Supporting Teachers' Use of Research-Based Instructional Sequences

Paul Cobb
Vanderbilt University
Department of Teaching and Learning
1930 South Drive, 240 Wyatt Center
Peabody Box 330
Nashville, TN 37203
USA
615.343.1492 (office)
paul.cobb@vanderbilt.edu

Kara Jackson
University of Washington
Miller Hall 115E
Box 353600
Seattle, WA 98195
USA
206.221.4794 (office)
karajack@uw.edu

Abstract

In this paper, we frame the dissemination of the products of classroom design studies as a process of supporting the learning of large numbers of teachers. We argue that high-quality pull-out professional development is essential but not sufficient, and go on to consider teacher collaboration and one-on-one coaching in the classroom as additional supports. We then illustrate that it is also important to identify other aspects of the school context that need to be addressed by a dissemination design. In doing so, we propose a backward mapping approach that brings to the fore aspects of school context that might need to be taken into account, such as school instructional leadership and teachers' access to colleagues who have already developed relatively accomplished instructional practices. We conclude by arguing for the importance of making the design and improvement of dissemination processes a focus of investigation in its own right. In addition, we consider how researchers who conduct classroom design studies can make the challenges of dissemination more tractable by designing with an eye towards large-scale implementation from the outset.

1.0 Introduction

The overall focus of this special issue is on design studies conducted to investigate processes of students' learning and the means of supporting them in particular mathematical domains. The prior articles clarify key aspects of the design research methodology and illustrate its value in producing empirically grounded designs for supporting students' mathematical learning. The editors have asked us to step back from the specifics of the methodology and discuss some of the issues that should be considered when disseminating the products of design studies conducted to investigate and support students' learning.

For the purposes of this article, we limit our focus to classroom design studies in which a research team collaborates with a mathematics teacher (who might be a research team member) to assume responsibility for the instruction of a group of students for an extended period of time. The intended products of such studies typically include a sequence of instructional activities and associated instructional resources together with a local or domain-specific instructional theory that constitutes the rationale for the instructional sequence (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Stephan & Akyuz, 2012). Instructional theories of this type comprise a substantiated learning trajectory or pathway, and the demonstrated means of supporting that learning (Gravemeijer, 1994). It is important to note that the means of support considered extend beyond tasks and physical, symbolic, and computer-based tools, and include, at a minimum, the types of questions the teacher should ask students to spur their thinking. A number of researchers take a more expansive view and include the establishment of particular classroom norms and forms of classroom discourse as crucial supports (Doorman, Drijvers, Gravemeijer, Boon, & Reed, 2013; Kwon, Ju, Kim, Park, & Park, 2013; Lehrer, Kim, & Jones, 2011; Stephan & Akyuz, 2012). In both cases, the supports considered therefore attend to how the tasks and tools are enacted in the classroom, thereby implicating teachers' instructional practices. This observation is clearly consequential as it indicates that dissemination involves supporting the development of teachers' classroom practices.

The student learning goals addressed by the vast majority of classroom design studies conducted in recent years have included conceptual understanding of central mathematical ideas, making connections among multiple representations, and communicating and justifying mathematical ideas by making mathematical arguments (Plomp & Nieveen, 2013). Furthermore, the designs developed, tested, and refined in the course of these design studies typically reflect the assumption that students' learning is an active constructive process, though researchers conducting design studies differ in the extent to which they view these as primarily individual processes or collective, communal processes (Cobb, Jackson, & Dunlap, in press). In addition, the view of high quality mathematics instruction inherent in these designs typically involves the teacher introducing cognitively challenging tasks, monitoring the range of solution strategies that students produce while working individually or in small groups, and then building on these strategies during a concluding whole-class discussion by pressing students to justify their reasoning and to make connections between their own and others' solutions (Plomp & Nieveen, 2013). The critical point to note for our purposes is that these suppositions and assumptions are at odds with typical mathematics instruction in many of the

countries in which design research is a prominent methodology (Krainer & Zehetmeier, 2013; Maaß & Artigue, 2013). As a consequence, disseminating the products of classroom design studies will often involve supporting large numbers of teachers not merely to extend or elaborate their current instructional practices, but to reorganize those practices. The teacher learning involved is substantial as teachers' development of instructional practices that place students' mathematical reasoning at the center of decision making involves reconceptualizing both student learning goals and the process of students' mathematical learning (Franke, Carpenter, Levi, & Fennema, 2001; Kazemi & Franke, 2004).

