This chapter focuses on research that can inform the improvement of mathematics teaching and learning at scale. In educational contexts, improvement at scale refers to the process of taking an instructional innovation that has proved effective in supporting students’ learning in a small number of classrooms and reproducing that success in a large number of classrooms. We first argue that such research should view mathematics teachers’ instructional practices as situated in the institutional settings of the schools and broader administrative jurisdictions in which they work. We then discuss a series of hypotheses about structures that might support teachers’ ongoing improvement of their classroom practices. These support structures range from teacher networks whose activities focus on instructional issues to relations of assistance and accountability between teachers, school leaders, and leaders of broader administrative jurisdictions. In describing support structures, we also attend to equity in students’ access to high quality instruction by considering both the tracking or grouping of students in terms of current achievement and the category systems that teachers and administrators use for classifying students. In the latter part of the chapter, we outline an analytic approach for documenting the institutional setting of mathematics teaching that can feed back to inform instructional improvement efforts at scale.

INTRODUCTION

In educational contexts, improvement at scale refers to the process of taking an instructional innovation that has proved effective in supporting students’ learning in a small number of classrooms and reproducing that success in a large number of classrooms. In countries with centralized educational systems, it might be feasible to propose taking an instructional innovation to scale at the national level. However, proposals for instructional improvement at the national level are usually impractical in countries with decentralized education systems because the infrastructure that would be needed to support coordinated improvement at the national level does not exist. The case of instructional improvement at scale that we consider in this chapter is located in a country with a decentralized education system, the US, in which there is a long history of local control of schooling. Each US state is divided into a number of independent school districts. In rural areas, districts might serve less than 1,000 students whereas a number of urban districts serve more than 100,000 students. In the context of the US educational system,
when we speak of scale we have in mind the improvement of mathematics teaching and learning in urban districts as they are the largest jurisdictions in which it is feasible to design for improvement in the quality of instruction (Supovitz, 2006). In this chapter, we speak of instructional improvement at the level of the school and the district with the understanding that the appropriate organizational unit or administrative jurisdiction beyond the school needs to be adjusted depending on the structure of the educational system in a particular country.

The central problem that we address in this chapter is how mathematics education research can generate knowledge that contributes to the ongoing improvement of mathematics teaching and learning at scale. The daunting nature of "the problem of scale" is indicated by the well-documented finding that prior large-scale improvement efforts in mathematics and other subject matter areas have rarely produced lasting changes in either teachers’ instructional practices or the organization of schools (Elmore, 2004; Gamoran, Anderson, Quiroz, Secada, Williams, & Ashman, 2003). Schools frequently experience external pressure to change, a condition that Hesse (1999) has termed policy churn. However, in most countries, classroom teaching and learning processes have proven to be remarkably stable amidst the flux. Cuban (1988), a historian of education, likened the situation to that of an ocean tossed by a storm in which all is calm on the sea floor even as the tempest whips up waves at the surface.

Researchers who work closely with teachers to support and understand their learning will probably not be surprised by Elmore’s (1996) succinct synopsis of the results of educational policy research on large-scale reform: the closer that an instructional innovation gets to the core of what takes place between teachers and students in classrooms, the less likely it is that it will implemented and sustained on a large scale. This policy research emphasizes that although research-based curricula and high-quality teacher professional development are necessary, they are not sufficient to support the improvement of mathematics instruction at scale.

Instructional improvement at scale also has to be framed as a problem of organizational learning for schools and larger administrative jurisdictions such as districts (Blumenfeld, Fishman, Krajcik, Marx, & Soloway, 2000; Coburn, 2003; McLaughlin & Mitra, 2004; Stein, 2004; Tyack & Tobin, 1995). This in turn implies that in addition to developing new approaches for supporting students’ and teachers’ learning, reformers also need to view themselves as institution-changing agents who seek to influence the institutional settings in which teachers develop and refine their instructional practices (Elmore, 1996; Stein, 2004). We capitalize on this insight in our chapter by emphasizing the importance of coming to view mathematics teachers’ instructional practices as situated within the institutional setting of the school and larger jurisdictions such as districts. This perspective implies that supporting teachers’ improvement of their instructional practices requires changing these settings in fundamental ways.

In the US context, the institutional setting of mathematics teaching, as we conceptualize it, encompasses district and school policies for instruction in mathematics. It therefore includes both the adoption of curriculum materials and guidelines for the use of those materials (e.g., pacing guides that specify a timeline for completing instructional units) (Ferrini-Mundy & Floden, 2007; Remillard,
The institutional setting also includes the people to whom teachers are accountable and what they are held accountable for (e.g., expectations for the structure of lessons, the nature of students’ engagement, and assessed progress of students’ learning) (Cobb & McClain, 2006; Elmore, 2004). In addition, the institutional setting includes social supports that give teachers access to new tools and forms of knowledge (e.g., opportunities to participate in formal professional development activities and in informal professional networks, assistance from a school-based mathematics coach or a principal who is an effective instructional leader) (Bryk & Schneider, 2002; Coburn, 2001; Cohen & Hill, 2000; Horn, 2005; Nelson & Sassi, 2005), as well as incentives for teachers to take advantage of these social supports.

