

Towards an Empirically Grounded Theory of Action for Improving the Quality of Mathematics Teaching at Scale

Paul Cobb
Vanderbilt University

Kara Jackson
McGill University

Our purpose in this article is to propose a comprehensive, empirically grounded theory of action for improving the quality of mathematics teaching at scale. In doing so, we summarise current research findings that can inform efforts to improve the quality of mathematics instruction on a large scale, and identify questions that are yet to be addressed. We draw on an ongoing collaboration with mathematics teachers, school leaders, and district leaders in four urban school districts in the US. The provisional theory of action that we report encompasses a coherent system of supports for ambitious instruction that includes both formal and job-embedded teacher professional development, teacher networks, mathematics coaches' practices in providing job-embedded support for teachers' learning, school leaders' practices as instructional leaders in mathematics, and district leaders' practices in supporting the development of school-level capacity for instructional improvement.

What does it take to improve the quality of mathematics instruction on a large scale? In this article, we address this question by proposing a comprehensive, empirically grounded theory of action for improving the quality of mathematics teaching at scale. In doing so, we draw on our ongoing collaboration with mathematics teachers, school leaders, and district leaders in four urban school districts in the US that serve a total of 360,000 students. Some components of the theory of action are necessarily provisional, given the limited research base on which we can draw in some areas.

Improving the quality of mathematics instruction across classrooms, schools, and broader educational jurisdictions is a pressing issue for both researchers and practitioners. School leaders in a number of countries are under increasing pressure to improve student learning opportunities in mathematics (Even & Ball, 2010). However, the issue of how to support instructional improvement on a large scale continues to be under-researched (Coburn, 2003; Cohen, Moffitt, & Goldin, 2007; Stein, 2004). As a consequence, research can currently provide only limited guidance to district and school leaders who aim to improve the quality of mathematics teaching.

In many respects, mathematics education researchers are well positioned to investigate the improvement of the quality of instruction at scale. Over the past 20 years, research in mathematics education and related fields has made considerable progress in documenting learning progressions in specific mathematical domains (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Lehrer & Lesh, 2003) and in identifying a common set of learning goals that focus on enduring understandings of central mathematical ideas (Kilpatrick, Swafford,

& Findell, 2001). Recent research efforts have also delineated a set of instructional practices that support students' development of these mathematical ideas (Franke, Kazemi, & Battey, 2007; National Council of Teachers of Mathematics, 2000). Additionally, research-based instructional materials that can support teachers' enactment of these instructional practices have been developed in several countries, including the US (Remillard, 2005; Senk & Thompson, 2003). The mathematics education research community has also learned a great deal about professional development that supports teachers' development of practices aimed at ambitious learning goals for students (Ball, Sleep, Boerst, & Bass, 2009; Kazemi & Hubbard, 2008; Lampert, Beasley, Ghouseini, Kazemi, & Franke, 2010). There have been a number of successful professional development programs in the US that have supported improvement in teachers' knowledge and instructional practices (Borko, 2004), including Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996), Developing Mathematical Ideas (Bell, Wilson, Higgins, & McCoach, 2010), and Quantitative Understanding: Amplifying Student Achievement and Reasoning (Stein, Grover, & Henningsen, 1996).

However, these advances have had limited impact on instruction in most US classrooms, which continues to focus primarily on performing procedures at the expense of understanding mathematical ideas and relationships (Stigler & Hiebert, 1999). More generally, studies in educational policy indicate that large-scale improvement efforts in mathematics and other subject matter areas have rarely produced lasting changes in teachers' instructional practices (Elmore, 2004; Gamoran et al., 2003; McLaughlin, 2006). To date, research on instructional improvement in mathematics education has focused primarily on supporting groups of teachers' learning. However, the challenge of instructional improvement at scale involves supporting schools' and broader educational jurisdictions' development of the capacity to scaffold teachers' (and others') ongoing learning (Bryk, Sebring, Allensworth, Luppesco, & Easton, 2010; Elmore, 2004; Sebring, Allensworth, Bryk, Easton, & Luppesco, 2006). In other words, instructional improvement at scale is a problem of organisational learning as well as teacher learning. As Coburn (2003) observed, "[B]ecause classrooms are situated in and inextricably linked to the broader school and system, teachers are better able to sustain change when there are mechanisms in place at multiple levels of the system to support their efforts" (p. 6). Instructional improvement across a large number of classrooms therefore requires that the settings in which teachers work be organised to support their learning. This in turn implies that members of other role groups, including mathematics coaches and school leaders, will need to reorganise their practices. A comprehensive theory of action for instructional improvement at scale therefore aims to inform schools' and broader jurisdictions' development of the capacity to support and coordinate the learning of the members of multiple role groups.

Research on large-scale instructional improvement has traditionally been the province of educational policy and educational leadership. While much can be learned from these studies, most of this work does not take a position on what

counts as high-quality teaching but instead operationalises it in terms of increasing student test scores irrespective of the quality of the tests. In the course of our work with the four collaborating school districts, it has become increasingly evident that views on what counts as high-quality mathematics teaching matter when formulating strategies or policies for instructional improvement. As Hiebert and Grouws (2007) noted, the notion of instructional quality is not absolute, but is instead grounded in views of what is worth knowing and doing mathematically, and thus in goals for students' mathematical learning. The learning goals that we and the leaders in the four collaborating districts view as worthwhile include that students should develop both conceptual understanding of key mathematical ideas and procedural fluency in a range of domains (e.g., number and operations, algebra, geometry, measurement, data analysis and probability), and that they should learn to communicate their mathematical reasoning effectively by mastering increasingly sophisticated forms of mathematics argumentation (including methods of proof) and by using and making connections between multiple representations (e.g., symbolic expressions, graphs, tables) (Kilpatrick, et al., 2001; National Council of Teachers of Mathematics, 2000).

The National Council of Teachers of Mathematics (NCTM, 2000) drew on available research to articulate a broad vision of instruction that aims at these goals. This vision is often referred to as ambitious teaching (Lampert, et al., 2010). In this vision, teachers support students to solve cognitively-demanding tasks (Stein, Smith, Henningsen, & Silver, 2000), press students to provide evidence for their reasoning and to make connections between their own and their peers' solutions (McClain, 2002), and orchestrate whole class discussions in which they build on students' contributions to achieve their mathematical agendas for students' learning (Franke et al., 2007; Stein, Engle, Smith, & Hughes, 2008). Instructional practices of this type contrast sharply with typical teaching in most US classrooms and require teachers to anticipate and respond to students' thinking (Kazemi, Franke, & Lampert, 2009).

