The Cognitive Demand of Mathematical Tasks: Investigating Links to Teacher Characteristics and Contextual Factors

Anne L. Garrison

Vanderbilt University
Department of Teaching and Learning
1930 South Drive, 240 Wyatt Center
Peabody Box 330
Nashville, TN 37203

The author is supported by the IES pre-doctoral research training program, grant number R305B080025. The data comes from a larger study, supported by the National Science Foundation under grants No. ESI-0554535 and No. DRL-0830029. The opinions expressed do not necessarily reflect the views of the U.S. Department of Education or the National Science Foundation. I would like to thank members of the research team for the larger project from which this data comes: Paul Cobb, Tom Smith, Kara Jackson, Erin Henrick, Chuck Munter, Glenn Colby, Lynsey Gibbons, Rebecca Schmidt, Jonee Wilson, Christy Larson, Dan Berebitsky, Adrian Larbi-Cherif, and Jennifer Baumann.
Abstract

This study explores relationships between teacher characteristics and contextual factors and teachers’ choice of tasks and the maintenance of the cognitive demand of those tasks. The sample consists of 56 middle school mathematics teachers across two large, urban school districts that began implementing the same NSF-funded curriculum at the same time. Results from logistic regression analyses suggest factors that are related to the cognitive demand of mathematical tasks in the classroom including whether the teacher is in the first year of using the curriculum, mathematical knowledge for teaching, beliefs about the role of the teacher, and beliefs about students’ mathematical capabilities. In addition, further analyses reveal complex interrelationships between mathematics teachers’ knowledge and beliefs and the cognitive demand of mathematical tasks. These findings suggest that professional development aimed at supporting teachers to implement high cognitive demand tasks in classroom may need to simultaneously focus on developing several different aspects of mathematics teachers’ knowledge and beliefs.
The Cognitive Demand of Mathematical Tasks: Investigating Links to Teacher Characteristics and Contextual Factors

Over the last 25 years, mathematics educators and researchers have suggested new approaches for teaching mathematics that change the nature of activity in the classroom. The *Curriculum and Evaluation Standards* and *Principles and Standards for School Mathematics* documents published by the National Council of Teachers of Mathematics (NCTM; 1989, 2000) reflect a consensus within the mathematics education research community for comprehensive reforms to traditional mathematics instruction. Two fundamental aspects of high quality mathematics instruction outlined in these documents are genuine, challenging problems and opportunities, in both small groups and as a whole class, for discourse about the key mathematical ideas that emerge from individual and collective efforts to solve problems. While both of these aspects place students at the center of these investigatory experiences, there are clear implications for the role of the teacher as well (Hiebert et al., 1997). For example, the teacher is expected to choose and set up the challenging tasks for students and orchestrate productive opportunities for discourse within the classroom (Stein, Engle, Smith, & Hughes, 2008). While classroom discourse related to the task provides critical learning opportunities for students, the level of challenge of the tasks chosen and posed to students is the foundation for those learning opportunities. For example, a task that requires students to reproduce memorized facts is unlikely to provide conceptual learning opportunities for students, no matter how well-orchestrated the classroom discourse. Hence, the cognitive demand, or level of challenge, of tasks is a critical aspect of high-quality mathematics instruction that requires further investigation.
The cognitive demand of a task refers to “the cognitive processes students are required to use in accomplishing it” (Doyle, 1988, p. 170). When examining the cognitive demand of mathematical tasks, Doyle chose familiar and novel as descriptors of two categories of mathematical tasks. While familiar tasks ask students to engage in routinized activities, novel tasks are deliberately ambiguous with regard to how to carry out the task. In 1996, Stein, Grover, and Henningsen set out to build on Doyle’s work and systematically delineate the cognitive demand of different mathematical tasks. They classified tasks into those with low and high cognitive demand (with parallels to familiar and novel tasks, respectively). Tasks with low cognitive demand require students to memorize or reproduce facts, or perform relatively routine procedures without making connections to the underlying mathematical ideas. Tasks with high cognitive demand require students to make connections to the underlying mathematical ideas. In addition, students are asked to engage in disciplinary activities of explanation, justification and generalization, or using procedures to solve tasks that are somewhat ambiguous in nature. While implied in the definition, it is important to emphasize that the distinctions between familiar and novel tasks and between high and low cognitive demand are relative to students’ current understanding and, thus, are situation-dependent.