The findings of a substantial and growing number of studies document that teachers’ instructional practices are partially constituted by the materials and resources that they use in their classroom practice, the institutional constraints that they attempt to satisfy, and the formal and informal sources of assistance on which they draw (Cobb, McClain, Lamberg, & Dean, 2003; Coburn, 2005; Spillane, 2005; Stein & Spillane, 2005). The findings of these studies call into question an implicit assumption that underpins many reform efforts, that teachers are autonomous agents in their classrooms who are unaffected by what takes place outside the classroom door (e.g., Krainer, 2005). In making this assumption, reformers are, in a very real sense, flying blind with little if any knowledge of how to adjust to the settings in which they are working as they collaborate with teachers to support their learning. In contrast, the empirical finding that teachers’ instructional practices are partially constituted by the settings in which they work orients us to anticipate and plan for the school support structures that need to be developed to support and sustain teachers’ ongoing learning.

INVESTIGATING INSTRUCTIONAL IMPROVEMENT AT SCALE

One of the primary goals of our current research, which is still in its early stages, is to generate knowledge that can inform the ongoing improvement of mathematics teaching and learning at scale. To this end, we are collaborating with four large, urban districts that have formulated and are implementing comprehensive initiatives for improving the teaching and learning of middle-school mathematics. We will follow 30 middle-school mathematics teachers and approximately 17 instructional leaders in each of the four districts for four years to understand how the districts’ instructional improvement initiatives are playing out in practice. In doing so, we will conduct one round of data collection and analysis in each district each year for four years to document: 1) the institutional setting of mathematics teaching, including formal and informal leaders’ instructional leadership practices, 2) the quality of the professional development activities in which the teachers participate, 3) the teachers’ instructional practices and mathematical knowledge for teaching, and 4) student mathematics achievement. The resulting longitudinal data on 120 teachers and approximately 68 school and district leaders in 24 schools in four districts will enable us to test a series of hypotheses that we have developed about school and district support structures that might enhance the effectiveness of
mathematics professional development. We will outline these hypothesized support structures later in the next section of this chapter.

In addition to formally testing our initial hypotheses, we will share our analysis of each annual round of data with the districts to provide them with feedback about the institutional settings in which mathematics teachers are developing and revising their instructional practices, and we will collaborate with them to identify any adjustments that might make the districts’ improvement designs for middle-school mathematics more effective. We will then document the consequences of these adjustments in subsequent rounds of data collection. In addition, we will attempt to augment our hypotheses in the course of the repeated cycles of analysis and design by identifying additional support structures and by specifying the conditions under which particular support structures are important. In doing so, we seek to address a pressing issue identified by Stein (2004): the proactive design of school and district institutional settings for mathematics teachers' ongoing learning.

In the remainder of this chapter, we focus on two types of conceptual tools that, we contend, are central to the improvement of mathematics teaching and learning at scale. The first is a theory of action for designing schools and larger administrative jurisdictions as learning organizations for instructional improvement in mathematics. The second is an analytic approach for documenting the institutional setting of mathematics teaching that can produce analyses that inform the ongoing improvement effort.

DESIGNING FOR INSTRUCTIONAL IMPROVEMENT IN MATHEMATICS

In preparing for our collaboration with the four urban school districts, we formulated a series of hypotheses about school and district support structures that we conjecture will be associated with improvement in middle-school mathematics teachers’ instructional practices and student learning. In developing these hypotheses, we assumed that a school or district has adopted a research-based instructional programme for middle-school mathematics and that the programme was aligned with district standards and assessments. In addition, we assume that mathematics teachers have opportunities to participate in sustained professional development that is organized around the instructional materials they use with students. The proposed support structures, which are summarized in Table 1, therefore fall outside mathematics educators’ traditional focus on designing high-quality curricula and teacher professional development. To the extent that the hypotheses prove viable, they specify the types of institutional structures that a school or district organizational design might aim to engender as it attempts to improve the quality of mathematics teaching across the organization.

As background for non-US readers, we should clarify that large school districts such as those with which we are collaborating have a central office whose staff are responsible for selecting curricula and for providing teacher professional development in various subject matter areas including mathematics. In this chapter, we use the designation district leaders to refer to members of the central office staff whose responsibilities focus on instruction. We speak of district mathematics leaders to refer to central office staff whose responsibilities focus specifically on
the teaching and learning of mathematics.

Table 1. The Proposed Support Structures

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**Teacher Networks**

We developed our hypotheses about potential support structures by taking as our starting point forms of classroom instructional practice that are consistent with current research on mathematics learning and teaching (Kilpatrick, Martin, & Schifter, 2003). Teachers who have developed high quality instructional practices of this type attempt to achieve a significant mathematical agenda by building on students’ current mathematical reasoning. To this end, they engage students in mathematically challenging tasks, maintain the level of challenge as tasks are enacted in the classroom (Stein & Lane, 1996; Stein, Smith, Henningsen, & Silver, 2000), and support students’ efforts to communicate their mathematical thinking in classroom discussions (Cobb, Boufi, McClain, & Whitenack, 1997; Hiebert et al., 1997; Lampert, 2001). These forms of instructional practice are complex, demanding, uncertain, and not reducible to predictable routines (Ball & Cohen, 1999; Lampert, 2001; McClain, 2002; Schifter, 1995; Smith, 1996). The findings of a number of investigations indicate that strong professional networks (see also Lerman & Zehetmeier, and Borba & Gadanidis, this volume) in which teachers participate voluntarily can be a crucial resource as they attempt to develop instructional practices in which they place students’ reasoning at the centre of their instructional decision making (Cobb & McClain, 2001; Franke & Kazemi, 2001b; Gamoran, Secada, & Marrett, 2000; Kazemi & Franke, 2004; Little, 2002; Stein, Silver, & Smith, 1998).