These observations about the depth of teachers' learning serve to clarify the challenges involved in disseminating the products of classroom design studies at a level of scale beyond researchers working directly with groups of teachers. The resulting perspective on the process of dissemination is broadly consistent with Rogers' (2003) seminal analysis of the features of an innovation and of the mechanisms of communication that influence the success or failure of diffusion. In summarizing Rogers' findings, Zaritsky, Kelly, Flowers, Rogers, and O'Neill (2003) noted that "among the factors relevant to successful innovation is perceived relative advantage: the degree to which an innovation is perceived as better than the idea, product, or technique it hopes to supersede" (p. 33). Zaritsky et al. explained that measures of relative advantage include simplicity, or the degree to which the innovation is perceived to be relatively easy to understand and use, and compatibility, or the extent to which the innovation is perceived to be compatible with existing values and needs. In our terms, simplicity and compatibility are measures of the extent to which the instructional practices inherent in the successful enactment of the products of a design study are aligned with teachers' current practices. More generally, the cases of successful innovation that Rogers discussed to illustrate his findings involve the extension and elaboration of current practices, but not the reorganization of those practices. As Zaritsky et al. observed, the application of Rogers' analysis of diffusion to educational settings would restrict dissemination efforts to innovations that fit to a considerable extent with teachers' current practices. In contrast, the dissemination of the products of classroom design studies problematizes many teachers' assumptions about learning, teaching, and mathematics. This strongly suggests that Rogers's analysis cannot, by itself, provide adequate guidance for the dissemination of the instructional activities and associated domain-specific instructional theory developed in the course of most classroom design studies.

The framework that Coburn (2003) proposes for thinking about the dissemination of educational innovations at scale is more useful for our purposes as it takes account of the depth of the teacher learning. As Coburn noted, standard definitions of dissemination emphasize the spread or dispersal of an idea or innovation, thereby orienting us to focus on the number of teachers using particular instructional sequences in their classrooms. She then went on to observe that these definitions say nothing "about the nature of the change envisioned or enacted or the degree to which it is sustained, or the degree to which schools and teachers have the knowledge and authority to continue to grow the reform over time" (p. 4).

The first dimension of Coburn's framework, *depth*, refers to the nature and quality of the enactment of instructional tasks and tools in classrooms. It is therefore "important to look beyond the presence or absence of specific materials or tasks to the underlying

pedagogical principles embodied in the ways that teacher engage students in using the materials and tasks" (p. 5). These pedagogical principles include teachers' decisions about which specific tasks and tools to use with students in particular lessons based on ongoing assessments of their reasoning with respect to the learning trajectory that underpins the instructional sequence. The principles might also include asking students particular types of questions and guiding the establishment of particular norms of mathematical argumentation in the classroom. The enactment of these principles would therefore seem to require that teachers have the opportunity to reconstruct the domain-specific instructional theory that constitutes the rationale for the instructional sequence.

Coburn's concern for the second dimension, *sustainability*, reflects her observation that schools that implement instructional innovations successfully usually "find it difficult to sustain them in the face of competing priorities, changing demands, and teacher and administrator turnover" (p. 6). As Coburn noted, depth might be important for sustainability because teachers who have a deep understanding of the underlying instructional theory are better able to adjust how they use an instructional sequence with their students in a principled manner as priorities and expectations change. Coburn also suggested that aspects of the school contexts in which teachers work might be important for sustainability. These might include school leaders who support the use of the instructional sequence and ongoing teacher collaboration that centers on enacting the sequence more effectively.

Coburn's third dimension, *spread*, is not limited to the implementation of an instructional sequence in new classrooms, but also concerns the extent to which grounding principles and norms become embedded in school and school system policies and routines. Thus, the learning trajectory that underpins a sequence might inform the development of system-wide student assessments, and the associated pedagogical principles might be a focus of teacher professional development. As Coburn noted, this institutionalization of principles and norms requires that key school and system leaders be encouraged and supported to develop a relatively deep understanding of the underlying principles.

The final dimension of Coburn's framework, *shift in ownership*, highlights the importance of teachers and school and system leaders increasingly taking responsibility for improving the enactment of the sequence, and for drawing on the underlying principles in various aspects of their practice. This shift in ownership is crucial for long-term sustainability even as circumstances change. Indicators that such a shift is occurring include continued funding for dissemination activities, ongoing supports for teacher learning, and use of the underlying principles to inform school and system-level decision-making.

We draw on Coburn's framework throughout this article as we discuss three broad steps that we contend need to be thought through when developing, testing, and refining designs for disseminating the products of classroom design studies. The first of these steps is to assess the depth of the teacher learning required to enact particular research-based instructional sequences effectively. The second step is to develop and enact a design for supporting teachers' learning. The third step is to extend the design to the organizational level by identifying aspects of the school contexts in which the teachers work that delimit the extent to which designs for supporting their learning actually impact their classroom practices. Our discussion of each of these steps is informed by our

ongoing work in which we have partnered with several large urban school systems in the U.S. to investigate and support efforts to improve the quality of mathematics teaching on a large scale.

