, while Hall (1996) described representation as a situated action rather than a mental object. In our previous work, we have focused on the epistemological framing of physics students at the individual and group levels.

Ma focuses on learning as distributed across an ensemble of students and materials. While we have not used our approach to address the dynamics of learning per se, we expect the heuristics will apply. If we think of understanding as the current state of a cognitive ecology, i.e., the set of resources activated in the moment, then learning would be a persistent change in the cognitive ecology, whether that ecology resides within an individual, within a group, or both. Learning could be evident, for example, as a shift in the cluster of resources the individual or group cues in a context, or as a shift in such a pattern’s resistance to change.

How an Ensemble Learns to Dilate a Quadrilateral

Ma recounts a group of three students solving a math problem laid out on a lawn: The task is to make a copy of a quadrilateral, laid out in flagging tape, 1.5 times as large as the original. “Natalie quickly proposed a dilation strategy that she vaguely remembered learning,” but “Lauryn and Tahir wanted to scale each side of the quadrilateral 1.5 times.” Natalie resisted, noting that their strategy presented them with the challenge of copying the angles precisely. “After some disagreement and the prompting of a researcher (Nate), Lauryn and Tahir agreed to try Natalie’s strategy,” which was to pick a center point and extend a line from it through each vertex. By scaling those lines by 1.5, they could “avoid copying the angles.” Natalie was just unsure whether dilating out by 1.5 meant the sides would also be 1.5 bigger. As they carry out Natalie’s strategy, Lauryn and Tahir initially “did not have a sense” of her plan, and she directed their participation, asking them to hold things down so she could measure out a new vertex. But as they moved on to make subsequent vertices, Ma’s analysis of their interaction reveals gradual shifts in the group’s understanding of the plan and willingness to pursue it, as manifest in their increased anticipation of each step, and their shared investment in its unknown outcome. When they ultimately find out that the strategy was successful, Ma argues, it is the ensemble that has learned how to dilate a quadrilateral.

The physical scale of the task that Ma set out for the students prohibits any one individual from seeing the entire shape they are dilating, since the shape is so large. With different perspectives on the shape depending on where they stand, the students need to coordinate with each other to determine whether lengths and angles are really equivalent. Based on these factors, Ma takes this task to be
set up for ensemble learning – no one can easily do it on their own. By taking an explicitly sociocultural, interactional, and embodied account of knowing and learning, Ma’s analysis seems to presume that students in the group will take up the task in a way that is inherently distributed at all times. The evidence we find, however, shows a dynamic shift from an individual, Natalie, to the ensemble, with respect to understanding the dilation strategy. We detail each phase of this shift in what follows.

**Phase 1: From the Individual…**

In her analysis, Ma attributed several things to Natalie as an individual, including dissatisfaction, the start of an alternative strategy, as well as a more rigorous standard for what counts as scaling up the quadrilateral. This is an observable cluster in Natalie’s speech and behavior, and the data show its persistence and resistance to Lauryn’s and Tahir’s reasoning. Lauryn and Tahir, for their part, showed a different pattern of evidence, also persistent and resistant – it took a researcher’s prompting, presumably an authority figure for them, to persuade them to try Natalie’s approach.

Even before they began to try her approach, Natalie showed higher standards of precision in measurement. To extend a side by 1.5, Tahir asked Natalie to hold down the tape at one vertex, while Lauryn and Tahir rolled out the tape along the full length of a side of the quadrilateral. Tahir held the tape at the other vertex, then with the excess tape Lauryn doubled back until the tape made it about halfway back to Natalie’s vertex. Lauryn and Tahir were all set to rip the tape off where they had eyeballed it to be about halfway (Tahir: “Okay, rip it off there, I guess…”), when Natalie stopped them to insist that they “make it exact.” She told Tahir to hold down his vertex as she folded the tape in half. Handing him her vertex, she pulled the halves taut, and then asked Lauryn to rip off the excess in line with the halved tape. This ensured the tape was 1.5 times the original side. Thus, in resisting Lauryn and Tahir’s strategy for extending a side, Natalie constructed her own strategy for extending the side by 1.5, a strategy situated in the materials at hand.