Stein and Lane (1996) found that the use of tasks with high cognitive demand was related to greater student gains on an assessment requiring high levels of mathematical thinking and reasoning. In particular, the greatest gains occurred when teachers assigned tasks that were initially of high cognitive demand, and teachers and students maintained the cognitive demand throughout the lesson. Stein, Grover, and Henningsen (1996) focused specifically on the initial cognitive demand of mathematical tasks as written or verbally posed to students and examined how teachers and students maintained, increased, or decreased the demand across different
phases of a math lesson (e.g., the task as set up and the task as implemented). They found that in classrooms where tasks with the potential for high levels of cognitive demand were assigned, teachers and/or students often decreased the cognitive demand during implementation of the task. The results from the 1999 Third International Mathematics and Science Study (TIMSS) video study are consistent with those of Stein and colleagues in that they suggest that the mathematical activity in US middle school classrooms tends to be procedural in nature and when teachers do pose high-level tasks they are often implemented in low-level ways (Hiebert et al., 2003; Hiebert et al., 2005). Hence, if we wish to encourage more high-level mathematical activity, we need to understand more about the cognitive demand of mathematical tasks and how we can support teachers to pose high-level tasks and implement them in high-level ways.

Of central importance to analyzing how teachers implement tasks is a framework for examining the nature of classroom activity over the course of a lesson. The Math Tasks Framework proposed by Stein, Grover, and Henningsen (1996) divides a lesson up into phases which correspond to useful transitions in task activity (see Figure 1). The transitions that are relevant when considering whether the level of cognitive demand of a task is maintained are those of moving from the task as written to the task as set-up, and from the task as set-up to the task as implemented.

From the perspective of supporting teachers’ instructional practice, the notion of contextual factors refers to aspects of the teachers’ in-school situations that are relevant to their practice. Hence, what might be considered characteristics of particular individuals or groups could alternatively become contextual factors when they are aspects of the school situation in which teachers work. For example, when considering the practice of teaching, various student characteristics (e.g., the prior knowledge of students) are contextual factors. Overall, when
considering contextual factors relevant to teachers’ instructional practice, there are several
categories of factors that emerge: student-related factors (e.g., student backgrounds, prior student
knowledge), aspects of a teacher’s work (e.g., type of textbook, course/grade level taught),
supports (e.g., teacher collaboration, professional development), and accountability (e.g.,
principal’s expectations).

In this study I explore relationships between teacher characteristics and contextual factors
and teachers’ choice of tasks and their subsequent maintenance of the cognitive demand of those
tasks across two school districts that began implementing the same NSF-funded curriculum at
the same time. In particular, I address the following research questions:

1. How do teacher characteristics and contextual factors relate to the cognitive demand of
tasks that teachers pose to students?
2. How do teacher characteristics and contextual factors relate to maintenance of the
cognitive demand of high-level tasks?

Only a few studies have focused on particular teacher characteristics and contextual factors
as they relate to teachers’ maintenance of the cognitive demand (e.g., see Charalambous, 2010;
Son, 2008). From those studies, factors that have been linked to the maintenance of the
cognitive demand include: alignment between teaching goals and textbooks, the nature of the
textbook, teacher knowledge, teacher perceptions about student achievement, and professional
development. The findings of these studies suggest that teachers’ maintenance of the cognitive
demand is related to teacher characteristics as well as contextual factors, yet further investigation
is warranted.
In an effort to develop a strong set of hypothesized teacher characteristics and contextual factors that might influence the cognitive demand of mathematical tasks in these districts, I draw on an expanded literature base. While there are few studies investigating maintenance of the cognitive demand in this manner, there are many more that investigate relationships between teacher characteristics or contextual features and teachers’ instructional practice. In addition, many of those studies characterize aspects of teachers’ instructional practice that are closely related to the cognitive demand of mathematical tasks in the classroom. Such characterizations range from broad descriptions like the nature of the available learning opportunities for students and reform curriculum implementation to whether the activity in the classroom takes on a procedural or conceptual focus and involves problem-solving or inquiry. I use the phrase “the nature of classroom mathematical activity” to encapsulate these characterizations of mathematics teachers’ instructional practice that overlap with the cognitive demand of mathematical tasks but are broader in scope. In the following section, I summarize the set of hypothesized factors and interrelationships between factors that might be related to the cognitive demand of mathematical tasks in the classroom.

*Teacher Characteristics and Contextual Factors*

There are several different categories of factors that have emerged from the literature as being potentially related to the cognitive demand of mathematical tasks because of their relationships with the nature of classroom mathematical activity. These categories include: teacher characteristics, student-related factors, and supports.