There is abundant evidence that the mere presence of collegial support is not by itself sufficient: both the focus and the depth of teachers’ interactions matter. With regard to focus, it is clearly important that activities and exchanges in teacher networks centre on issues central to classroom instructional practice (Marks &
Furthermore, the findings of Coburn and Russell’s (in press) recent investigation indicate that the depth of interactions around classroom practice make a difference in terms of the support for teachers’ improvement of their classroom practices. Coburn and Russell clarify that interactions of greater depth involve discerning the mathematical intent of instructional tasks and identifying the relative sophistication of student reasoning strategies, whereas interactions of less depth involve determining how to use instructional materials and mapping the curriculum to district or state standards.

Teacher networks that focus on issues relevant to classroom instruction constitute our first hypothesized support structure. In addition, we anticipate that networks in which interactions of greater depth predominate will be more supportive social contexts for teachers’ development of ambitious instructional practices than those in which interactions are primarily of limited depth (Franke, Kazemi, Shih, Biagetti, & Battey, in press; Stein et al., 1998).

**Access of Teacher Networks to Key Resources**

Mathematics teacher networks do not emerge in an institutional vacuum. Gamoran et al.’s (2003) analysis reveals that to remain viable, teacher networks and communities need access to resources. The second and third hypothesized support structures concern two specific types of resources that facilitate the emergence and development of teacher networks (see Table 1).

**Time for collaboration.** The first resource is time built into the school schedule for mathematics teachers to collaborate. As Gamoran et al. (2003) make clear, time for collaboration is a necessary but not sufficient condition for the emergence of teacher networks. Although institutional arrangements such as teachers’ schedules do not directly determine interactions, they can enable and constrain the social relations that emerge between teachers (and between teachers and instructional leaders) (Smylie & Evans, 2006; Spillane, Reiser, & Gomez, 2006).

**Access to expertise.** The second resource for supporting the emergence of teacher networks of sufficient depth is access to colleagues who are already relatively accomplished in using the adopted instructional programme to support students’ mathematical learning. In the absence of this resource, it is difficult to envision how interactions within a teacher network will be of sufficient depth to support teachers’ development of ambitious instructional practices. In this regard, Penuel, Frank, and Krause (2006) found that improvement in mathematics teachers’ instructional practices was associated with access to mentors, mathematics coaches, and colleagues who were already expert in the reform initiative. Their results indicate that accomplished fellow teachers and coaches can share exemplars of instructional practice that are tangible to their less experienced colleagues, thus supporting their efforts to improve their instructional practices.
Shared Instructional Vision

In considering additional support structures, we step back to locate teacher networks first within the institutional context of the school, and then within the context of the broader administrative jurisdiction. At the school level, it seems reasonable to speculate that teacher networks will be more likely to emerge and sustain if the vision of high quality mathematics instruction that they promote is consistent with the instructional vision of formal or positional school leaders. Research in the field of educational leadership indicates that this intuition is well founded. The results of a number of studies reveal that professional development, collaboration between teachers, and collegiality between teachers and formal school leaders are rarely effective unless they are tied to a shared vision of high quality instruction that gives them meaning and purpose (Elmore, Peterson, & McCarthey, 1996; Newman & Associates, 1996; Rosenholtz, 1985, 1989; Rowan, 1990). In the case of US schools, formal school leaders might include the school principal, an assistant principal with responsibility for curriculum and instruction, a mathematics department head, and possibly a school-based mathematics coach.

The notion of a shared instructional vision encompasses agreement on instructional goals and thus on what it is important for students to know and be able to do mathematically, and on how students’ development of these forms of mathematical knowledgeability can be effectively supported.

Our argument for the importance of a shared instructional vision is not restricted to the school but also extends to broader administrative jurisdictions. We illustrate this point by taking the relevant administrative jurisdiction in the US context, the school district, as an example. As is the case for the relevant jurisdiction in most countries, there are typically a number of distinct departments or units within the administration of large districts whose work has direct consequences for the teaching and learning of mathematics. For example, one unit is typically responsible for selecting instructional materials in various subject matter areas including mathematics, and for providing teacher professional development. A separate unit is typically responsible for hiring and providing professional development for school leaders. The unit responsible for assessment and evaluation would also appear critical given the importance of the types of data that are collected to assess school, teacher, and student learning. In addition, depending on the district, the unit responsible for special education might also be influential to the extent that it focuses on how mainstream instruction serves groups of students identified as potentially at-risk. Spillane et al.’s (2006) findings indicate that staff in different administrative units whose work contributes to the district’s initiative to improve the quality of mathematics teaching and learning frequently understand district-wide initiatives differently. In such cases, the policies and practices of the various units are fragmented and often in conflict with each other. This has consequences both for the coherence of the district’s instructional improvement effort and for the degree to which the institutional settings of mathematics teaching support teachers’ ongoing improvement of their instructional practices. Our fourth hypothesized support structure therefore concerns the development of a shared instructional vision between participants in teacher networks, formal school leaders, and district leaders. We anticipate that mathematics teachers’ improvement...
of their instructional practices will be greater in schools and broader jurisdictions in which a shared instructional vision consistent with current reform recommendations has been established.