2.0 Assessing the Depth of Teachers' Learning

We have argued that disseminating the products of design studies is a problem of supporting significant teacher learning on a large scale (Pegg & Krainer, 2008). Taking a learning perspective on dissemination suggests that an important first step is to address the first dimension of Coburn's (2003) framework by assessing the depth of the learning for teachers to enact the instructional sequences effectively. As Coburn noted, depth is also important for her second dimension of sustainability because teachers who have a deep understanding of the underlying instructional theory are more likely to adjust how they enact an instructional sequence in a manner consistent with the underlying domain-specific instructional theory as priorities change. This first step of assessing the depth of teachers' learning is akin to how researchers typically proceed when conducting classroom design studies that attempt to support students' mathematical learning. For example, when conducting a classroom design study, researchers typically assess the nature of the student learning involved by a) clarifying the goals for students' mathematical reasoning, and b) documenting students' current mathematical reasoning with respect to such goals. The resulting assessment shapes the conjectures that a research team specifies regarding students' development of the intended learning goals and the means of supporting it.

Similarly, assessing the depth of teachers' learning involves a) clarifying the goals for teachers' learning with respect to the products of the classroom design study, and b) documenting teachers' current instructional practices and relevant forms of knowledge and conceptions about teaching and learning. Clarifying goals for teachers' learning entails specifying what teachers need to know and be able to do in order to enact instructional sequences in a manner that effectively supports' students' learning. As classroom design studies rarely focus explicitly on teachers' instructional routines and practices (Cobb et al., in press), specifying the necessary forms of knowledge and practice is likely to require additional work. In some cases, it may be possible to turn to current research on mathematics teaching to identify instructional practices that have been shown to support students' attainment of the specific mathematics learning goals. In cases where the relevant research is thin, it may be necessary to analyze existing data collected during the classroom design study in order to identify relevant aspects of the design study teacher's knowledge and practice.

Documenting teachers' current instructional practices involves either directly observing or video-recording a representative group of teachers' classroom instruction. Documenting relevant forms of knowledge and conceptions about teaching and learning might include assessing, for example, teachers' mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008) and their understanding of the development of students' reasoning in the targeted mathematical domain. In our ongoing work, we have also found it useful to assess both teachers' visions of what counts as high-quality mathematics instruction (Munter, 2014) and their conceptions of what their students are capable of mathematically (Jackson & Gibbons, 2014).

It is important that the assessment of teachers' current practice and knowledge is viewed as a starting point for a trajectory that aims towards the effective enactment of the instructional sequences. In our work, we have found it important to identify aspects of teachers' current practice that can be leveraged, as well as those that might be at odds with the intended goals. Both then inform the design of supports for teachers' learning.

3.0 Developing and Enacting Designs for Supporting Teacher Learning

The second step when disseminating the products of classroom design studies involves formulating and enacting a design for supporting teachers' learning. In other words, the goals for teachers' learning and the identified starting points frame the challenge of generating a hypothetical learning trajectory or pathway together with the specific means of supporting this learning. Again, we note that this is akin to the work a research team undertakes when developing an initial design for a classroom design study by specifying an envisioned or hypothetical learning trajectory that comprises testable conjectures about both significant developments in students' reasoning and the specific means of supporting these developments (Simon, 1995). In our view, a major goal of the design for supporting teachers should be to enable them to reconstruct the domain-specific instructional theory that constitutes the rationale for the instructional sequence and that corresponds to what Coburn termed the underlying pedagogical principles.

A detailed discussion of designs for supporting teachers' learning is beyond the scope of this article. However, in what follows, we first make some general observations about what the current research on teacher learning and high-quality professional development (PD) suggests for supporting teachers' reorganization of their current practice. We then suggest that it will be important to extend supports for teachers' learning beyond traditional forms of pull-out PD in which a facilitator works with a group of teachers outside the classroom context (and the teachers may or may not work in the same school context). Our goal in doing so is to propose an initial set of principles on which others can draw when developing designs to support teachers in enacting instructional sequences effectively.