The primary teaching characteristics that appear likely to be related to the cognitive demand of mathematics tasks that teachers use in the classroom are teachers’ experience,
knowledge, and beliefs. Several studies have noted the potential importance of experience teaching with regard to the nature of classroom mathematical activity (Charalambous, 2010; Escudero & Sánchez, 2007; Remillard & Bryans, 2004). Also, from the perspective of reform implementation, there is evidence that experience with the curriculum matters in that it takes time to faithfully implement a new program (Fullan, 2000).

There is considerable evidence that mathematical knowledge for teaching (MKT), which consists of content knowledge and pedagogical content knowledge (PCK; Shulman, 1986; Shulman, 1987), is related to the nature of classroom mathematical activity. While some studies have demonstrated that a general form of PCK is related to the nature of classroom mathematical activity (Baumert et al., 2010; Sherin, 2002; Son, 2008), others have found that more specific aspects of PCK, including knowledge of student thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) and knowledge of students (Peterson, Carpenter, & Fennema, 1989), are related to the nature of classroom mathematical activity. Several other studies have demonstrated that mathematics content knowledge is related to the nature of classroom mathematical activity (Manouchehri & Goodman, 2000; Putnam, Heaton, Prawat, & Remillard, 1992; Sherin, 2002). Baumert and colleagues separated teachers’ PCK and content knowledge and modeled their simultaneous effect on the cognitive challenge of math tasks and found that while PCK was related to the cognitive challenge of math tasks, math content knowledge was unrelated. Finally, several studies that have examined MKT, as a combination of PCK and content knowledge, have found that it is related to the nature of classroom mathematical activity (Charalambous, 2010; Escudero & Sánchez, 2007; Hill, Ball, Blunk, Goffney, & Rowan, 2007; Hill et al., 2008; Shechtman, Roschelle, Haertel, & Knudsen, 2010). Hence, there is clear
Another teacher characteristic that has been linked to the nature of classroom mathematical activity is mathematics teachers’ beliefs. In particular, teachers’ beliefs about teaching and learning mathematics are related to the nature of classroom mathematical activity. Several subsets of beliefs about teaching and learning mathematics have emerged from the literature as being particularly salient for the nature of the classroom mathematical activity: 1) beliefs about curriculum, 2) beliefs about the role of the teacher, and 3) beliefs about students. Within each of these subsets, there has still been considerable variation with regard to particular aspects of beliefs that have been studied. For example, beliefs about curriculum include beliefs about the role of curriculum (Manouchehri & Goodman, 2000), teachers’ orientations toward the curriculum (Collopy, 2003; Remillard & Bryans, 2004; Son, 2008), and teachers’ reliance on the curriculum/supplementary materials (Hill, et al., 2008; Remillard & Bryans, 2004; Son, 2008; Sowder, Philipp, Armstrong, & Schappelle, 1998) are all related to the nature of the classroom mathematical activity. With regard to beliefs about the role of the teacher, characterizations of such beliefs that are related to the nature of classroom mathematical activity range from broad, as in beliefs about sources of classroom authority, to narrow, as in beliefs about the role of the teacher within classroom discourse. In general, it appears that teachers’ beliefs about the role of the teacher are related to the nature of classroom mathematical activity (Cohen, 1990; Lambdin & Preston, 1995; Lloyd, 1999; Thompson, 1984).

Lastly, with regard to beliefs about students, a variety of different aspects of this subset of beliefs about teaching and learning math have been linked to the nature of classroom mathematical activity. These different aspects include: 1) beliefs about students’ capabilities
(Charalambous, 2010; Prawat & Jennings, 1997; Stipek, Givvin, Salmon, & MacGyvers, 2001; Stodolsky & Grossman, 2000; Thompson, 1984), 2) beliefs about equity (Hill, et al., 2008), 3) beliefs about motivating students (Stipek, et al., 2001; Turner, Warzon, & Christensen, 2011), and 4) beliefs about students’ needs (Sztajn, 2003). Hence, there is considerable evidence that mathematics teachers’ beliefs about teaching and learning mathematics are related to the nature of classroom mathematical activity.

Several student-related, contextual factors have emerged from the literature as related to the nature of classroom mathematical activity. These include: background knowledge of students (Stein, et al., 1996), background characteristics of students (Hand, 2010; Jackson, 2009) and struggles with motivation (Cooney, 1985). In particular, Stein and colleagues found that students’ prior mathematical knowledge influenced whether the cognitive demand of a task was maintained in the classroom. Also, research suggests that teachers in urban settings, with high numbers of minority and low-SES students, tend to teach in procedural ways (Bryk, Sebring, Allensworth, Luppesco, & Easton, 2010; Hand, 2010; Jackson, 2009).