Brokers

The development of a shared instructional vision of high quality mathematics instruction in a school and a broader jurisdiction such as a district is a non-trivial accomplishment. This becomes apparent when we note that mathematics teachers, principals, and district curriculum specialists, and so forth constitute distinct occupational groups that have different charges, engage in different forms of practice, and have different professional affiliations (Spillane et al., 2006). The fifth support structure concerns the presence of brokers who can facilitate the development of a shared instructional vision by bridging between perspectives and agendas of different role groups (see Table 1). Brokers are people who participate at least peripherally in the activities of two or more groups, and thus have access to the perspectives and meanings of each group (Wenger, 1998). For example, a principal who participates in professional development with mathematics teachers might be able to act as a broker between school leaders and mathematics teachers in the district, thereby facilitating the alignment of perspectives on mathematics teaching and learning across these two groups (e.g., Wenger, 1998). Extending our focus beyond the school, we anticipate that brokers who can bridge between school and district leaders and between units of the district central office will also be critical in supporting the development of a shared instructional vision across the district. Brokers who can help bring coherence to the reform effort in a relatively large jurisdiction such as an urban district by grounding it in a shared instructional vision constitute our fifth support structure.

Negotiating the Meaning of Key Boundary Objects

The sixth hypothesized support structure also facilitates the development of a shared instructional vision (see Table 1). Mathematics teachers and instructional leaders use a range of tools as an integral aspect of their practices. Star and Griesemer (1989) call tools that are used by members of two or more groups boundary objects. For example, mathematics teachers and instructional leaders in most US schools use state mathematics standards and test scores, thereby constituting them as boundary objects. Tools that are produced within a school or district might also be constituted as boundary objects. For example, the district leaders in one of the districts in which we are working are developing detailed curriculum frameworks for middle-school mathematics teachers to use as well as a simplified version for school leaders. It is important to note that boundary objects such as state and district standards, test scores, and curriculum frameworks can be and are frequently used differently and come to have different meanings as members of different groups such as teachers and school leaders incorporate them into their practices (Star & Griesemer, 1989; Wenger, 1998). Boundary objects do not therefore carry meanings across group boundaries. However, they can serve as
important focal points for the negotiation of meaning and thus the development of a shared instructional vision. The value of boundary objects in this regard stems from the fact that they are integral to the practices of different groups and are therefore directly relevant to the concerns and interests of the members of the groups. From the point of view of organizational design, this observation points to the importance of developing venues in which members of different role groups engage together in activities that relate directly to teaching and instructional leadership in mathematics.

Our sixth hypothesis is therefore that a shared vision of high quality mathematics instruction will emerge more readily in schools and districts in which members of various groups explicitly negotiate the meaning and use of key boundary objects. In speaking of key boundary objects, we are referring to tools that are used when developing an agenda for mathematics instruction (e.g., curriculum frameworks) and when making mathematics teaching and learning visible (e.g., formative assessments, student work), as well as tools that are used while actually teaching.

Accountability Relations between Teachers, School leaders, and District Leaders

The picture that emerges from the support structures we have discussed thus far is that of a coherent reform effort grounded in a shared instructional vision, in which networks characterized by relatively deep interactions support teachers’ ongoing learning. Although the activities of teachers as well as of school and district leaders are aligned in this picture, we have not specified the relationships between members of these different role groups. The next two potential support structures address this issue.

The seventh hypothesized support structure concerns accountability relations between teachers, school leaders, and district leaders. At the classroom level, instruction that supports students’ understanding of central mathematical ideas involves what Kazemi and Stipek (2001) term a high press for conceptual thinking. Kazemi and Stipek clarify that teachers maintain a high conceptual press by 1) holding students accountable for developing explanations that consist of a mathematical argument rather than simply a procedural description, 2) attempting to understand relations among multiple solution strategies, and 3) using errors as opportunities to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies. Analogously, we hypothesize that the following accountability relations will contribute to instructional improvement:

– Formal school instructional leaders (e.g., principals, assistant principals, mathematics coaches) hold mathematics teachers accountable for maintaining conceptual press for students and, more generally, for developing ambitious instructional practices.
– District leaders hold school leaders accountable for assisting mathematics teachers in improving their instructional practices.

We anticipate that the potential of these accountability relations to support instructional improvement will both depend on and contribute to the development of a shared instructional vision. In the absence of a shared vision, different school
leaders might well hold teachers accountable to different criteria, some of which are at odds with the intent of the district’s instructional improvement effort (Coburn & Russell, in press).

Relations of Assistance between Teachers, School leaders, and District Leaders

Elmore (2000; 2004) argues, correctly in our view, that it is unethical to hold people accountable for developing particular forms of practice unless their learning of those practices is adequately supported. We would, for example, question a teacher who holds students accountable for producing mathematical arguments to explain their thinking but does little to support the students’ development of mathematical argumentation. In Elmore’s terms, the teacher has violated the principle of mutual accountability, wherein leaders are accountable to support the learning of those who they hold accountable. The eighth hypothesized support structure comprises the following relations of support and assistance:

- Formal school instructional leaders (e.g., principals, assistant principals, mathematics coaches) are accountable to teachers for assisting them in understanding the mathematical intent of the curriculum, in maintaining conceptual press for students and, more generally, in developing ambitious instructional practices.
- District leaders are accountable to school leaders to provide the material resources needed to facilitate high quality mathematics instruction, and to support school leaders’ development as instructional leaders.