A central goal of ambitious teaching is that learning opportunities are distributed equitably (Lampert & Graziani, 2009; NCTM, 2000). In this context, equity implies that all students should be able to participate substantially in all phases of classroom activities. However, in the course of our work with the four collaborating districts, it has become clear that research in mathematics education currently provides only limited guidance about concrete instructional practices that result in equitable learning opportunities. Although there is a sizable body of literature on equity and mathematics achievement, only a small proportion of these studies focus on classroom teaching and learning (Jackson & Cobb, 2010), and only a handful describe and provide empirical evidence for concrete instructional practices that support all students' substantial participation in each phase of classroom lessons (e.g., Boaler & Staples, 2008; Franke, et al., 2007; Gutiérrez, 2000; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2011; Moschkovich, 2007). Identifying specific instructional practices that result in equitable learning opportunities and that are learnable in the

context of high-quality teacher professional development is an important goal for future research.

The learning goals and vision of ambitious instruction we have outlined oriented the development of the theory of action for instructional improvement in mathematics reported on in the remainder of this article. We organise our presentation of this theory of action by considering its five key components: a coherent system of supports for ambitious instruction that encompasses both formal and job-embedded teacher professional development; teacher networks; mathematics coaches' practices in providing job-embedded support for teachers' learning; school leaders' practices as instructional leaders in mathematics; and district leaders' practices in supporting the development of school-level capacity for instructional improvement. As our focus is on instructional improvement at the level of large US school districts, we give an overview of the US educational system before discussing each of these components in turn. We anticipate that most, if not all, of the components will be relevant to non-US readers but acknowledge that the appropriate organisational unit or educational jurisdiction beyond the school will differ depending on the structure of the educational system in a particular country.

The US Educational System

The US educational system is decentralised, and there is a long history of the local control of schooling. Each US state is divided into a number of independent school districts. In rural areas, districts might serve less than 1,000 students whereas a number of urban districts serve more than 100,000 students. In the context of the US educational system, urban districts are the largest jurisdictions in which it is feasible to design for improvement in the quality of instruction (Supovitz, 2006).

Large school districts such as those with which we are collaborating have a central office whose staff are responsible for selecting curricula and for providing teacher professional development in various subject matter areas including mathematics. In this article, we use the designation district leaders to refer to members of the central office staff whose responsibilities focus on either classroom instruction or school leadership. We use the term *district mathematics specialists* to refer to central office staff whose responsibilities focus specifically on the teaching and learning of mathematics.

The role of the US national government in education has been quite limited historically when compared with most other industrialised countries. However, in 2001, the US Congress passed a national policy called the No Child Left Behind (NCLB) act. The intent of NCLB is to enable all students to meet high performance standards in language arts and mathematics. States are given financial incentives to design and enact the three central components of NCLB policy: content standards, tests aligned with the standards, and mechanisms for holding schools accountable for increasing scores on those tests and for closing gaps in achievement between particular student sub-populations. Historically,

students of color, students from economically disadvantaged backgrounds, and students for whom English is not their first language have performed at significantly lower levels than white students and students from economically advantaged backgrounds on mathematics assessments (Darling-Hammond, 2007).

Most impartial commentators consider that NCLB policy is flawed in two important respects. First, most states lacked the capacity to respond effectively to the assessment and accountability mandates of the policy (Elmore, 2004). As a consequence, the tests used in these states to assess student achievement emphasise procedural skills at the expense of understanding central mathematical ideas (Shepard, 2002). Second, it is becoming increasingly clear that most district and school leaders have little, if any, knowledge of how to respond effectively to state accountability policies (Elmore, 2006). The majority of districts are implementing strategies that involve “teaching to the test”, and some are attempting to “game the system” (Heilig & Darling-Hammond, 2008). As a consequence, standards-based reform has, for the most part, become assessment-driven reform in most districts (cf. Resnick & Zurawsky, 2005). However, a minority of schools and districts has developed at least moderately “worked out” strategies that have the potential to support teachers in improving the quality of their instructional practices (Elmore, 2006).

The four districts with which we are collaborating are typical of urban districts in most respects and have to cope with a number of challenges including significant numbers of low-performing students, limited funding, high teacher turnover, and a significant proportion of novice teachers. However, they are atypical in one respect: they are amongst the minority identified by Elmore and are responding to high-stakes accountability pressures by attempting to support teachers’ development of ambitious instructional practices that aim at rigorous learning goals for students.

Investigating and Supporting Large-scale Instructional Improvement

We identified the key components of a theory of action for instructional improvement at scale in the course of our current work with the four collaborating districts. The overall goal of this work is to test, revise, and elaborate a comprehensive set of hypotheses and conjectures about district and school supports for improving the quality of classroom instructional practice. The 50 participants in each of the four districts include 30 middle-school mathematics teachers from between six and ten schools who teach 12 to 14-year-old students, and 20 coaches, school leaders, and district leaders.

Thus far, we have completed four annual rounds of data collection and analysis. Each October, we interview leaders in each district to document their current strategies for supporting the improvement of middle-school mathematics instruction. In January-March of each year, we document how these strategies are actually playing out in schools and classrooms. The data we collect

include: audio-recorded interviews conducted with the 200 participants that focus on the school and district settings in which the teachers and instructional leaders work (e.g., formal and informal sources of support, to whom they are accountable and for what they are accountable); on-line surveys for teachers, coaches, and school leaders (interview protocols and surveys are downloadable at <http://bit.ly/MISTtools>); video-recordings of two consecutive lessons in the 120 participating teachers' classrooms, coded using the *Instructional Quality Assessment* (IQA) (Matsumura, Garnier, Slater, & Boston, 2008); teachers' and coaches' scores on the *Mathematics Knowledge for Teaching* (MKT) instrument (Hill, Schilling, & Ball, 2004); video-recordings of select district professional development; audio-recordings of teacher collaborative planning meetings; and an on-line assessment of teacher networks completed by all 300 middle-school mathematics teachers in the participating schools. In addition, the four districts have provided us access to mathematics achievement data for students in the participating teachers' classrooms. Thus far, we have achieved almost 100% participation in our data collection efforts in each district each year.