Teacher supports also appear to be related to the nature of classroom mathematical activity. Several supports that have been linked to the nature of classroom mathematical activity include formal professional development (Desimone, Porter, Garet, Yoon, & Birman, 2002; Parise & Spillane, 2010), informal learning opportunities (Parise & Spillane, 2010), the culture of the mathematics department (Lloyd, 1999; Stodolsky & Grossman, 2000) and images of expert teaching (Son, 2008). With regard to professional development, both Desimone and colleagues and Parise and Spillane found that participating in formal professional development related to instructional practices had an impact on those instructional practices in the classroom. Hence, if teachers participate in formal professional development pertaining to the nature of classroom
mathematical activity then they are likely to make changes in the nature of classroom mathematical activity. Parise and Spillane also investigated informal learning opportunities which they called “on-the-job learning” and they found that both formal professional development and these less formal opportunities (e.g., collaborative discussions with colleagues, advice-seeking activities, etc.) contributed to teachers’ reported change in their instructional practice.

Interrelationships between Factors

The relationships between the teacher characteristics and contextual factors, described above, and the nature of classroom mathematical activity are very complex. In fact, the literature suggests specific interrelationships between these factors that might also be related to the cognitive demand of mathematical tasks. In this case, the interrelationships are moderating relationships. In other words, the interplay of the two factors is likely to have a strengthening or weakening effect on one of the factors and the outcome, the cognitive demand of mathematical tasks. In the following section, the extensions to hypothesized moderating relationships for cognitive demand of mathematical tasks are summarized following the synopsis of the literature upon which the hypotheses are based.

Studies suggest complex interrelationships between mathematics teacher knowledge and beliefs about teaching and learning mathematics. In addition, those studies discuss interrelationships specific to particular beliefs about teaching and learning, including beliefs about the role of the teacher and beliefs about students’ mathematical capabilities. First, a number of different studies suggest that there is a complex relationship between beliefs about the role of the teacher and mathematical knowledge for teaching in how they relate to the nature of
classroom mathematical activity. A subset of those studies suggests that the interrelationship between beliefs about the role of the teacher and mathematical knowledge for teaching manifests itself as unsophisticated beliefs about the role of the teacher moderating the relationship between mathematical knowledge for teaching and the nature of classroom mathematical activity. In particular, those studies suggest that beliefs about teaching and learning mathematics can keep teachers from enacting mathematical tasks in conceptual ways. Schoenfeld (2011) demonstrates that Mr. Nelson is a teacher who seemed to have sufficient mathematical knowledge but his belief that "what he said should be an elaboration or clarification of what a student had said," (p.82) limited the nature of the mathematical activity in the classroom. Ball (1991) generally describes the same phenomenon by arguing that beliefs about teaching and learning mathematics might intervene in whether or not teachers draw on the mathematical knowledge they possess.

_Hypothesis 1: Beliefs about the role of the teacher moderate the relationship between mathematical knowledge for teaching and the cognitive demand of mathematical tasks: Without some alignment between beliefs about the role of the teacher and the NCTM recommendations, the positive relationship between MKT and cognitive demand will be diminished._

The remainder of the studies suggesting a complex relationship between beliefs about the role of the teacher and mathematical knowledge for teaching indicate that the interrelationship manifests itself as a requisite baseline level of mathematical knowledge for teaching in order for beliefs about the role of the teacher to be related to the nature of classroom mathematical activity in the expected manner. In fact, several studies of the relationship between mathematics teachers’ beliefs and the nature of classroom mathematical activity suggest that mathematical knowledge for teaching moderates that relationship (Ball, 1991; Putnam, et al., 1992).
Hypothesis 2: MKT moderates the relationship between beliefs about the role of the teacher and the cognitive demand of mathematical tasks: Without a sufficient level of mathematical knowledge for teaching, the positive relationship between beliefs about the role of the teacher and cognitive demand will be diminished.

To clarify, both hypothesis 1 and hypothesis 2 pertain to a moderating relationship (i.e., a statistical interaction) between mathematical knowledge for teaching and beliefs about the role of the teacher, but the two hypotheses suggest different chains of causality.