First, at the most general level, research suggests the importance of sustained PD over time, including the opportunity for the same group of teachers to continue to work together (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). Second, it is important that the supports are close to instructional practice (Ball & Cohen, 1999), implying the supports need to focus squarely on what teachers are expected to do in their professional work and, when possible, make use of tools that they already use. In the case of introducing teachers to a new instructional sequence, it is likely that teachers will be expected to use new tools (e.g., instructional tasks, lesson planning protocols). Research on tool use indicates that people frequently appropriate new tools to their current practice (Wenger, 1998). It will therefore prove crucial to support teachers to develop a principled way of understanding how the new tools fit with, or not, their existing tools and practices. Furthermore, it will be important that the teachers see a need for the tool when it is first introduced (Cobb, 2002). This implies either that the tool should be designed to address what teachers perceive as a problem of current practice or that it should be feasible to cultivate the need for the tool during PD. In addition, it is important that the intended users can begin to use the tool shortly after it has been

introduced in relatively elementary ways that are nonetheless compatible with the designers' intentions and do not involve what Brown (1992) termed lethal mutations.

Third, the literature on learning that involves a significant reorganization of current practice suggests it is crucial that teachers have opportunities to co-participate in activities that approximate the targeted practices with more accomplished others (Bruner, 1996; Forman, 2003; Lave & Wenger, 1991). This implies that teachers are unlikely to implement the products of classroom design studies in a principled manner unless they work closely with others who are already accomplished in enacting a particular instructional sequence and using the associated tools.

Fourth, based on empirical studies of the development of professional practice (including pre-service teacher education), Grossman and colleagues (Grossman, Compton, et al., 2009; Grossman, Hammerness, & McDonald, 2009) suggest the importance of ensuring that professional development activities include what they refer to as pedagogies of investigation and pedagogies of enactment organized around the target forms of practice and knowledge. Pedagogies of investigation involve analyzing and critiquing representations of practice such as student work and video-cases of teaching (Borko, Jacobs, Eiteljorg, & Pittman, 2009; Kazemi & Franke, 2004; Sherin & Han, 2004). PD activities such as these are somewhat common and can support teachers in developing "an image" or vision of the targeted "activity and embedded practices" (McDonald, Kazemi, & Kavanagh, 2013, p. 383).

However, as Grossman and her colleagues convincingly argue, pedagogies of investigation alone are unlikely to support teachers in significantly reorganizing their current practices. Teachers also need adequate opportunities to engage in pedagogies of enactment that involve actually trying out the intended forms of practice with support from more expert others. For example, a skilled facilitator might scaffold teachers and provide targeted feedback as they plan and then rehearse an upcoming lesson within an instructional sequence prior to actually enacting the lesson with students. The goals of the rehearsal might include refining questions the teacher will ask students, anticipating how students might respond to such questions, as well as reconstructing the domain-specific instructional theory that constitutes the rationale for the instructional sequence. It is advantageous to also document what happens when teachers actually enact the instructional activities with students, for example by collecting student work or video-recordings of instruction. These artifacts can then serve as close-to-practice objects of investigation in upcoming PD sessions (McDonald et al., 2013).

Thus far, we have focused our discussion of learning supports on typical forms of pull-out PD. The literature on teacher learning suggests that while pull-out PD is important, in and of itself, it is probably not sufficient to support teachers' reorganization of their classroom practices (Grossman, O'Keefe, Kantor, & Delgado, 2013). As we will discuss further in the next section, this is because relations of accountability and support in the settings in which teachers work dramatically shape the impact of pull-out PD on teachers' practice (Cobb, McClain, Lamberg, & Dean, 2003). It is therefore crucial to expand our view of PD to include, for example, opportunities for mathematics teachers from the same school to collaborate with one another and to receive one-on-one classroom support from a coach (Maaß & Artigue, 2013; Maaß & Doorman, 2013).

As we have already noted, there is evidence that it can be productive for groups of teachers to work together over time. Factors that increase the likelihood that teacher

collaboration will give rise to significant learning opportunities include that the teachers work in the same school context (Darling-Hammond et al., 2009), that these meetings are regularly scheduled during the school day, and that the group has strong leadership and includes teachers who are already relatively accomplished in enacting the instructional sequences (Cobb, Zhao, & Dean, 2009; Horn & Little, 2010). Briefly, relatively accomplished leaders can ensure that the teacher group engages in activities that have the potential to support the development of teachers' knowledge and practice. Such activities include solving mathematical tasks together to determine their mathematical potential, and planning lessons by anticipating students' solutions and thinking through how to build on them to achieve a mathematical agenda. In addition, the leaders of collaborative meetings can influence how these collective activities are enacted by pressing on key issues (e.g., Where is the mathematics? Which student solutions should be included in a whole class discussion?) (Coburn & Russell, 2008; Stein, Engle, Smith, & Hughes, 2008).