Leadership Content Knowledge

The ninth hypothesized support structure follows directly from the relations of accountability and assistance that we have outlined and concerns the leadership content knowledge of school and district leaders (see Table 1). Leadership content knowledge encompasses leaders’ understanding of the mathematical intent of the adopted instructional materials, the challenges that teachers face in using these materials effectively, and the challenges in supporting teachers’ reorganization of their instructional practices (Stein & Nelson, 2003). Ball, Bass, Hill, and colleagues have demonstrated convincingly that ambitious instructional practices involve the enactment of a specific type of mathematical knowledge that enables teachers to address effectively the problems, questions, and decisions that arise in the course of teaching (Ball & Bass, 2000; Hill & Ball, 2004; Hill, Rowan, & Ball, 2005). Analogously, Stein and Nelson (2003) argue that effective school and district instructional leadership in mathematics involves the enactment of a subject-matter-specific type of mathematical knowledge, leadership content knowledge, that enables instructional leaders to recognize high-quality mathematics instruction when they see it, support its development, and organize the conditions for continuous learning among school and district staff. Stein and Nelson go on to argue that the leadership content knowledge that principals require to be effective instructional leaders in mathematics includes a relatively deep understanding of mathematical knowledge for teaching, of what is known about how to teach
mathematics effectively, and of how students learn mathematics, as well as “knowing something about teachers-as-learners and about effective ways of teaching teachers” (p. 416). They extend this line of reasoning by proposing that district leaders who provide professional development for principals should know everything that principals need to know and should also have knowledge of how principals learn.

We see considerable merit in Stein and Nelson’s arguments about the value of leadership content knowledge in mathematics. However, the demands on principals seem overwhelming if they are to develop deep leadership content knowledge in all core subject matter areas including mathematics. This is particularly the case for principals of middle and high schools. We therefore suggest that it might be more productive to conceptualize this type of expertise as being distributed across formal and informal school leaders rather than residing exclusively with the principal. In other words, we suggest that the depth of leadership content knowledge that principals require is situational and depends in large measure on the expertise of others in the school. In cases where principals can capitalize on the expertise of a core group of relatively accomplished mathematics teachers or an effective school-based mathematics coach, for example, the extent of principals’ leadership content knowledge in mathematics might not need to be particularly extensive. In such cases, it might suffice for principals to understand the characteristics of high quality instruction that hold across core subject matter areas provided they also understand the overall mathematical intent of the instructional programme and appreciate that using the programme effectively is a non-trivial accomplishment that requires ongoing support for an extended period of time. We speculate that this limited knowledge might enable principals to collaborate effectively with accomplished teachers and possibly school-based coaches. Stein and Nelson (2003, p. 444) acknowledge the viability of this approach when they observe that

where individual administrators do not have the requisite knowledge for the task at hand they can count on the knowledge of others, if teams or task groups are composed with the recognition that such knowledge will be requisite and someone, or some combination of people and supportive materials, will need to have it.

The ninth support structure is therefore leadership content knowledge in mathematics that is distributed across the principal, teachers, and the coach. This hypothesized support structure implies that it will be important for principal professional development to attend explicitly to the issue of leveraging teachers’ and coaches’ expertise effectively.

*Equity in Students’ Access to Ambitious Instructional Practices*

The student population is becoming increasingly diverse racially and ethnically in most industrialized countries and in a number of developing countries. An established research base indicates that access to ambitious instructional practices for students who are members of historically under-served populations (e.g., students of colour, students from low-income backgrounds, students who are not
native language speakers, students with special needs) is rarely achieved (see Darling-Hammond, 2007). In addition, a small but growing body of research that suggests that ambitious instructional practices are not enough to support all students’ mathematical learning unless they also take account of the social and cultural differences and needs of historically marginalized groups of students (see Nasir & Cobb, 2007). This work indicates the importance of professional development for teachers and instructional leaders in mathematics that focuses squarely on meeting the needs of underserved groups of students. In addition, it has implications for the establishment of institutional support structures that are likely to result in access to appropriate instructional practices for historically marginalized groups of students. The final two support structures that we discuss concern equity in students’ learning opportunities.

**De-tracked instructional programme.** Tracking, or the grouping of students according to current achievement, often prevails in schools that serve students from marginalized groups. However, current research indicates that “tracking does not substantially benefit high achievers and tends to put low achievers at a serious disadvantage” (Darling-Hammond, 2007, p. 324; see also Gamoran, Nystrand, Berends, & LePore, 1995; Horn, 2007; Oakes, Wells, Jones, & Datnow, 1997). The tenth support structure is therefore a rigorously de-tracked instructional programme in mathematics.

**Category system for classifying students.** The final support structure concerns the categories of mathematics students that are integral to teachers’ and instructional leaders’ practices. Horn’s (2007) analysis of the contrasting systems for classifying students constructed by the mathematics teachers in two US high schools is relevant in this regard because it indicates that these classification systems were related to the two groups of teachers’ views about whether mathematics should be tracked (see Table 1). Significantly, Horn’s analysis also indicates that the contrasting classification systems also reflected differing views of mathematics as a school subject. The teachers in one of the schools differentiated between formal and informal solution methods, and viewed the latter as illegitimate. They also took a sequential view of school mathematics and assumed that students had to first master prior topics if they were to make adequate progress. This conception of school mathematics was reflected in the teachers’ classification of students as more or less motivated to master mathematical formalisms, and as faster and slower in doing so. The teachers’ classification of students in terms of stable levels of motivation and ability grounded their perceived need for separate mathematics courses for different types of students.