With regard to more specific beliefs about learning mathematics, as opposed to teaching mathematics, there is also evidence that teachers’ beliefs about students’ capabilities might moderate the relationship between mathematical knowledge for teaching and the nature of classroom mathematical activity. Turner, Warzon, and Christensen (2011) found that, Helen, one teacher with a high level of mathematical knowledge for teaching was unable to enact conceptually-rich mathematical activity in the classroom due to her negative views of students’ abilities. In other words, she did not believe that her students were motivated or capable of being motivated to engage in conceptually-rich mathematical tasks. Hence, no matter how advanced her mathematical knowledge for teaching, she was still limited by unproductive beliefs about students.

Hypothesis 3: Beliefs about students’ mathematical capabilities moderate the relationship between mathematical knowledge for teaching and the cognitive demand of mathematical tasks: With unproductive beliefs about students’ mathematical capabilities, the positive relationship between MKT and cognitive demand will be diminished.
Lastly, beyond interrelationships between beliefs about teaching and learning mathematics and mathematical knowledge for teaching, another complex relationship is suggested by the literature. It appears that alignment between different aspects of teachers’ beliefs moderates the relationship between those beliefs and the nature of classroom mathematical activity. In particular, one study suggests that beliefs about students moderate the relationship between teachers’ beliefs and the nature of classroom mathematical activity.

Cooney (1985) studied one teacher, Fred, and found that despite the fact that Fred’s beliefs about mathematics were consistent with a problem-solving approach to teaching, his beliefs about students and motivation appeared to more directly influence his choice of tasks for students. Fred believed that he needed to pose “recreational” mathematics problems to catch his students’ interest and, therefore, he used superficially interesting problems, unconnected to key mathematical ideas, to capture students’ attention, rather than mathematically-rich problems. Hence, his beliefs about students limited the strength of the relationship between his beliefs about mathematics and the nature of the classroom mathematical activity.

Hypothesis 4: Alignment between beliefs moderates the relationship between beliefs about teaching and learning and the cognitive demand of mathematical tasks: If beliefs about the role of the teacher and beliefs about students’ capabilities are not well aligned, then the positive relationship between either of those beliefs and cognitive demand will be diminished.

It is clear that investigating relationships between mathematics teacher characteristics and contextual factors, and the cognitive demand of mathematical tasks is critical for our understanding as a field, but without considering the effects of the moderating relationships, our understanding of the primary relationships is likely to be incomplete. In particular, I investigate these hypothesized moderators for two different outcomes pertaining to the cognitive demand of
mathematical tasks: the cognitive demand of the task posed to students and maintenance of the cognitive demand of high level tasks.

Method

Sample

This investigation draws on data from a five-year study designed to address the question of what it takes to improve the quality of middle-grades mathematics teaching, and thus student achievement, at the scale of a large urban district. In the larger study, the research team collaborated with four urban districts that were attempting to achieve a vision of high quality mathematics instruction that is broadly compatible with the National Council of Teachers of Mathematics’ (2000) recommendations. In this investigation, I make use of data from the two school districts whose adoption of the Connected Mathematics Project 2 (CMP2) coincided with the first year of the larger study to explore relationships between teacher characteristics, contextual factors and the cognitive demand of mathematical tasks.

The two school districts, District B and District D, upon which I focus in this study are large, urban districts located in the southern region of the United States. Both districts began implementation of CMP2 in the same year, and both districts devoted considerable resources (e.g., high-quality professional development, instructional coaches, and collaboration time) to supporting teachers to implement the curriculum.

District B serves approximately 80,000 students, over 50% of whom are Hispanic, over 25% are African American, and about 15% are White. Over 25% of all students are classified as Limited English Proficient (LEP). The majority of the students are eligible for free or reduced-price lunches. District B’s student achievement patterns in middle-school mathematics are
typical for large, urban districts. For example, on a recent state assessment in eighth grade mathematics, less than 40% of the African American students met the eighth grade mathematics standards, as compared to 55% of the Hispanic students and about 75% of the White students. Only about 25% of the LEP students met the eighth grade standards in mathematics.

District D serves over 90,000 students, 55% of whom are White, about 35% are African American, and about 5% are Hispanic. Less than 1% of students are classified as LEP. In 2007, about 40% of the Hispanic students, 30% of the LEP students, and about 25% of the African American students met the eighth grade standards in mathematics as compared to 50% of the White students.

In each of the districts, the research team selected a sample of 6 to 10 middle grades schools that reflected variation in student performance and in capacity for improvement across the district. Within each school, we randomly selected up to five mathematics teachers to participate in the study, for a total of approximately 30 teachers per district. My analytic sample includes 56 middle school mathematics teachers: with 30 in District B, and 26 in District D.