Lauryn and Tahir then walked this new extended side out away from the quadrilateral to eyeball where it would sit for the new quadrilateral. Again, Natalie resisted by questioning how they would make sure the angles were congruent, to insure that the new side was precisely parallel. With Tahir and Lauryn showing no signs of sharing her concern, Natalie reached out to the researchers/facilitators. She asked them what they considered to be a loaded question: “What would be like, how much would we dilate it if we wanted it to be like one and a half?” Nate, the researcher, encouraged her to try it. She then walked to what she selected as the “point of dilation” in the quadrilateral and called the others over.
In Excerpts 1 and 2, Natalie, Tahir, and Lauryn were trying to construct the first vertex. Tahir and Lauryn showed resistance, apparently because they did not understand Natalie’s strategy. Natalie directed them around, telling Tahir to hold down the tape at the old vertex, to put the yellow rope down, which involved untangling it, etc. When Tahir noted that the point Natalie was using to dilate from was not the center of the quadrilateral, Natalie asserted that it did not have to be, without offering justification, and Tahir did not press further. Perhaps as a result of their passive direction-following without sign of agreement, Natalie shared that she felt like she was “doing something really stupid.” This allowed Tahir and Lauryn to say they didn’t understand her strategy. Natalie tried to explain, but she had a hard time putting it into words.

Throughout, the evidence shows Natalie’s individual persistence and resistance to Tahir’s and Lauryn’s suggestions and behavior, and theirs, as a pair, to hers. While there are certainly aspects of the group’s work that reflect ensemble thinking — they are out on the lawn, they are taking turns speaking and listening, etc. — applying our heuristics to the data allows us to attribute distinct understandings of the dilation strategy to Natalie, as an individual, and to Lauryn and Tahir as a pair. Natalie’s shows greater persistence, perhaps in part due to the adult’s support. Lauryn and Tahir transition from pressing their idea to implementing Natalie’s.

Phase 2: To the Ensemble…

As the group was finishing the first vertex (~9:30), there is evidence of a group-level transition. Natalie was applying her folding strategy for extending the side by 1.5, using the same tape, only now she was using it to find the location of the new vertex. Tahir was observing Natalie folding the tape, at first confused by the extra tape left over from the side they had already extended, but suddenly realizing what Natalie was doing: “I know how it’s 1.5 now.” It also became apparent that Lauryn had thought they were creating a side to the new quadrilateral, which was not what Natalie had in mind. When Natalie explained they weren’t creating a new side at all but a new vertex, Lauryn also expressed a sudden realization, saying “Ohhh, I get it now.”

Tahir spoke to express ongoing doubt, but signaled his willingness to follow through, and Lauryn offered support (lines 356–359 of Ma’s transcript):

[356] Tahir: Are you sure it’s gonna be proportionate to the other, ah, angles? I guess we can just try it and see what happens.
[357] Jasmine: Yeah let’s find out.
[358] Natalie: Yeah, Ok, so now,
[359] Lauryn: Yeah cause if we go that way ((points right hand to where the next new vertex will be)) and then connect them here ((points left hand to the new vertex they just made)), or wherever the flag is over there, then it should be.
Lauryn’s gestures helped clarify her meaning: She pointed to the new vertex with her right hand while pointing to the dilated vertex they just constructed with her left, indicating she was sizing up the new side to see that it “should be” about 1.5 times as large. In this way, the evidence shows Tahir’s and Lauryn’s shift from resistance to acquiescence, as a pair, a transition that signals a shift toward a shared understanding of the dilation strategy, and a shared willingness to try it.

From this moment forward, Tahir and Lauryn supported the dilation strategy, and the evidence increasingly suggests the ensemble is now the unit that understands the strategy. As they worked on the second vertex, Lauryn and Tahir started to anticipate Natalie’s instructions, a shift from their earlier stances of resistance and then passive obedience. Thus Lauryn walked to the center of dilation and asks, “do you want me to hold this here?” Tahir corrected Natalie on not ripping the tape at the old vertex, suggested she tie it back together, and Natalie followed his suggestion. Tahir anticipated the next step, which was to have Lauryn fold over her end of the tape from the center, finding half the distance to the vertex “and now she [Lauryn] has to bring it over, like.” Lauryn finished his sentence with “half.”