In sharp contrast, the mathematics teachers at the second school that Horn (2007) studied tended to take a non-sequential view of school mathematics and conceptualized it as a web of ideas rather than an accumulation of formal procedures. These teachers also rejected the categorization of students as fast or slow because it emphasized task completion at the expense of considering multiple strategies. In addition, the teachers in this school viewed it as their responsibility to support students’ engagement both by selecting appropriate tasks and by
influencing students’ learning agendas. Thus, these teachers addressed the challenge of teaching mathematics to all their students in the context of a rigorously de-tracked mathematics programme by focusing primarily on their instructional practices rather than on perceived mismatches between students and the curriculum. In doing so, they constructed categories for classifying students that characterized them in relation to their current instructional practices rather than in terms of stable traits. Building on Horn’s analysis, the eleventh support structure is a category system that classifies students in relation to current instructional practices rather than in terms of seemingly stable traits.

Reflection

We developed the proposed support structures summarized in Table 1 by mapping backwards from the classroom and, in particular, from a research-based view of high quality mathematics instruction. In doing so, we have limited our focus to the establishment of institutional settings that support school and district staff’s ongoing improvement of their practices. This backward mapping process could be extended to develop conjectures that are directly related to the traditional concerns of policy researchers. For example, several of the hypothesized support structures involve conjectures about the role of mathematics coaches and school leaders. These conjectures have implications for district hiring and retention policies. In addition, the hypotheses imply that the allocation of frequently scarce material resources should be weighted towards what Elmore (2006) terms the bottom of the system (see also Gamoran et al., 2003). As the notion of distributed leadership is currently fashionable, it is worth noting that the hypotheses do not treat the distribution of instructional leadership as a necessary good. In the absence of a common discourse about mathematics, learning, and teaching, the distribution of leadership can result in a lack of coordination and alignment (Elmore, 2000). As Elmore (2006) observes, effective schools and districts do not merely distribute leadership. They also support people’s development of leadership capabilities, in part by structuring settings in which they learn and enact leadership. As the proposed support structures indicate, important outcomes of an initiative to improve the quality of mathematics learning and teaching include “the system capacity developed to sustain, extend, and deepen a successful initiative” (Elmore, 2006, p. 219).

DOCUMENTING THE INSTITUTIONAL SETTING OF MATHEMATICS TEACHING

The hypothesized support structures that we have discussed constitute a theory of action for designing schools and larger administrative jurisdictions such as school districts as learning organizations for instructional improvement in mathematics. We now consider a second conceptual tool that is central to the improvement of mathematics teaching and learning at scale, an analytic approach for documenting the institutional setting of mathematics teaching. In addition to formally testing our hypotheses about potential support structures, we will share our analysis of the data collected each year with the four districts and collaborate with them to identify any
adjustments that might make the districts’ improvement designs for middle-school mathematics more effective. To accomplish this, we require an analytic approach for documenting the institutional setting of mathematics teaching that can feed back to inform the districts’ ongoing improvement efforts.

The analytic approach that we will take makes a fundamental distinction between schools and districts viewed as designed organizations and as lived organizations. A school or district viewed as a designed organization consists of formally designated roles and divisions of labour together with official policies, procedures, routines, management systems, and the like. Wenger (1998) uses the term designed organization to indicate that its various elements were designed to carry out specific tasks or to perform particular functions. In contrast, a school or school district viewed as a lived organization comprises the groups within which work is actually accomplished together with the interconnections between them. As Brown and Duguid (1991; 2000) clarify, people frequently adjust prescribed organizational routines and procedures to the exigencies of their circumstances (see also Kawatoko, 2000; Ueno, 2000; Wenger, 1998). In doing so, they often develop collaborative relationships that do not correspond to formally appointed groups, committees, task forces, and teams (e.g., Krainer, 2003). Instead, the groups within which work is actually organized are sometimes non-canonical and not officially recognized. These non-canonical groups are important elements of a school or district viewed as a lived organization.

Given the goals of our research, we find it essential to document the districts in which we are working as both designed organizations and as lived organizations. One of our first steps has been to document the districts as designed organizations by interviewing district leaders about their plans or designs for supporting the improvement of mathematics teaching and learning. In analyzing these interviews, we have teased out the suppositions and assumptions and have framed them as testable conjectures. The process of testing these conjectures requires that we document how the districts’ improvement designs are playing out in practice, thereby documenting the schools and districts in which we are working as lived organizations.

Methodologically, we will use what Hornby and Symon (1994) and Spillane (2000) refer to as a snowballing strategy and Talbert and McLaughlin (1999) term a bottom-up strategy to identify groups within the schools and districts whose agendas are concerned with the teaching and learning of mathematics. The first step in this process involves conducting audio-recorded semi-structured interviews with the participating 30 middle-school mathematics teachers in each district to identify people within the district who influence how the teachers teach mathematics in some significant way. The issues that we will address in these interviews include the professional development activities in which the teachers have participated, their understanding of the district’s policies for mathematics instruction, the people to whom they are accountable, their informal professional networks, and the official sources of assistance on which they draw.