**Measures**

For the larger study, each year (2007-2011), several types of data were collected to test and refine a set of hypotheses and conjectures about district and school organizational arrangements, social relations, and material resources that might support mathematics teachers’ development of high-quality instructional practices at scale. Data collected include interviews with all participants (teachers, coaches, principals, and district personnel), video-recordings of classroom instruction, assessments of teachers’ mathematical knowledge for teaching (MKT; Hill, Schilling, & Ball, 2004), video- and/or audio-recordings of professional development
sessions, participant surveys, and student achievement data. For the purpose of this investigation, I draw on data from the 2008-2009 school year, the second year of the larger study. The primary data sources are video-recordings of two consecutive days of classroom instruction, assessments of teachers’ MKT, surveys, interviews, and student demographic and achievement data. The measures are described in detail below and descriptive statistics, by district, are given in Table 1.

Cognitive demand of mathematical tasks. The primary outcome variables of interest were derived from the video-recordings of classroom instruction, coded using the Instructional Quality Assessment (IQA; Boston & Wolf, 2006). Among other measures of instructional quality, the potential cognitive demand of the task as set-up (Task Potential) and the cognitive demand of the task as implemented (Implementation) correspond to phases of instruction within the Math Tasks Framework (see Figure 1) and were assessed by coders. Inter-coder reliability for the Task Potential rubric was 56.9% agreement, with a corresponding Cohen’s kappa score of 0.29. The inter-coder reliability for the Implementation rubric was 78.5% agreement, with a corresponding kappa score of 0.37. While these are lower than desirable, it is important to note that the percent agreement within 1 score category was 92% and 96.1%, respectively, and that these instruments are quite complex. These two scores, Task Potential and Implementation, are the primary measures used in this study to determine the quality of task choice and maintenance of the cognitive demand.

The IQA was developed by a team of researchers at the University of Pittsburgh, and the Task Potential and Implementation rubrics were based on the earlier work by Stein and colleagues (e.g., see Stein, et al., 1996; Stein & Lane, 1996), described above. The instrument describes five levels of cognitive demand (with four of them being mathematical in nature).
a level 0, the task is not mathematical in nature. Tasks at levels 1 and 2 are low in cognitive
demand, with a level 1 task requiring only memorization or the reproduction of facts and a level
2 task requiring students to perform relatively routine procedures without making connections to
the underlying mathematical ideas. Levels 3 and 4 represent tasks of high cognitive demand. At
a level 3, students are required to make connections to underlying mathematical ideas, but tasks
may lack explicit requests for generalization or justification. At the highest level, a level 4 task
asks students to engage in disciplinary activities of explanation, justification and generalization,
or using procedures to solve tasks that are somewhat ambiguous in nature. The full Task
Potential and Implementation rubrics are given in Appendix A. Lastly, maintenance of the
cognitive demand of a task is a measure of whether the score for Implementation is at least as
high as the score for the Task Potential. In this case, maintenance is just a measure of whether
the score for Task Potential is equal to the score for Implementation because in my sample tasks
did not increase from set-up to implementation.

Teacher characteristics. The primary teacher characteristics of interest include:
experience teaching mathematics, experience with the curriculum, mathematical knowledge for
teaching, and beliefs about teaching and learning mathematics. Two basic teacher characteristics
were included in these models: their years of experience teaching mathematics (Yrs Experience)
and whether or not they are new to the curriculum (New to CMP2). The first measure was
collected as a part of the survey and the second is derived from survey information and
information about district curriculum implementation¹. Mathematical knowledge for teaching

¹ In District D the curriculum was “rolled out” so some teachers were in their first year of curriculum
implementation despite the fact that it was officially adopted the prior year. The other reason teachers were new to
the curriculum is if it was their first year teaching math in that school district.
was assessed using the paper-and-pencil test developed by Hill and colleagues (Hill & Ball, 2004; Hill, et al., 2007), and administered to teachers.

Several measures of teachers’ beliefs about teaching and learning mathematics were created by coding transcripts of interviews. First, teachers’ visions of high quality mathematics instruction (VHQMI; Munter & Correnti, 2011) were determined based on teachers’ responses to a question about what they would look for during an indefinite observation to determine if the mathematics instruction were high quality. For the purpose of this study, I utilize only one aspect of VHQMI, beliefs about the role of the teacher (Beliefs about RT), because of their suggested importance within the literature. Scores for beliefs about the role of the teacher ranged from 0 representing “teacher as motivator” and 1 representing “teacher as deliverer of knowledge”, to 4 representing “teacher as more knowledgeable other”. See Appendix B for details of the entire score range of the VHQMI role of the teacher rubric.