Each February-May, we analyse transcripts of the 200 interviews to identify and explain gaps between each district's intended and implemented improvement strategies. On this basis, we develop a detailed report for leaders in each district in which we share our findings and make actionable recommendations on how they might adjust their improvement strategies to make them more effective. Each May, we visit the districts to discuss our findings and recommendations with district leaders. The interviews conducted the following October reveal that the district leaders are indeed acting on our recommendations to a remarkable degree (Cobb & Jackson, in press). As a consequence, we are, in effect, conducting four parallel design experiments at the level of large districts in which we are testing and revising our hypotheses about supports for instructional improvement at scale (Cobb & Smith, 2008).

Key Components of a Comprehensive Theory of Action for Improving the Quality of Mathematics Instruction at Scale

The initial hypotheses and conjectures about district and school supports for instructional improvement that we formulated prior to working with the four districts (Cobb & Smith, 2008) were relatively abstract. We refined and elaborated these hypotheses while conducting the four rounds of data collection, analysis, and feedback by endeavoring to identify concrete, potentially learnable practices for members of specific role groups (e.g., teachers, coaches, school leaders). The resulting theory of action for district-level instructional improvement comprises the following five components: coherent instructional system, teacher networks, coaching, school instructional leadership, and district instructional leadership. Although we present each component separately, we contend that instructional improvement at scale requires the coordination of all five components.

Coherent Instructional System. The first component of the theory of action concerns the construction of a coherent instructional system for supporting

mathematics teachers' development of ambitious teaching practices. Drawing on Newmann, Smith, Allensworth, and Bryk's (2001) and Bryk et al.'s (2010) seminal work and our own findings (Cobb & Jackson, in press), we differentiate between the following elements of a coherent instructional system:

1. explicit goals for students' mathematical learning;
2. a detailed vision of high-quality instruction that specifies particular instructional practices that will lead to students' attainment of the learning goals;
3. instructional materials and associated tools designed to support teachers' development of these practices;
4. district teacher professional development that focuses on the specific practices, is organised around the above materials, and is sustained over time;
5. school-based professional learning communities (PLCs) that provide ongoing opportunities for mathematics teachers to discuss, rehearse, and adapt the practices that have been introduced in district professional development;
6. assessments aligned with the goals for students' mathematical learning that can inform the ongoing improvement of instruction and the identification of students who are currently struggling; and
7. additional supports for struggling students to enable them to succeed in mainstream mathematics classes.

In the mathematics education research literature, various aspects of this system are often investigated separately. For example, research on the design of sequences of instructional tasks typically draws on research on student learning and often fails to make contact with research on either mathematics teaching or teacher professional development. However, prior research (Bryk et al., 2010; Newmann et al., 2001) and our ongoing analyses indicate that instructional improvement at scale is only possible in practice when district leaders deliberately coordinate the above elements so that they constitute a system in the true sense of the term.

Explicit goals for students' mathematical learning. When attempting to improve instruction at any level of scale, it is imperative to identify the learning goals toward which the instruction aims (Hiebert & Morris, 2009; Jansen, Bartell, & Berk, 2009; Wiggins & McTighe, 1998). In Jansen et al.'s (2009) terms, the learning goals should be: 1) targeted, or "sufficiently well specified to suggest the interventions for supporting learners in achieving them and to indicate the types of evidence needed to determine if the goals have been achieved," and 2) shared, or "mutually understood and committed to by all participants in the knowledge-building process" (p. 525). Our current work suggests the importance of district leaders supporting teachers and members of other role groups in coming to understand the goals for students' mathematical learning. As we describe when we discuss district leadership, if different units within the district central office have different goals for students' learning, it is likely that they will, in turn, hold members of different role groups (e.g., teachers, school leaders) accountable for

developing practices that are at odds with each other. The goals for students' mathematical learning (in conjunction with a vision of high-quality instruction) should drive the design of the remaining elements of the instructional system.

Detailed vision of high-quality mathematics instruction. A second aspect of a coherent instructional system is a detailed vision of high-quality instruction that specifies concrete instructional practices that have the potential to lead to the attainment of the learning goals. This vision articulates the goals for teachers' learning. Consistent with current research on teacher professional development, we have found it important that the guiding vision of instruction specifies a relatively small set of high-leverage instructional practices that are learnable in the context of high-quality professional development (Ball et al., 2009; Grossman et al., 2009; Lampert et al., 2010; Lampert & Graziani, 2009). We have come also to appreciate the value of specifying practices that are specific to particular phases of lessons (e.g., introducing tasks to support all students' substantial engagement while maintaining the level of cognitive challenge, planning and conducting whole class discussions to further all students' understandings). The resulting set of high-leverage instructional practices orients both the design of teachers' professional development and the delineation of high-quality coaching and school instructional leadership practices that aim to support and press for instructional improvement (as described below).

Instructional materials and associated tools designed to support teachers' development of these practices. A third aspect of a coherent instructional system is the provision of instructional materials and tools designed to support teachers' development of the focal practices. Particularly over the past few decades, it has become increasingly common for US school districts to adopt a single mathematics text to guide instruction in mathematics (Remillard, 2005). Available mathematics curricula vary significantly in terms of the goals for students' learning, nature of tasks, sequencing of tasks, and support provided for teachers (Stein & Kim, 2009; Stein, Remillard, & Smith, 2007). The districts with which we work have all adopted middle-grades mathematics texts that reflect ambitious instructional visions and aim at rigorous goals for students' learning. The findings of a number of studies indicate that curricula of this type, when implemented effectively, support similar levels of improvement in procedural fluency as instruction using traditional curricula, and support greater improvements in conceptual understanding and problem solving (Cai, Nie, & Moyer, 2010; Schoenfeld, 2002). However, there is also strong evidence that teachers' use of texts of this type does not ensure that they will enact the intended practices in their classrooms (Remillard, 2005; Riordan & Noyce, 2001; Tarr et al., 2008). For example, it is common for US teachers to proceduralise the cognitively demanding tasks in these texts when they introduce them to students, thus reducing the cognitive demand (Boston & Smith, 2009; Henningsen & Stein, 1997; Stein et al., 2000). In this regard, Tarr et al. (2008) found that the classroom learning environment impacts the effectiveness of ambitious curricula, with an achievement advantage occurring only in classrooms where students are pressed to explain their solutions, multiple

strategies are encouraged, and lessons foster conceptual understanding. Investigations of the implementation of ambitious curricula also indicate that although sub-population achievement gaps tend to diminish, they are not eliminated (Riordan & Noyce, 2001; Schoenfeld, 2002). This suggests that these curricula can be a valuable resource for addressing issues of equity but are not by themselves sufficient (Darling-Hammond, 2007).