The second step in this bottom-up or snowballing process involves interviewing the formal and informal instructional leaders identified in the teacher interviews as influencing their classroom practices. The purpose of these interviews is to
understand formal and informal leaders’ agendas as they relate to mathematics instruction and the means by which they attempt to achieve those agendas. We will then continue this snowballing process by interviewing people identified in the second round of interviews as influencing instruction and instructional leadership in the district. In terms familiar to policy researchers, this bottom-up methodology focuses squarely on the activity of what Weatherley and Lipsky (1977) term street-level bureaucrats whose roles in interpreting and responding to district efforts to improve mathematics instruction are as important as those of district leaders who designed the improvement initiative. The methodology therefore operationalizes the view that what ultimately matters is how district initiatives are enacted in schools and classrooms (e.g., McLaughlin, 2006).

In addition to identifying the groups in which the work of instructional improvement is accomplished and documenting aspects of each group’s practices, our analysis of the schools and districts as lived organizations will also involve documenting the interconnections between the groups. To do so, we will focus on three types of interconnections, two of which we introduced when describing potential support structures. Interconnections of the first type are constituted by the activities of brokers who are at least peripheral members of two or more groups. As we noted, brokers can bridge between the perspectives of different groups, thereby facilitating the alignment of their agendas. As our hypotheses indicate, our analysis of brokers will be relatively comprehensive and will seek to clarify whether there are brokers between various groups in the school (e.g., mathematics teachers and school leaders), between school leaders and district leaders, and between key units of the district central office. Boundary objects that members of two or more groups use routinely as integral aspects of their practices constitute interconnections of the second type. As we have noted, there is the very real possibility that members of different groups will use boundary objects differently and imbue them with different meanings (Wenger, 1998). Our analysis will therefore seek to identify boundary objects and to document whether members of different groups used them in compatible ways.

The third type of interconnection is constituted by boundary encounters in which members of two or more groups engage in activities together as a routine part of their respective practices. Three of the hypothesized support structures focus explicitly on boundary encounters: the explicit negotiation of the meaning of boundary objects, relations of accountability, and relations of assistance. In addition to documenting the frequency of boundary encounters between members of different groups, our analysis will focus on the nature of their interactions.

A recent finding reported by Coburn and Russell (in press) indicates the importance of pushing for this level of detail. They studied the implementation of elementary mathematics curricula designed to support ambitious instruction in two school districts. As part of their instructional improvement efforts, both districts hired and provided professional development for a cadre of school-based mathematics coaches (see also Nickerson, this volume). Coburn and Russell found that there were significant differences in the depth of the interactions between the coaches and the professional development facilitators in the two districts. In the first district, interactions were relatively deep and focused on issues such as discerning the mathematical intent of instructional tasks and on identifying and
building on student reasoning strategies. In the second district, interactions were typically of limited depth and focused primarily on how to use instructional materials and on mapping the curriculum to district or state standards. Coburn and Russell also documented the nature of interactions between coaches and teachers in the two districts. They found that teacher-coach interactions increased in depth to a far greater extent in the first district than in the second district. In addition, interactions between teachers when a coach was not present also increased in depth in the first but not the second district. In other words, the contrasting routines of interaction in coach professional development sessions became important features of interactions in teacher networks in the two districts.

In our view, Coburn and Russell’s analysis represents a significant advance in research on instructional improvement at scale. To this point, policy researchers have tended to frame social networks as conduits for information about instructional and instructional leadership practices. However, research in mathematics education makes it abundantly clear that information about ambitious instructional practices is, by itself, insufficient to support teachers’ development of this form of practice. Coburn and Russell’s analysis focuses more broadly on interactions across groups as well as within social networks, and highlights the importance of co-participation in collective activities. In addition, their findings demonstrate that the depth of co-participation matters. Their analysis therefore establishes a valuable point of contact between research on policy implementation and research on mathematics teachers’ learning. This latter body of work documents that teachers’ co-participation in activities of sufficient depth with an accomplished colleague or instructional leader is a critical source of support for teachers’ development of ambitious practices (e.g., Borko, 2004; Fennema et al., 1996; Franke & Kazemi, 2001a; Goldsmith & Shifter, 1997; Kazemi & Franke, 2004; Wilson & Berne, 1999). We anticipate that Coburn and Russell’s (in press) notion of routines of interaction will prove to be a useful analytic tool as we seek to understand whether the nature of the boundary encounters in which school and district staff engage in activities together influences how they subsequently interact with others in different settings.

PROVIDING FEEDBACK TO INFORM INSTRUCTIONAL IMPROVEMENT

In the approach that we have outlined, the analysis of a school or district as a lived organization involves identifying the groups in which the work of instructional improvement is actually accomplished and documenting interconnections between these groups. An analysis of the lived organization therefore focuses on what people actually do and the consequences for teachers’ instructional practices and students’ mathematical learning. In contrast, an analysis of a school or district as a designed organization involves documenting the school or district plan or design for supporting instructional improvement in mathematics. This design specifies organizational units and positional roles as well as organizational routines, and involves conjectures about how the enactment of the design will result in the improvement of teachers’ instructional practices and student learning. An analysis of the designed organization documents both this design and the tools and
activities that will be employed to realize the design by enabling people to improve their practices. In giving feedback to the four collaborating districts to inform their improvement efforts, we will necessarily draw on our analyses of the districts as both designed and lived organizations.