At this scale of analysis, the clustering was evident at the level of the group, to the point of the students finishing of each other’s sentences, and this continued. As they started to construct the third vertex, Tahir anticipated the results of their strategy:

[399] Tahir: It’s gonna look like a, wheel or something like a, four-sided wheel.
[401] Tahir: It’s gonna have spokes.

Once Natalie had stretched the pink tape out to the vertex, Lauryn started to walk the pink tape in from the center to fold it in half so they could rip the excess off at 1.5. As Lauryn walked, Natalie acknowledged what she was doing: “bring it in, yeah.”

A researcher (Rogers) started asking questions, and Lauryn was answering first, before Natalie had a chance to answer. Although she attributed the strategy to Natalie, Lauryn’s responses showed it was no longer just Natalie’s. Working on vertex 4 (~14min) the group showed more evidence of ensemble thinking: Natalie was no longer giving directions, and in fact it seemed they no longer needed to talk to each other to coordinate. The facilitator noted they were “much more efficient,” and the evidence shows all three students involved in these patterns of activity and (lack of) speech that persist over time.

When they had all four vertices dilated, the ensemble transitioned seamlessly to making a new side, both to finish the construction and to test that the strategy worked. Together, they constructed a new side to test whether their dilating of the vertices by 1.5 would mean that the sides were also scaled by 1.5. Using the side they had originally extended, they repeated the folding procedure to again
measure out a new side that was 1.5 times bigger. The same measurement now meant something very different: they were checking for coherence between the new strategy and the original strategy. When they stretched out the new side and found it perfectly connected the new vertices, they exulted. Satisfied, they completed the other three sides of the new quadrilateral without much suspense.

Throughout this, Natalie, Tahir, and Lauryn all participated in the cluster of activity of construction as well as the transition to the new cluster of testing and celebration, and finally the falling action of completing the dilated quadrilateral.

Even though Natalie was originally directing the activity, they succeeded as a team. The episode reminds us of team-building exercises; for example, the challenge of getting a group over a wall that is too tall and smooth for any individual to climb. Even if the team uses one person’s strategy, it is still the team that succeeds or fails in making it over the wall. In the end, we agree with Ma that it is the ensemble that succeeded in dilating the quadrilateral.

Conclusion

Our analysis supports Ma’s claim that the ensemble learns to dilate a quadrilateral. It also supports her analysis, including her attributions to members within the ensemble: What the ensemble learned started with Natalie, with Lauryn and Tahir first in opposition, then in acquiescence, and finally in support, invested, and participating.

If we differ with Ma, it is in our view that attention to this finer grain size – to individuals within the group – is important. For one, it contributes to understanding the ensemble, as evident in Ma’s analysis and ours. For another, we argue in the next section, education research must ultimately care about individual students.

From the Individual to the Ensemble

Ma’s and our analyses of the group involve looking within it, recognizing and describing individuals within it as epistemic agents. Viewing the ensemble as the unit of analysis in this way is continuous with analyses of individuals, from “society of mind” (Minsky, 1988) perspectives of “individuals” as ensembles themselves, ensembles of various perceptual, affective, cognitive, meta-cognitive “agents.” There are various models of that ensemble in the literature, with various theoretical commitments and foci of attention, such as of modules (Fodor, 1983) and p-prims (diSessa, 1993) and schemata (Rumelhart, 1980). We have used the generic term “resources.”

Many of these accounts, including Minsky’s of agents and Rumelhart’s and many others’ of schemas, conceptualize resources as manifold themselves, comprised of resources interacting in generally stable patterns. Knowledge Analysis and many other methodologies identify evidence of these resources and the
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dynamics of their interactions, including how they may coordinate into larger patterns and the “soft assembly” of existing resources in local time, sometimes coming to stabilize into new resources (Thelen & Smith, 1994).

All of which is to say that Ma’s “Interaction Analysis” of the ensemble is itself a kind of “Knowledge Analysis,” of an “individual mind” at a larger scale. It is analysis of an “individual” made up of multiple agents, which are themselves made up of multiple agents.