It is important to clarify that US teachers are expected to use the text adopted by their school or district to address state mathematics standards. Districts frequently develop curriculum frameworks to assist teachers in making this coordination when they plan for instruction. In their most elementary forms, curriculum frameworks specify the sequencing and pacing of instruction both within and across grade levels. However, some districts, including two of the four with which we are collaborating, have developed elaborate curriculum frameworks that provide information about student solutions to particular tasks, what is likely to be linguistically demanding in particular lessons for students whose first language is not English, and strategies for meeting the needs of these and other groups of students.

Newmann et al. (2001) argue that the use of a common curriculum framework is a key element of a coherent instructional system. However, there is little if any research on how teachers actually use such frameworks. If the two districts with which we are working are in any way typical, it could be the case that most districts provide only limited professional development on using the framework, and that most teachers use the framework developed by their district primarily as a pacing guide. Teachers in the two collaborating districts rarely draw on other information in the framework to anticipate students' solutions or differentiate instruction. Changing the pacing of instruction involves adjusting current instructional practices whereas the effective use of other information in curriculum frameworks requires that teachers reorganise their current instructional practices. These observations suggest that teachers need sustained support to learn how to use frameworks productively.

Taken together, the findings we have discussed indicate that effective use of ambitious curricula and associated curriculum frameworks requires significant teacher learning, and that the equity dimension of ambitious teaching requires explicit attention. Ongoing professional development is required if teachers are to learn how to use these resources effectively as part of the process of developing the intended instructional practices (e.g., introducing tasks to support all students' substantial engagement while maintaining the cognitive challenge).

District teacher professional development. The research literature indicates that teacher professional development is more likely to influence classroom practice when it is sustained over time, involves the same group of teachers working together, is focused on issues central to instruction, and is organised around the instructional materials that teachers use in their classrooms (Darling-Hammond, Wei, & Orphanos, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Kazemi & Franke, 2004; Little, 2003). In addition, recent work by Grossman and

colleagues (Grossman et al., 2009; Grossman & McDonald, 2008) indicates the value of distinguishing between two types of activities, both of which contribute to effective professional development: *pedagogies of investigation* and *pedagogies of enactment*. Pedagogies of investigation involve analysing and critiquing representations of practice such as student work and video-cases of teaching (Borko, Jacobs, Eiteljorg, & Pittman, 2009; Sherin & Han, 2004). Pedagogies of enactment involve planning for, rehearsing, and enacting high-leverage practices in a graduated sequence of increasingly complex settings (e.g., teaching other teachers who play the role of students, working with a small group of students, teaching an entire class). Pedagogies of investigation are far more common in mathematics teacher professional development than pedagogies of enactment. However, Grossman et al. argue convincingly that both are necessary if teachers are to develop ambitious forms of practice.

In considering the role of pedagogies of investigation and of enactment in a coherent professional development program, it is useful to distinguish between professional development offered by a district for all mathematics teachers at particular grade levels and school-based professional development. Typically, teachers are released from teaching to attend district-wide professional development for a few days each year, whereas school-based professional development is often ongoing and involves only the mathematics teachers in a particular school. We conjecture that district professional development is better suited for pedagogies of investigation because large numbers of teachers are involved, whereas school-based professional development is suited for both types of pedagogies. The importance of coordinating the two forms of professional development such that they focus on the same high-leverage practices has become apparent in our current work. For example, teachers might first analyse video-recordings of teachers introducing cognitively demanding tasks in district professional development, and then enact introducing similar tasks with their colleagues in school-based professional development. This coordinated approach focuses on supporting teachers' development of the same practices over time, using materials that are central to their instruction.

Research in mathematics education has begun to made headway in delineating effective professional development practices (Borko et al., 2009; Elliott et al., 2009). This work suggests the value of supporting facilitators' development of what Coburn and Russell (2008) call "routines of interaction" (p. 213). Routines of interaction are questions that professional development leaders pose routinely to press participating teachers on key issues (e.g., identifying the key mathematical ideas in a set of tasks, identifying aspects of the task scenario that might be unfamiliar to some students, anticipating student solutions to particular tasks). Coburn and Russell present evidence that coaches who had been pressed on issues of this type in coach professional development subsequently pressed teachers on the same issues, and that teachers then began pressing each other on these issues. Based on this finding, we conjecture that the routines of interaction enacted in district professional development might influence the nature and depth of interactions in school-based professional

development, and thus the extent to which the latter supports teachers' development of ambitious instructional practices.

School-based professional learning communities. Professional learning communities (PLCs) in which the mathematics teachers at a school meet together on a regular basis to work on problems of practice are a central aspect of school-based teacher professional development. It is becoming common for US districts to mandate that schools schedule time during the school day for teacher collaboration. These are costly initiatives, given that PLCs vary in the extent to which they support instructional improvement (Little, 1993). A growing number of studies indicate that when PLCs function well, they provide opportunities for teachers to address problems that arise in the course of instruction, integrate ideas and tools introduced in district professional development into practice, and rehearse specific practices (Cobb, Zhao, & Dean, 2009; Horn & Little, 2010). The potential links between pedagogies of investigation and of enactment suggest that the work of PLCs should follow up on district professional development by focusing on the same high-leverage instructional practices. Research in teacher professional development suggests that potentially productive PLC activities might include doing mathematics problems and comparing solution strategies, analysing student work and classroom video-recordings, and rehearsing high-leverage instructional practices (Ball et al., 2009; Borko et al., 2009; Kazemi & Hubbard, 2008; Sherin & Han, 2004). In addition, this research indicates the importance of leadership for PLCs in setting an agenda, initiating and guiding activities, and enacting routines of interaction. As we clarify below when we discuss mathematics coaching, we view coaches as the most likely candidates for providing this leadership.