To develop this feedback, we will identify gaps between the districts’ designs for instructional improvement and the ways in which those designs are actually playing out in practice by comparing our analyses of each district as a designed organization and as a lived organization. This approach will enable us to differentiate cases in which a theory of action proposed by a district is not enacted in practice from cases in which the enactment of the theory of action does not lead to the anticipated improvements in the quality of teachers’ instructional practices (Supovitz & Weathers, 2004). As an illustration, one of the districts with which we are collaborating is investing some of its limited resources in mathematics coaches with half-time release from teaching for each middle school. The district’s theory of action specifies that the coaches’ primary responsibilities are to facilitate teacher collaboration and to support individual teachers’ learning by co-teaching with them and by observing their instruction and providing constructive feedback. Suppose that the district’s investment in mathematics coaches does not result in a noticeable improvement in teachers’ instructional practices. It could be the case that the theory of action of the district has not been enacted. For example, the coaches might be tutoring individual students or preparing instructional materials for the mathematics teachers in their schools rather than working with teachers in their classrooms. In attempting to understand why this is occurring, we would initially focus on coaches’ and school leaders’ understanding of the coaches’ role in supporting teachers’ improvement of their instructional practices. Alternatively, it could be the case that the coaches are working with teachers in their classrooms, but their efforts to support instructional improvement are not effective. In this case, we would initially seek to understand how, specifically, the coaches are attempting to support teachers’ learning and would take account of the process by which the coaches were selected and the quality of the professional development in which they participated.

As this illustration indicates, our goal when giving feedback is not merely to assess whether the district’s design is being implemented with fidelity, although our analysis will necessarily address this issue. We also seek to understand why the district’s theory of action is playing out in a particular way in practice by taking seriously the perspectives and practices of street-level bureaucrats such as teachers, coaches, and school leaders. In doing so, we will draw on both an analysis of the district design as a potential resource for action and an analysis of the district as a lived organization that foregrounds people’s agency as they develop their practices within the context of others’ institutionally situated actions (e.g., Feldman & Pentland, 2003).

**DISCUSSION**

In this chapter, we have focused on the question of how mathematics education research might contribute to the improvement of mathematics teaching and learning at scale. We addressed this question by first clarifying the value of
viewing mathematics teachers’ instructional practices as situated in the institutional settings of the schools and districts in which they work. Against this background, we presented a series of hypotheses about school and district structures that might support teachers’ ongoing improvement of their classroom practices. We then went on to outline an analytic approach for documenting the institutional settings of mathematics teaching established in particular schools and districts that can feed back to inform the instructional improvement effort.

We conclude this chapter by returning to the *relation between research in educational policy and leadership and in mathematics education*. To this point, researchers in these fields have conducted largely independent lines of work on the improvement of teaching and learning (e.g., Engeström, 1998; Franke, Carpenter, Levi, & Fennema, 2001). Research in educational policy and leadership tends to focus on the designed structural features of schools and how changes in these structures can result in changes in classroom instructional practices. In contrast, research in mathematics education tends to focus on the role of curriculum and professional development in supporting teachers’ improvement of their instructional practices and their views of themselves as learners. In this chapter, we have argued that mathematics education research that seeks to contribute to the improvement of teaching and learning at scale will have to transcend this dichotomy by drawing on analyses of schools and districts viewed both as designed organizations and as lived organizations. In the interventionist genre of research that we favour, organizational design is at the service of large-scale improvement in the quality of teachers’ instructional practices. In research of this type, the attempt to contribute to improvement efforts in particular schools and administrative jurisdictions constitutes the context for the generation of useful knowledge about the relations between the institutional settings in which teachers’ work, the instructional practices they develop in those settings, and their students’ mathematical learning. This genre of research therefore reflects de Corte, Greer, and Verschaffel’s (1996) adage that if you want to understand something try to change it, and if you want to change something try to understand it.

**NOTES**

1 The analysis reported in this chapter was supported by the National Science Foundation under grant No. ESI 0554535. The opinions expressed do not necessarily reflect the views of the Foundation. The hypotheses that we discuss in this chapter were developed in collaboration with Sarah Green, Erin Henrick, Chuck Munter, John Murphy, Jana Visnovska, and Qing Zhao. We are grateful to Kara Jackson for her constructive comments on a previous draft of this chapter.

2 In engaging in these repeated cycles of analysis and design, we will, in effect, attempt to conduct a design experiment at the level of the school and district.

3 The term theory of action was coined by Argyris and Schön (1974, 1978) and is central to most current perspectives on organizational learning. A theory of action establishes the rationale for an improvement design and consists of conjectures about both a trajectory of organizational improvement and the specific means of supporting the envisioned improvement process.

4 These two types of conceptual tools serve to ground the two aspects of the design research cycle, namely design and analysis (e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collaborative, 2003).

Mathematics coaches are teachers who have been released from some or all of their instructional responsibilities in order to assist the mathematics teachers in a school in improving the quality of their instruction. Ideally, coaches should be selected on the basis of their competence as mathematics teachers and should receive professional development that focuses on both mathematics teaching and on supporting other teachers' learning.

A focus on instructional goals takes us onto the slippery terrain of mathematical values (Hiebert, 1999). It is important to note that values are not a matter of mere subjective whim or taste but are instead subject to justification and debate (Rorty, 1982).

Spillane and colleagues (Spillane, 2005; Spillane, Halverson, & Diamond, 2001, 2004) proposed distributed leadership as an analytic perspective that focuses on how the functions of leadership are accomplished rather than on the characteristics and actions of individual positional leaders. However, as so often happens in education, the basic tenets of this analytic approach have been translated into prescriptions for practitioners’ actions. In our view, this is a fundamental category error that, if past experience is any guide, might well have unfortunate consequences (e.g., Cobb, 1994, 2002).

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