One-on-one coaching in teachers' classrooms is another valuable form of PD. Coaches who have sufficient expertise in both supporting teachers' learning and in enacting the instructional sequence can provide "live, in-the-moment coaching" or co-teach to "provide in-the-moment modeling" (McDonald et al., 2013, p. 383). This kind of extended support enables teachers to try out what they are learning in pull-out PD in their classrooms with the direct support of a more expert other. A coach might then use her observations of teachers' practices to inform the focus of upcoming teacher collaborative time or pull-out PD (either as the leader herself or through conversation with the leader). In our view, coordinating supports for teachers' learning across pull-out PD, teacher collaborative meetings, and one-on-one coaching in this manner is a key characteristic of a potentially productive PD design (Jackson & Cobb, 2013).

To this point, our discussion of teacher PD has emphasized Coburn's first dimension of depth. It is also important to take account of the fourth dimension of her framework, shift in ownership, when designing PD to support the dissemination of research-based instructional sequences (cf. Fullan, 2011). This dimension of the framework orients us to consider why teachers might view it as reasonable and be motivated to learn to use the instructional sequences effectively. In our view, it is essential that PD designs aim to support not only teachers' increasingly effective enactment of instructional sequences, but also their development of reasons and motivations to work to improve their instruction in this way. In formulating, testing, and improving such designs, it appears important to differentiate between engendering teachers' initial buy-in such that they are willing to try using the instructional sequences in their classrooms, and fostering their identification with and ownership of the instructional sequences such that they assume increasing responsibility for improving both the sequences and how they enact them in their classrooms (Akerlof & Kranton, 2005; Battey & Franke, 2008; Gresalfi & Cobb, 2011). Analyses of development of professional identities in the context of communities of practice in general (Wenger, 1998), and in the context of teacher collaboration in particular (Franke & Kazemi, 2001), can inform the formulation of initial designs. Nonetheless, the initial designs will likely be provisional because the issue of cultivating teachers' initial buy-in and subsequent ownership of instructional improvement initiatives has been rarely addressed directly in the mathematics teacher learning literature.

4.0 Extending Designs to the Organizational Level

To this point, we have limited our focus to direct supports for teachers' learning such as pull-out PD, teacher collaboration, and one-on-one coaching that are generally viewed as falling within the purview of mathematics education research. Coordinating these supports so that teacher PD is coherent across contexts is clearly an ambitious but essential undertaking. However, it might not by itself be sufficient to support teachers' principled enactment of research-based instructional sequences. Coburn (2003) alluded to this possibility by repeatedly indicating other aspects of the school contexts in which teachers work that can influence the extent to which instructional innovations are disseminated successfully. In particular, she suggested that the work of school leaders and system leaders influences sustainability, spread, and shift in ownership, and argued that it is important for these leaders to develop a relatively deep understanding of the core principles of the innovation.

The contention that it might be necessary to extend the dissemination design beyond teacher PD by taking account of other aspects of the school context is generally consistent with research in educational policy and educational leadership (Bryk, Sebring, Allensworth, Luppescu, & Easton, 2010; Elmore, 2004; Spillane, Mesler, Croegaert, & Sherer, 2007). The findings of a substantial and growing number of studies indicate the importance of the culture of the school and, in particular, whether relations of trust have been established between teachers and school leaders (Bryk & Schneider, 2002), the scope and depth of teachers informal advice networks (Sun & Frank, 2011), and to whom and for what teachers perceive themselves accountable (Cobb, McClain, et al., 2003). In addition, there is ample evidence that school and system leaders can influence whether and how particular supports for teachers' learning such as, for example, regularly scheduled time for mathematics teachers to collaborate, are implemented (Coburn, 2005; Coburn & Russell, 2008; Confrey, Castro-Filho, & Wilhelm, 2000; Elmore, 2006; Spillane, 2005).

The specific aspects of the school context that need to be considered when disseminating the products of design studies vary depending on both the overall structure of educational systems in particular countries and on the capacity for instructional improvement of individual school systems and schools. Rather than attempting to identify a comprehensive list of contextual factors that should be considered, we illustrate a general approach for identifying relevant aspects of the school contexts in which teachers work and in which they modify, adjust, and perhaps reorganize their instructional practices. We have in fact used this approach, which involves mapping out from the classroom (Elmore, 1979-80), to organize this article. We first clarified a set of relatively ambitious goals for students' mathematical learning, and then outlined a research-based view of high-quality mathematics instruction that can be justified in terms of those learning goals (Hiebert & Grouws, 2007). This view of high-quality instruction is consistent with the intent of most classroom design studies (Plomp & Nieveen, 2013), and serves to delineate goals for teacher' learning. These goals for the development of teachers' knowledge and practice then orient the design of supports for their learning, which might include but are not necessarily limited to the three types of PD that we have discussed: pull-out PD, teacher collaboration, and on-on-one coaching. Crucially, each

of the intended supports should be justified in terms of their potential to scaffold teachers' effective enactment of particular instructional sequences. The next step in this backward mapping process involves identifying aspects of the school context that might influence the provision and enactment of each proposed support, and thus the extent to which teachers become committed to and are able to improve how they enact the instructional sequences with their students.