For the most part, current research on PLCs has treated them as existing in an institutional vacuum and has not taken account of the school and district settings in which the participating teachers work. However, several studies indicate that these settings can profoundly affect the extent to which PLCs are productive for teacher learning (Cobb & McClain, 2006; Stein & Coburn, 2008). For example, one of the basic requirements for a PLC to be productive is that the participating teachers have deprivatised their instructional practices and are willing to discuss openly the problems that they encounter in practice. Cobb, McClain, Lamberg, and Dean (2003) reported a case in which school leaders made frequent classroom visits in order to monitor that instruction addressed state mathematics objectives and that students were on task. In this setting, teaching was highly privatised and it was 18 months before the teacher group became a genuine PLC with a common agenda that focused on problems of practice (Dean, 2005). This study indicates that it is important to consider the role of school leaders in both organising the conditions for PLCs (e.g., scheduling time for meetings) and shaping PLC interactions (e.g., the focus of their classroom observations and the intent of the feedback that they give teachers).

Assessments aligned with the goals for students' mathematical learning. The sixth element of a coherent instructional system concerns assessments that can inform the ongoing improvement of classroom instruction and the identification of

students who are currently struggling and may need additional support (Newmann et al., 2001). As we have noted, the four districts with which we are working are among the minority that are responding productively to high-stake accountability demands. Three of the four districts have given priority to assessments that enable them to identify students who are unlikely to be successful on state assessments and thus need additional support.

Mathematics education research on student learning trajectories or progressions is well situated to inform the development of classroom assessments that can also be used to improve the quality of classroom instruction (e.g., Barrett, Clements, Klanderma, Pennisi, & Polaki, 2006; Clements & Sarama, 2004; Confrey, 2011; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). As Confrey (2011) observes, the use of diagnostic assessments that are based on substantiated learning progressions in particular mathematical domains can support teachers to “identify student proficiency levels,... diagnos[e] problems,... and scaffold student learning” (p. 11). However, the effective use of such assessments involves a significant reorganisation rather than a mere adjustment of current practices for most US teachers. We therefore conjecture that sustained professional development that is organised around such assessments as well as instructional materials and curriculum frameworks in an integrated manner will be essential.

Additional supports for struggling students. The seventh aspect of a coherent instructional program concerns additional supports for struggling students (Newmann et al., 2001). From the point of equity, the aim of these supports should not be limited to passing state assessments, but should also include enabling struggling students to succeed in mainstream mathematics classes. It is common for districts to provide supplemental supports for struggling students (e.g., tutoring, second or “double dose” mathematics classes). A recent study found that “nearly half of large [US] urban districts report double-period math instruction as the most common form of support for students with lower skills” (Durwood, Krone, & Mazzeo, 2010, p. 7). However, the research base on instruction in additional mathematics classes for struggling students is extremely limited (cf. Confrey, 2011; Nomi & Allensworth, 2009). Based on our work with the four collaborating districts, we strongly suspect that supplemental instruction in most districts currently focuses on the procedural competencies assessed by state tests. It is therefore important that districts provide teachers who conduct this instruction with professional development and associated curricular resources that are specific to meeting the needs of struggling students.

Teacher Networks

Networks of professional relations between mathematics teachers constitute the second component of the proposed theory of action for instructional improvement. The establishment of the first component, a coherent instructional system, can support the development of strong relationships among mathematics teachers in a school (Bryk et al., 2010). The findings of several

studies indicate that the resulting trust, mutual accountability for student learning, and access to others' expertise are at least as important as teachers' perceptions of the value of the improvement initiative in driving improvements in classroom practice (Bryk & Schneider, 2002; Frank, Zhao, & Borman, 2004; Penuel, Riel, Krause, & Frank, 2009; Spillane & Thompson, 1997). Teachers' social networks are therefore a key support for school-wide instructional improvement (Coburn, 2001; Penuel, Frank, & Krause, 2006).

Although teacher networks are emergent phenomena and cannot simply be mandated into existence (Smylie & Evans, 2006; Spillane, Reiser, & Gomez, 2006), district and school improvement policies can influence the conditions under which teachers decide whether to turn to a colleague for instructional advice and the types of advice they seek (Coburn & Russell, 2008). For example, it seems reasonable to conjecture that teachers' participation in district professional development and in follow-up school-based PLCs that both focus on specific instructional practices and are organised around the instructional materials the teachers are using might support the emergence of professional relations among the teachers in a school.

The extent to which a teacher network does in fact support the participating teachers' learning depends crucially on the nature of their interactions with one another. Building on the work of Coburn and Russell (2008), we have found it essential to distinguish between low-depth interactions that focus on "surface structures and procedures" (e.g., sharing materials, pacing) and high-depth interactions that focus on "underlying pedagogical principles of the approach, the nature of the mathematics and how students learn" (e.g., discussing different solution strategies to mathematical problems, analysing student work) (p. 212). Consistent with our expectations, our initial analyses of teacher networks in the four collaborating districts indicate that the allocation of time for teacher collaboration is not, by itself, sufficient to influence the depth of teachers' advice-seeking interactions (Sun & Frank, 2011). Instead, the presence of one or more teachers who have already developed relatively accomplished practices in a network appears to be crucial. Our findings indicate that the presence of a mathematics coach in a school influences the overall depth of teachers' interactions (Garrison, Smith, Cobb, & Green, 2011). Furthermore, the level of sophistication of the practices of the most accomplished teacher in a school is related to overall improvement in the quality of instruction in the school. In addition, teachers' interactions with more accomplished colleagues are related to significant improvements in their mathematical knowledge for teaching and in the quality of their instructional practices (Sun & Frank, 2011).

We conjecture that the extent to which network interactions support instructional improvement is also related to the nature of activities in which teachers engage during district and school-based professional development and, in particular, the role of coaches in pressing teachers on specific high-leverage issues. We plan to investigate this conjecture in future analyses.

Mathematics Coaching

The third component of the proposed theory of action concerns mathematics coaching.¹ Although US districts are increasingly using coaches as a primary means of supporting teachers' learning, the designs of their coaching programs vary considerably. For example, one of the districts we work with is implementing a school-based coaching design in which a mathematics teacher in each middle-grades school serves as a part-time coach (i.e., the coach works with colleagues half of the day and teaches the other half of the day). Another district has created a cadre of full-time coaches, each of whom serves three or four schools. Across the four districts, there are also differences in the extent to which coaches are expected to work with individual teachers in their classrooms and with groups of teachers.