The second measure of teachers’ beliefs about teaching and learning mathematics, coded from interview transcripts is teachers’ views of students’ mathematical capabilities (VSMC). Generally, this measure captures whether the teacher holds unproductive views of students’ mathematical capabilities. In particular, unproductive VSMC are ones that position the teacher as unable to effect change that will in turn support struggling students to substantially participate in rigorous mathematical activity. For the purpose of this study, unproductive VSMC was coded with a dummy variable (Unproductive VSMC) with 1 indicating that the teacher espoused unproductive views of students’ mathematical capabilities. It is important to note that for some teachers we were unable to code for this variable within the interviews, so some teachers in the sample are missing a VSMC score. In Table 1, the descriptive statistics for all measures are given for the sample with VSMC scores (“Complete Sample”) and the sample without VSMC
scores ("Larger Sample"), for both districts. For both of these interview-based measures, training reliability was over 80% and ongoing reliability was assessed to check for rater drift.

**Contextual factors.** The review of the literature suggested several key contextual factors that might be related to the cognitive demand of mathematical tasks. One set of contextual factors included in this study are related to demographics and background knowledge of students. These measures are derived from student achievement data. In particular, I include aggregate measures of the percentage of students in the class who were classified as receiving special education services (%SPED), the percentage of students in the class who were classified as limited English proficiency (%LEP), the percentage of students in the class who were classified as eligible for free or reduced price lunch (%FRL), mean prior student achievement (M_PRIOR) and the standard deviation of prior student achievement (SD_PRIOR)

Another category of contextual factors included in this study are supports. This category includes both formal and informal professional development and both measures of supports come from the teacher survey. The measure of formal professional development is based on teachers’ responses to a question about the extent to which challenging, problem-solving tasks were addressed in professional development sessions. Scores range from 0 = not at all to 3 = to a great extent. The measure of informal professional development is a scale that was created based on 10 items pertaining to collaboration with other math teachers (alpha = 0.93). Scores range from 0 = never to 4 = at least weekly. For more information about both of these survey-based professional development measures see Appendix C.

---

2 For 4 teachers in this sample, very limited student achievement data was provided (i.e., under 5 students per class) so the average prior student achievement and standard deviation of prior student achievement for those teachers were imputed using multiple imputation in STATA.
Hypothesized mediators. In order to test out the four hypotheses described above, several different threshold variables and interaction terms were created. To test out hypothesis 1, pertaining to the need for some basic level of beliefs about the role of the teacher in order for the positive relationship between MKT and cognitive demand to be sustained, I first created two threshold variables for beliefs about the role of the teacher. While scores of 1 indicate traditional beliefs about the role of the teacher, scores of 2 indicate that the teachers’ beliefs allow for group work yet still indicate a level of control on the part of the teacher that is not very conductive to exploration of mathematical ideas. I created threshold variables indicating whether or not a teacher held beliefs at a level 1 (RT=1), and whether or not a teacher held beliefs at a level 2 or below (RT<=2), since none of the teachers in our sample were scored at a level 0. To test out the hypothesized moderating relationship, I created interaction terms between the threshold variables and MKT scores.

Hypothesis 2 is similar to hypothesis 1 in that it involves an interaction between MKT and beliefs about the role of the teacher. But, in this case, it is a threshold for mathematical knowledge for teaching that is interacted with beliefs about the role of the teacher. Given that the IRT scores for MKT are based on a nationally representative sample, the threshold of 0 seems like an appropriate cut-off given that enacting high quality mathematics instruction is very challenging. The threshold variable, MKT<0, was created to indicate that teachers are below the national average in their mathematical knowledge for teaching. To test out the moderating hypothesis, this threshold variable for below average MKT was interacted with beliefs about the role of the teacher.

In a similar fashion to the previous two hypotheses, in order to test out hypothesis 3, pertaining to whether unproductive beliefs about students’ mathematical capabilities has a
diminishing effect on the relationship between MKT and cognitive demand, I used the existing dummy variable for unproductive VSMC and created an interaction term with that variable and MKT scores.