As a first illustration of this backward mapping process, we focus on the implementation of regularly scheduled teacher collaborative meetings across a large number of schools as a key support for teachers' learning. In the U.S. context, the provision of time for these meetings usually implicates principals' practices as instructional leaders, as they typically control school schedules. This suggests that it is important that they come to see value in teacher collaboration, which in turn has implications for their development as instructional leaders.

We noted earlier that the extent to which collaborative meetings give rise to teacher learning opportunities depends on the quality of the leadership of these meetings and the inclusion of teachers who are already relatively accomplished in enacting the instructional sequences. In this regard, we found in the course of our ongoing partnerships with several large, urban school districts in the U.S. that most of the districts had very few accomplished teachers. These districts therefore had to address the challenge of developing a cadre of mathematics coaches who could both lead teacher collaborative meetings and work with individual teachers in their classrooms (Cobb & Jackson, 2011; see also Maaß & Doorman, 2013). This in turn had implications for the practices of both district mathematics specialists who were charged with supporting coaches and principals, as there is evidence that coaches' effectiveness in supporting teachers' learning is influenced by their relationship with the school principal. In particular, it appears to be important that principals' and coaches' visions of high-quality mathematics instruction are aligned if an instructional improvement effort is to be coherent at the school level (Mangin, 2007; Matsumura, Sartoris, Bickel, & Garnier, 2009). This finding has implications for the practices of district leadership specialists who are expected to monitor and support principals' work.

Thus, in the case of our partner districts, mapping backwards from teacher collaborative time as a support for teachers' learning led us to identify the influence of school principals' and of specific district leaders' practices on the provision and the enactment of this support. Clearly, the aspects of school and system contexts that are identified by mapping out from the classrooms might differ in the case of U.S. districts that have a significant proportion of already accomplished teachers, and in other countries that have established an infrastructure for identifying and supporting the development of accomplished teacher leaders. We contend that the backward mapping approach is also useful in such cases even though the resulting dissemination designs might be less far reaching.

As a second illustration of the backward mapping process, we focus on a further aspect of principals' roles. In the U.S., principals are increasingly expected to act as instructional leaders in all content areas including mathematics even though the majority were not mathematics teachers. In all of our partner districts, principals were expected to observe classroom instruction and give teachers feedback that communicated appropriate instructional expectations. Our analyses indicated that most were not able to distinguish

between strong and weak inquiry-oriented mathematics lessons (Cobb & Jackson, 2012). This was the case even though they had received extensive PD that focused on general, content-independent aspects of high-quality instruction. It was therefore important that the dissemination designs in these districts included the provision of mathematics-specific PD that aimed to support the development of principals' visions of high-quality mathematics instruction. This in turn had implications for the knowledge and practices of the district leadership specialists who were responsible for supporting and supervising the work of principals (Honig, 2012). As a consequence, we routinely included them in PD sessions for school leaders that we co-designed and co-led with the district mathematics specialists (Jackson et al., 2015).

As a final illustration of backward mapping, we consider the extent to which system leaders need to understand the instructional innovations they are attempting to implement. In the U.S. case, we have already referred to the practices of district mathematics specialists and leadership specialists who work directly with coaches and principals respectively. Our observations indicate that senior district leaders to whom these specialists report do not need to develop a similarly principled understanding of instructional innovations in particular content areas (Cobb & Jackson, 2011). However, a backward mapping analysis suggests that it is important for senior leaders to appreciate that the effective enactment of the instructional sequences involves significant teacher learning if they are to allocate adequate resources over the long-term to support implementation. In addition, this backward mapping analysis suggests that it is important for senior leaders to ensure that the instructional improvement effort is coherent at the system level by pressing the district mathematics and leadership specialists to work towards compatible goals for students' mathematical learning and for teachers' pedagogical learning (Cobb & Jackson, 2011).

Our intent in presenting these illustrations has been to exemplify the backward mapping process and to clarify its value in identifying relevant aspects of school and system contexts that might otherwise be overlooked. We contend that this process is broadly applicable, but readily acknowledge that the specific aspects of school and system contexts that prove relevant are influenced by the organization of the education system in a particular country. For example, the implications we discussed for principals' practices reflect the current emphasis on principals as instructional leaders in U.S. In a number of other countries including China, the principal's role is primarily administrative, and a group of teacher leaders is responsible for supporting improvements in the quality of instruction in all content areas including mathematics. A backward mapping analysis conducted to inform the dissemination of the products of design studies in countries where the educational system is organized in this manner will likely indicate the importance of developing teacher leaders' capacity to communicate appropriate expectations for instructional improvement to their colleagues, and to support them in improving their instructional practices.