The various coaching designs share a common rationale, namely that coaches are more accomplished colleagues who can work with teachers on problems that are close to practice. Given the importance that we attribute to ensuring that PLCs are productive and to fostering the emergence of supportive teacher networks we currently recommend to the four collaborating districts that they give priority to coaches leading PLC meetings. Current research on teacher professional development provides some insight into the types of activities in which coaches might engage with groups of teachers to support their learning (Franke et al., 2007; Kazemi & Hubbard, 2008). In contrast, the research base on how coaches might work with individual teachers in their classrooms and on what constitutes high-quality coach professional development is limited. However, more general research on the development of complex practices is relevant and suggests that it is critical for novices to co-participate in activities that approximate the targeted practices with more accomplished others (Bruner, 1996; Forman, 2003; Lave & Wenger, 1991). This suggests that potentially productive coaching activities might include co-teaching and enacting the coaching cycle of jointly planning a lesson, observing the enactment of the lesson, and then jointly analysing the lesson (Bradley, 2007; Neufeld & Roper, 2003; Olson & Barrett, 2004). We conjecture that the effectiveness of these activities might be enhanced if they foreground the specific instructional practices that have been the focus of district professional development. It is noteworthy that, to this point, coaches in the four collaborating districts spend the bulk of their time observing instruction and giving feedback when they work with individual teachers. This coaching practice might be useful at specific points in teachers' development (e.g., when they have become relatively accomplished in enacting particular instructional practices and need assistance in fine-tuning those practices). However, there is little reason to believe that the practice of observing and providing feedback will, by itself, be sufficient to support teachers' development of ambitious instructional practices. Coburn and Russell's

1 We recognize that districts might not employ coaches as a strategy for supporting instructional improvement at scale. Our key point is that people who have already developed accomplished instructional practices work with teachers directly on their classroom practices.

(2008) findings indicate that it is also important for coaches to enact specific routines of interaction (e.g., pressing teachers to identify the mathematical relationships that students need to understand in order to engage in a given task productively) when working both with groups of teachers and with individual teachers.

The experience of the district with which we are working that is attempting to implement a school-based coaching design indicates that it can be challenging to support coaches in becoming more accomplished than the teachers they are expected to support. The findings of several studies also indicate that the development of relatively accomplished instructional practices is a necessary but not sufficient condition for developing effective coaching practices (Borko et al., 2009; Elliott et al., 2009). As is the case for teachers, coaches' learning needs to be scaffolded by co-participating in activities close to practice with more accomplished others, namely district mathematics specialists. Extrapolating from research in teacher professional development, we also conjecture that coach professional development should be ongoing, should include both pedagogies of investigation and of enactment, and should focus on specific coaching practices. For example, coaches might work with district mathematics specialists on how to support teachers in learning to introduce tasks, or to orchestrate a whole class discussion effectively. It might also be important that district mathematics specialists enact specific routines of interactions with coaches, and then support them in enacting those same routines with teachers.

School Instructional Leadership

The fourth component of the theory of action for improving the quality of mathematics instruction at scale concerns school instructional leadership. Historically, the principalship in US schools has focused on administration and management (e.g., scheduling classes, school finances, student discipline, relations with the community served by the school) (Elmore, 2000; Glennan & Resnick, 2004; Honig, 2006; Leithwood, Louis, Anderson, & Wahlstrom, 2004). However, increasing accountability demands have resulted in the widespread expectation that principals should act as instructional leaders in mathematics and other disciplines (Fink & Resnick, 2001; Murphy, Elliott, Goldring, & Porter, 2007; Robinson, Lloyd, & Rowe, 2008). The findings of a number of investigations indicate that the principal's role as an instructional leader can be critical in driving instructional improvement efforts (Bryk et al., 2010; Spillane, Hallett, & Diamond, 2003).

Current research on school instructional leadership provides contradictory guidance on what principals need to know and do in order to be effective instructional leaders in mathematics. Some researchers argue that it is sufficient for school leaders to understand general, content-independent principles of learning and instruction (Resnick & Glennan, 2002; Resnick & Zurawsky, 2005) whereas other researchers contend that school leaders need a deep understanding of mathematics, students' mathematical learning, and teacher learning (Nelson & Sassi, 2005; Stein & Nelson, 2003).

All four districts in our study are attempting to support school leaders' (principals' and/or assistant principals') development as instructional leaders in mathematics. In three of the districts, school leaders have received extensive professional development that was designed to support them in understanding content-independent principles of learning and instruction. Our initial findings suggest that professional development of this type is too global in that most of the school leaders in these three districts are not able to distinguish between strong and weak inquiry-oriented mathematics lessons (Cobb & Jackson, in press). In addition, the provision of professional development based on the view that school leaders need to develop a deep understanding of mathematics, students' mathematical learning, and teacher learning appears to be beyond the capacity of most districts.

As part of our current work, we conducted three half-day professional development sessions for 80 school leaders and mathematics coaches in the fourth district in August-October 2009. The goals of the sessions were to support principals in distinguishing between cognitively low- and high-demand mathematics tasks and in recognising the value of key aspects of ambitious instruction (e.g., whole class discussions that support the development of conceptual understanding). The results from this pilot study are encouraging. School leaders' ability to distinguish between high- and low-level mathematics tasks increased significantly as a consequence of the sessions (Colby, Gibbons, Henrick, Wong, & Boston, 2010). Their ability to recognise key aspects of ambitious mathematics instruction also increased moderately.

In the context of our current work, we have come to see value in a distributed model of school instructional leadership in which coaches and district mathematics specialists are primarily responsible for supporting teachers' learning, and school leaders are responsible for pressing and holding teachers accountable for developing the intended instructional practices (Elmore, 2006; Printy & Marks, 2006; Spillane, Halverson, & Diamond, 2004). We have also drawn on our initial findings and the available literature to identify three leadership practices that might be feasible goals for school leaders' learning. Two of these practices aim at pressing teachers to develop the intended forms of practice and providing teachers with adequate support: observing mathematics instruction and providing feedback, and participating in mathematics PLCs. The third practice concerns the development of productive relationships with coaches.

By observing instruction and providing teachers with informed feedback, school leaders can both communicate expectations and hold teachers accountable for improving classroom instruction. It is important that the feedback be specific to particular phases of lessons and to instructional practices on which teacher professional development focuses. However, the extent to which school leaders' feedback accomplishes these goals depends crucially on the nature of professional development school leaders have received (see below).