Whether considered at the individual or ensemble level, these agents can be at odds with each other, and this is part of what drives learning. There are times when an individual person may be “of two minds” – having multiple conflicting ways of understanding something. Learning occurs when these conflicts are somehow resolved, or at least coordinated. The same holds true for an ensemble. The ensemble of Lauryn, Tahir, and Natalie progressed from having individual, conflicting understandings of the dilation strategy to having a shared understanding. The conflicts serve as evidence of individual understandings, but also of ensemble learning: It was only through resolving and repairing these conflicting understandings that the ensemble learned the strategy.

…And Back Again

But what about the individual students? Would Natalie or Tahir or Lauryn have been able to carry out this task on her or his own, or lead another group in the strategy? How about on a paper-and-pencil task? It is difficult to make claims about the individual students’ learning with the data given, partly due to the decision to take data (and even set up the activity) in ways that were attuned to the theoretical commitment to take the ensemble as the unit of analysis. This is an example of how video data are theory-laden (Hall, 2000), which motivates our seeking a broader theoretical perspective.

Still, from what we see in the data, we believe Natalie would be able to dilate a given shape on a paper-and-pencil task. She had introduced the strategy, and over the course of the episode she came to articulate her reasoning clearly. We are not as sure about Lauryn and Tahir, but there is some evidence to suggest they had progressed as well, each as individuals: By the fourth vertex, they each seemed to know how to carry out the procedure, without direction. Even so, the extent to which the stability of their understanding was bound up in their interactions with this ensemble and these materials remains an open question.

But these questions beg a bigger one: Should we care whether these students individually learn how to dilate a quadrilateral, out in a field or on a piece of paper? For us, the answer is emphatically yes, we should care. This particular ensemble will not work together very often, if ever again, but the “individual” ensembles that we think of as Natalie, Tahir, and Lauryn will go on to other things.
It is important to question whether their experience of working together to dilate a quadrilateral will find any relevance in their future learning and experiences. We hope and suspect that it will. We would hope, for instance, they learned that by working together they can accomplish something none of them could do alone, rather than, say, learning how to follow instructions from someone who knows the answer. The choice to focus solely on the ensemble makes this sort of question more difficult to answer.

Hall (1996) described IA as focused less on transferring content knowledge from task to task and more on engaging students in authentic practices of the discipline, and we share this preference. Cognitivist accounts, Hall pointed out, have generally focused on what individual students might learn and be able to bring to new situations, with pedagogical activities often explicitly focusing on these content goals. Situative accounts, on the other hand, tend to want to create opportunities for communities of learners to take part in the disciplinary practices of a larger community, e.g., scientists or mathematicians.

We want both, and for both individual learners and the ensembles they comprise. Pedagogical activities such as Ma’s provide wonderful opportunities for both conceptual learning and participation in disciplinary practices, for the ensemble of students as well as for its members. The dynamics of what takes place are important at both scales.

Notes

1 Transcripts follow a modified version of Jefferson’s transcription convention (Atkinson & Heritage, 2006). Turns at talk and new lines, determined by topic of talk or activity, are labeled with [Line#], in accordance with the original transcript in the data corpus. Non-talk activity is enclosed in ((italicized, double parentheses)). Overlapping talk across turns is signified by vertically aligned [left open brackets. Emphasis is underlined, louder utterances are CAPITALIZED, and drawn-out speech with colons. Transcriber uncertainty is indicated with (parentheses). Duration of pauses in speech is indicated in seconds by (#s). When frame-by-frame images are provided, they are labeled <Figure#>.

2 Aspects of Natalie’s understanding of the dilation strategy are bound up in her use of the materials; for instance, folding the tape and ripping off excess to extend the side by 1.5. Still, the overall approach of dilating the quadrilateral from a point through each vertex is a strategy she learned in geometry class, and with different materials.

3 There are many possible senses in which an understanding could be thought of as “shared,” which may or may not include multiple individuals’ “having the same thing in mind.” For instance, it may be that each individual understands only their piece of the strategy, but an ensemble-level understanding emerges when these pieces are woven together in coordinated action. Whether the individuals have anything like a shared mental representation that could guide future action apart from the other ensemble members is a different but important question. The data here are insufficient to make any strong claims on this matter. For present purposes, we take the ensemble’s understanding to be “shared” in the sense that the locus of stability of the students’ carrying out of the strategy resides at the group level, as suggested by the heuristics.
References


Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), Handbook of research on teaching (pp. 119–161). New York: Macmillan.