The backward mapping approach we have illustrated is generally consistent with the principles of sound designs for supporting learning, including the influential tenets of instructional design proposed by Wiggins and McTighe (1998). As we have illustrated, the aspects of school and system contexts that come to the fore extend beyond the typical interests of mathematics education researchers. We suspect that this is because few mathematics education researchers have, to this point, investigated dissemination designs.

[The work of Confrey et al. (2000), Krainer and Zehetmeier (2013), Maaß and Artigue (2013), and Stein and Nelson (2003) are significant exceptions.] This is unfortunate given the importance of extending dissemination designs to the organizational level if they are to address Coburn's dimensions of sustainability, spread, and shift in ownership adequately. Regardless of how an education system is organized in a particular country, it is unlikely that the use of instructional sequences developed during a design experiment will be sustained unless underlying pedagogical principles become institutionalized in school and system policies and routines, and key school and system leaders increasingly take responsibility for supporting teachers in improving their enactments of the sequences.

As our discussion of the backward mapping process makes clear, it is crucial that dissemination designs are content specific. Because few mathematics education researchers have investigated dissemination designs, the research base on which to draw when formulating such designs is relatively thin. For example, research on content-focused coaching in general and on mathematics coaching in particular is underdeveloped. In addition, there is little research on instructional leadership at the school and system levels that is specific to teachers' development of inquiry-oriented classroom practices that aim at ambitious goals for students' learning. In the absence of an adequate research base, we have found it essential to extrapolate from research on professional learning in general and on teacher PD and teacher education in particular when formulating dissemination designs. Many aspects of the resulting designs are therefore necessarily conjectural, and are thus open to revision. In our view, investigations of these conjectures that make dissemination an explicit object of study is a highly productive way in which to begin to build an adequate research base (Cobb, Jackson, Smith, Sorum, & Henrick, 2013; Fishman, Penuel, Allen, Cheng, & Sabelli, 2013; Maaß & Artigue, 2013).

5.0 Discussion

In this article, we have framed the process of disseminating the products of classroom design studies as one of supporting the learning of large numbers of teachers. We have drawn on current research on mathematics teachers' learning to argue that high-quality pull-out PD is essential but not sufficient, and went on to consider teacher collaboration and one-on-one coaching as additional direct supports. We also drew on Coburn's (2003) analysis of dissemination to clarify the importance of extending dissemination designs to the organizational level. In doing so, we proposed a backward mapping approach that brings to the fore aspects of the school and system contexts that influence the provision and enactment of supports for teachers' learning. We noted that the specific aspects of the school context that need to be considered will vary depending on both the overall structure of educational systems in particular countries, as well as on the capacity for instructional improvement of individual school systems and schools.

As Maaß and Artigue (2013) observed, approaches to large-scale instructional improvement are often characterized as either bottom-up or top-down in nature. They clarified that bottom-up approaches typically involve groups of teachers developing their own questions and working on them together, perhaps by engaging in action research. Accounts of groups of teachers in the U.S. who have collaborated to develop

sophisticated inquiry-oriented instructional practices can be found in the literature (e.g., Boaler & Staples, 2008; Stephan, this volume) The accomplishments of these teacher groups are both impressive and inspirational. However, consistent with Coburn's (2003) analysis, they did not sustain their work when new school or system leaders with different agendas and priorities took over. In our view, exclusively bottom-up approaches are unlikely to be effective in disseminating the products of classroom design studies on a large scale. Recall that these products typically comprise sequences of instructional activities together with a domain-specific instructional theory that constitutes their rationale. It is doubtful that, in the absence of external support and guidance, teacher groups will identify and attempt to address questions that are compatible with those that orient the development of the instructional sequences, or that they will have the capacity to enact the sequences in a manner consistent with the underlying instructional theory.

Maaß and Artigue (2013)clarified that

> within the top–down approach, it is assumed that innovation in an organization can be planned and implemented top–down. The so-called 'fidelity-perspective' assumes a linear transfer process from the intended innovation to the implementation. (p. 783)

As they observed, exclusively top-down initiatives such as requiring teachers to use a particular curriculum are rarely effective because "these efforts do not always draw on the current teaching practice and neglect supporting measures such as professional development courses" (p. 783). We would add that initiatives of this type also fail to consider why teachers might see it as reasonable to revise their instructional practices.