School leaders' participation in mathematics PLCs signals the importance of teacher collaboration, enables school leaders to hold teachers accountable for

using collaborative time productively, and constitutes a context for school leaders' learning, thus better positioning them to procure appropriate resources for teachers. In this regard, a meta-analysis conducted by Robinson et al.(2008) found that school leaders' participation in teacher professional development is strongly associated with improvements in the quality of instruction and student achievement.

The findings of several studies, including our own, indicate that coaches' effectiveness in supporting teachers' learning depends on school leaders assuming shared responsibility for instructional improvement with coaches (Gibbons & Cobb, 2010; Mangin, 2007; Matsumura, Sartoris, Bickel, & Garnier, 2009). This requires that school leaders understand the district-wide goals for students' mathematical learning and the guiding vision of high-quality instruction, and appreciate the critical role of coaches in supporting teachers' learning. In the context of our current work, we have documented several cases in which principals assigned additional duties to coaches that took them away from their work with teachers (e.g., analysing data to identify struggling students, tutoring struggling students). Our observations also indicate that principals protect coaches' time when they understand the coaches' role in the improvement effort.

The development of shared responsibility for instructional improvement appears to be facilitated if school leaders and coaches meet regularly to share their observations about the quality of teachers' instructional practices, discuss how work with teachers is progressing, jointly select teachers with whom the coach should work, and plan for future work with groups of teachers (Gibbons & Cobb, 2010). These meetings provide opportunities for the school leader to both communicate expectations to the coach, and to hold the coach accountable for working with individual and groups of teachers individually as planned. These meetings can also give rise to opportunities for school leaders to deepen their understanding of high-quality mathematics instruction and the means of supporting teachers' learning (Cobb & Jackson, in press). To further support school leaders' learning, we recommend to the four collaborating districts that coaches and school leaders observe instruction together and then discuss their observations and the nature of feedback that they might give the teacher. Clearly, it is important that teachers understand that the purpose of the observations is not evaluative in nature, lest the observations jeopardise the coach's relationship with teachers.

It is a non-trivial undertaking for school leaders, most of whom are not mathematics specialists, to develop the three instructional leadership practices that we have described. In our view, the principles of high-quality professional development that we have discussed should guide the design of professional development for school leaders as well as for teachers and coaches. These principles include that professional development should involve ongoing work with more accomplished others that is organised in terms of pedagogies of investigation and enactment, that focuses on concrete high-leverage practices, and that includes a consistent press on a small number of key issues. The

feedback that we give to the collaborating districts is based on the conjecture that professional development with the following foci will support school leaders' development of the intended instructional leadership practices.

First, if school leaders are to effectively and realistically press teachers to improve the quality of instruction, professional development for school leaders should enable them to recognise the instructional practices that are the focus of teacher professional development, and to distinguish between low- and high-quality enactments of those practices. We conjecture that a consistent emphasis on instructional practices across teacher, coach, and school leader professional development will contribute to the development of compatible visions of high-quality instruction and to the alignment of supports for teachers' learning.

Second, we conjecture that professional development should attend explicitly to how to provide feedback to teachers that communicates expectations for ambitious instruction. This might involve school leaders and district mathematics specialists observing instruction or watching video-recordings of specific phases of lessons and discussing the feedback they would provide with the goal of improving its quality.

Third, professional development should build on school leaders' developing understanding of high-quality mathematics instruction by clarifying the role of coaches and PLCs in supporting teachers' development of ambitious instructional practices. We have documented several cases in which a school leader has taken over the agenda for PLC meetings to the detriment of the participating teachers' learning. We therefore conjecture that it is important to give particular attention to how the distribution of instructional leadership between coaches and school leaders should follow the contours of their complementary areas of expertise (Elmore, 2006). The pilot professional development sessions that we conducted in one of the collaborating districts indicated that mathematics coaches' participation in professional development with principals can foster productive professional relationships.

District Instructional Leadership

The fifth and final component of the theory of action for improving the quality of mathematics instruction concerns district instructional leadership. The work of several district leaders has emerged as particularly significant in the course of our work with the four collaborating districts. These include the Superintendent, who is in charge of the entire district, the Chief Academic Officer (CAO) who is typically responsible for matters relating to curriculum and instruction in all content areas, and the leaders of several central office units including Curriculum and Instruction (C&I, responsible for teacher and coach professional development), Leadership (responsible for supporting and assessing school leaders), English Language Learners, Special Education, and Research, Evaluation, and Accountability (REA). The literature on the role of central office units in supporting instructional improvement is almost non-existent (Honig & Copland, 2009; Louis, 2008; Rorrer, Skrla, & Scheurich, 2008). However, at the

outset of our current work, we conjectured that the relationship between central office units would influence the success of the collaborating districts' instructional improvement efforts. This has proved to be the case. The alignment of the agendas of C&I and Leadership appear to be particularly critical (Cobb & Jackson, in press). In the following paragraphs, we share our findings about the role of district leaders in supporting instructional improvement.

It appears critical that district leaders in the central office units that we have listed share both goals for students' mathematical learning and a vision of ambitious instruction (i.e., goals for teachers' learning). However, we have found that compatible goals for students' and teachers' learning is not, by itself, sufficient for supporting district-wide instructional improvement. In addition, how district leaders frame the problem of supporting student learning also appears to be important. This framing influences what district leaders hold school leaders, coaches and teachers accountable for, and thus the prospects for district-wide instructional improvement.

On the basis of interviews that we have conducted with leaders in the four collaborating districts, we distinguish between two broad framings that we term *instructional improvement* and *instructional management*. As an illustration, if a CAO frames the problem of supporting student learning as one of fundamentally improving the quality of instruction, he or she might view mathematics instruction that is compatible with the NCTM Standards and that addresses state objectives as a viable way to both meet NCLB mandates and ensure that instruction attends to conceptual as well as procedural goals. We refer to this response as reflecting an *instructional improvement* orientation because it focuses on the quality of teachers' instructional practices and entails the provision of professional development and job-embedded supports for teachers' learning. Alternatively, a CAO might frame the problem of supporting student learning as one of ensuring that instruction focuses on state mathematics objectives, and by providing students who have not met particular objectives with additional instruction or tutoring that focuses on those objectives. We refer to this response as reflecting an *instructional management* orientation because it focuses on redeploying the district's current instructional resources and does not attempt to improve the quality of those resources. Based on our work with the four districts, we have come to the view that instructional improvement and instructional management are both important but need to be tightly coordinated so that instructional management aims specifically at enabling struggling students to succeed in their regular mathematics classes.

It appears to be particularly important that leaders of C&I and Leadership share common goals for students' learning and frame the problem of supporting students' achievement of those goals in similar ways (Cobb & Jackson, in press). When this is not the case, we have found that leaders tend to hold members of different role groups accountable for developing practices that are at odds with each other. For example, in one of the collaborating districts, we have found that while the efforts of leaders in C&I focus on supporting teachers' and coaches' development of ambitious practices, leaders in Leadership hold principals

accountable primarily for the improvement of students' mathematics achievement scores. In turn, principals communicate these expectations to teachers, and do not press teachers to improve the quality of their instruction. Additionally, principals typically direct resources toward providing supplemental supports for struggling students that are not aligned with mainstream classroom instruction (e.g., requiring coaches to coordinate tutoring programs that focus on basic computational skills rather than working with teachers to improve the quality of instruction).

In the course of our work it has become clear that the Superintendent plays a crucial role in setting direction for the improvement efforts, especially regarding how the problem of supporting student learning is framed. In addition, we conjecture that it is important for leaders in various central office units to have regular opportunities to collaborate together on the design and implementation of instructional improvement policies if they are to develop shared goals for students' learning and to frame the problem of supporting students' attainment of those goals in similar ways.

Finally, our analysis of each of the four district's instructional improvement plans and of district leader interviews indicates the importance of district leaders approaching instructional improvement at scale from a *learning perspective* (Hubbard, Mehan, & Stein, 2008). Leaders who take a learning perspective recognise that achieving an ambitious vision of mathematics instruction across district classrooms is not merely a matter of ensuring compliance with district policies, but instead requires significant learning on the part of teachers, coaches, and school leaders. Additionally, they view it as their responsibility to lead the design and implementation of supports for teachers', coaches', and school leaders' learning. Further, they recognise district personnel who have expertise in supporting mathematics teachers' or instructional leaders' learning and attempt to capitalise on that expertise (Spillane & Thompson, 1997). For example, the CAO in one of our districts regularly draws on the expertise of district mathematics specialists when formulating policies that are specific to instructional improvement in mathematics. The CAO recognises that he does not have the specific knowledge to make decisions regarding, for example, teacher professional development, and draws on the expertise of those who do.

Discussion and Conclusion

In this article, we have proposed an empirically grounded theory of action for improving the quality of mathematics instruction at scale. The theory of action comprises five components: a coherent system of supports for ambitious instruction that encompasses both formal and job-embedded teacher professional development; teacher networks; mathematics coaches' practices in supporting teachers' learning; school leaders' practices as instructional leaders in mathematics; and district leaders' practices in supporting the development of school-level capacity for instructional improvement. This theory of action is specific to the US educational context and reflects both the decentralised

structure of the US educational system and the demands of high-stakes accountability. We anticipate that the specific components of the theory of action and the more general approach of framing instructional improvement at scale as a problem of organisational learning will both prove relevant when considering the improvement of mathematics instruction at scale in the context of other educational systems.

We contend that all five components of the proposed theory of action are necessary for large-scale instructional improvement; the prospects for achieving and sustaining instructional improvement diminish significantly if any one of the components is neglected. As we have attempted to illustrate, improving instruction at scale involves aligning supports for the learning of members of multiple role groups. For example, we would question an improvement strategy that focuses on high-quality curriculum materials, teacher professional development, and mathematics coaching but does not attend to school leaders' development as instructional leaders. Such a strategy might well be ineffective because it is unlikely that school leaders will either press teachers to develop the intended practices or support coaches' work with teachers.

Due to space limitations, we have not given adequate attention to the development of tools designed to support the members of various role groups in reorganising their practices. Although we included curriculum frameworks for teachers as a key element of a coherent instructional system, we did not discuss tools for coaches and school leaders. Initial findings from our work suggest that it is important that any tool used by teachers or instructional leaders be aligned with the guiding vision of high-quality instruction. For example, we have found that the aspects of instruction that school leaders attend to when observing classroom instruction and giving feedback to teachers are significantly influenced by the classroom observation protocol they use. We have documented cases in which a lack of alignment with the vision of high-quality instruction impedes the extent to which school leaders' observations and feedback communicate appropriate expectations for instructional improvement to teachers. However, as we have made clear, the provision of tools will not, by itself, support the reorganisation of practice. Carefully designed professional development that focuses on learning to use the tools in the intended ways (and thus on developing the intended practices) is essential. We refer the reader to Cobb and Jackson (in press) for a detailed discussion of the design of tools aimed at supporting instructional improvement at scale, and the design of professional development specific to supporting various role groups in learning to use the tools.

We view the theory of action we have proposed as a work in progress and have indicated the aspects of the theory that are provisional and subject to revision. We intend to further refine these aspects of the theory as we conduct future empirical analyses. As we noted at the beginning of this article, the history of large-scale improvement efforts that involved significant changes in teachers' instructional practices is primarily one of failure. We contend that this unfortunate record is due in large part to the inability of research to inform the

design and implementation of comprehensive systems of supports aimed at building and sustaining district and school capacity for instructional improvement. In the course of our work, it has become only too apparent that district leaders necessarily have to venture into uncharted territory when they formulate and attempt to implement instructional improvement policies. The intent of the work we have reported in this article is to contribute to the development of a body of research that maps this territory and can provide district leaders with empirically grounded guidance.

Acknowledgements

This article is based on work conducted in collaboration with Thomas Smith, Erin Henrick, Glenn Colby, Annie Garrison, Lynsey Gibbons, Sarah Green, Karin Katterfeld, Chuck Munter, Rebecca Schmidt, and Jonee Wilson. The empirical work reported on in the article has been supported by the National Science Foundation under grants No. ESI-0554535 and No. DRL-0830029. In addition, the National Academy of Education/Spencer Postdoctoral Fellowship Program supported Kara Jackson's contributions to the article. The opinions expressed do not necessarily reflect the views of either Foundation.

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Authors

Paul Cobb, Vanderbilt University, Nashville, Tennessee, USA.

Email: <paul.cobb@vanderbilt.edu>

Kara Jackson, McGill University, Montreal, Quebec, Canada.

Email: <kara.jackson@mcgill.ca>