The approach for disseminating the products of designs studies that we have discussed in this article has both top-down and bottom-up elements, and exemplifies what Maaß and Artigue termed a symbiosis of top-down and bottom-up approaches. For example, the specification of the effective enactment of research-based instructional sequences as a goal for teachers' learning is clearly top-down. However, we have also indicated that designs for supporting teachers' attainment of these goals should have strong bottom-up elements. In this respect, a design for supporting teachers' pedagogical learning is analogous to designs for supporting students' mathematical learning that are developed and refined during a classroom study. The intent of these latter designs is to enable teachers to achieve a mathematical agenda by building on students' current mathematical reasoning. Similarly, designs for supporting teachers' learning should attempt to achieve a pedagogical agenda by building on teachers' current instructional practices (Gibbons & Cobb, 2013). As part of this process, the design should support teachers' reconstruction of the underlying domain-specific instructional theory so that they might be able to adapt the instructional sequences in a principled manner. Further, it is essential in our view that the design support teachers in coming to see reason and developing motivations to want to learn to enact the sequences effectively. Beyond this, as we have stressed throughout this article, the design should attend to Coburn's (2003) dimensions of sustainability and spread as well as to those of depth and shift in ownership by also supporting changes in the school and system contexts in which teachers develop and revise their instructional practices.

In our experience, considerable humility is appropriate when addressing what my be termed the "dissemination problem." Research in math education and related fields has made significant progress over the last 25 years or so by deepening our understanding of students' learning in particular mathematical domains, of high-quality mathematics instruction, and of promising PD designs. However, it is sobering to note that this work has had little impact on classroom mathematics instruction in many countries. In the case of the U.S., for example, the history of large-scale instructional improvement efforts in all content areas including mathematics is largely one of failure (Elmore, 2000). We recognize that the range of issues that we have suggested should be considered when developing dissemination designs might seem daunting. However, in our view, the prospects for successful dissemination improve significantly if we are realistic rather than unjustifiably optimistic.

We conclude this chapter by suggesting that researchers who conduct classroom design studies can make the challenges of dissemination more tractable if they design with an eye towards large-scale implementation from the outset. One of the strengths of the design research methodology is that it enables researchers to explore what is possible in students' learning. As a consequence, there is typically a significant discontinuity between typical forms of education and those that are the focus of classroom studies. However, it is also the case that the researchers conducting classroom design studies often give little consideration to the knowledge and skills that teachers would have to develop to enact the design for supporting students' learning effectively. As a consequence, the teacher learning involved frequently appears to be unrealistic given teachers' current instructional practices and the capacity of the educational systems in which they work to support their learning. A design for supporting students' learning developed and refined in the course of a classroom study is unlikely to contribute to improvements in classroom teaching and learning on a large scale unless researchers consider not merely their own but others' capacity to support students' and teachers' learning from the outset of the study.

This weakness of current classroom design studies might be addressed by giving at least as much weight to the problems of practice that school personnel identify as to researchers' assessments of what counts as theoretically interesting problems about students' or teachers' learning (cf. Bannan-Ritland, 2008). In other words, researchers might take practitioners' concerns as their starting point and negotiate how those issues are framed so that the study is pragmatically as well as theoretically significant. For example, a study that begins with teachers' concern about motivating students might reframe the focal issue in terms of cultivating students' mathematical interests or supporting their development of productive mathematical identities. In this and similar instances, the design research methodology would approach its full potential by exploring what is possible in students' or teachers' learning in a manner that is also likely to have implications for educational improvement more generally.

Earlier in the chapter, we noted a second weakness of many classroom design studies, namely the limited attention given to the instructional practices of the teacher in the study. Most researchers who conduct classroom design studies readily acknowledge that the study teacher plays a central role in supporting the participating students' learning. However, these teachers' instructional practices are rarely the focus of explicit analysis. This is unfortunate because analyses of their relatively sophisticated practices

can clarify the goals for teachers' learning that should be addressed by dissemination designs. Kwon et al.'s (2013) classroom study that focused on students' development of mathematical argumentation is a rare exception as they analyzed the study teacher's discursive moves.

In addition to keeping future dissemination in mind when conducting classroom studies, we also suggest that it is important to make dissemination processes a focus of investigation in their own right. Together with Penuel, Fishman, Cheng, and Sabelli (2011) and Maaß and Artigue (2013), we see considerable value in design studies that focus primarily on teachers' rather than students' learning, and indeed on design studies that investigate dissemination processes by focusing on the development of the capacity for instructional improvement at the school and system levels. To this point, the number of studies that have investigated teachers' learning is relatively small, and the development of design research methods for investigating organizational learning at the school and system level is still in its infancy. As we have noted, the research base on which to draw when formulating dissemination designs is currently very thin. Teacher development design studies and organizational design studies would go some way towards addressing this limitation.

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