Hypothesis 4 pertains to alignment between beliefs and its influence on the relationship between beliefs about teaching and learning and cognitive demand. I utilized the existing threshold variables for beliefs about the role of the teacher and beliefs about students’ mathematical capabilities to characterize alignment between teachers’ beliefs. Generally, we would expect that teachers with more productive views of students’ mathematical capabilities would also have more inquiry-oriented beliefs about the role of the teacher, but there are cases where teachers’ beliefs about the role of the teacher are more sophisticated than their beliefs about students’ mathematical capabilities, or the converse. The other important distinction is when beliefs are aligned whether they are generally productive or unproductive. Hence, three dummy variables were created to distinguish between the four different alignment cases: aligned unproductive (comparison case), aligned productive (Both High), unaligned with VSMC more productive than beliefs about RT (VSMC>RT), unaligned with beliefs about RT more productive than VSMC (VSMC<RT).

Analytic Models

In an effort to address the research questions, several different sets of analyses were conducted. I estimated statistical models with teacher characteristics, contextual factors and moderators suggested by the review of the literature. Also, given that our data consists of groups of teachers nested within schools, all of the models were adjusted for clustering at the school level. Given the small size of the sample, I was required to make careful decisions about which
variables to include at which times. For this reason, I used a set of models for each research question, rather than one all-inclusive model, to look for trends in relationships and to highlight particular features. For example, hypothesized moderators were included only in models testing those specific moderating relationships. Also, since the data come from two different schools districts and there is the possibility of district-related omitted variable bias, I include a dummy variable (District B) to account for district membership in each of the models. Lastly, for ease of interpretation, standardized versions of all student background variables and the standard deviation of prior achievement are used in the models.

With regard to the first research question, pertaining to tasks posed to students, I estimated multinominal logistic regression models of the level of cognitive demand of the task (scored as the Task Potential in the IQA) comparing the score categories. The multinominal logistic regression model allows for differences in impact of the predictor variables when moving between different score categories. In other words, the estimated impact of a particular factor can vary between score categories 2 and 3 when compared to moving from a 3 to a 4 and the multinominal logistic regression model allowed me to examine that variation.

To address the second research question, pertaining to maintenance of the cognitive demand of high-level tasks, I estimated a series of logistic regression models of the maintenance of the cognitive demand (i.e., was the cognitive demand maintained or did it drop from task potential to task implementation?), limited to teachers who posed high-level tasks. For both research questions, the same teacher characteristics, contextual factors, and moderating relationships were investigated.
Results

Cognitive Demand of Tasks Posed to Students

With regard to factors related to the cognitive demand of tasks teachers pose to students, results from estimation of 8 different models are given in Tables 2 and 3. For each model, two columns of coefficients and standard errors are shown: 2 v. 3 and 4 v. 3. They are labeled in this manner to denote that a Task Potential score of 3 is the base outcome, with a positive value suggesting that the teacher is more likely to choose the other outcome over the 3 and a negative value suggesting that the teacher is less likely to choose the other outcomes. All results are presented in the tables but only a subset of those results is highlighted in the text. Given the limited sample size, statistical results with p<.1 are reported as statistically significant.

First, due to the reduced sample of teachers with scores for VSMC, two baseline models were estimated. The results from the baseline model with all factors except for unproductive VSMC are labeled model (1a) in Table 2, and the results from the analogous model with the reduced, complete sample (n=37) are labeled model (1b) in Table 3. The results from the complete baseline model (including all factors from previous models and unproductive VSMC) are labeled model (1c) in Table 3. Results from those models make it clear that being new to the curriculum seems to have an impact on the cognitive demand of tasks posed to students. All three models suggest that a teacher who is new to CMP2 is more likely to pose a level 2 task over a level 3 task (p<.05), and the baseline model without VSMC (1a) also suggests that being new to the curriculum makes it more likely that you will pose a level 4 task over a level 3 task, when you have not controlled for VSMC (p<.05). Other findings are less consistent across all three models. Comparisons between models (1a) and (1b) suggest that some of the differences
may be attributable to differences in the sample. For example, for the complete sample (N=37), the model suggests that as the class percentage of students eligible for free or reduced price lunch increases, teachers are more likely to pose a level 3 task over a level 2 or a level 4 for both model (1c) which controls for unproductive VSMC (r = -1.50, p<.05; r = -1.94, p<.1), and model (1b) which does not (r = -1.51, p<.05; r = -1.90, p<.1). Other factors that are related to the task potential for just the smaller Complete Sample are the class percentage of limited English proficiency (LEP) students and the time spent in collaborative discussion with other teachers in their school: as the percentage of LEP students increases, teachers are more likely to pose a level 2 task over a level 3 task (r = 3.64, p<.05); and if teachers spend more time in collaborative discussion with other teachers in their school, they are more likely to pose a level 2 tasks over a level 3 task (r = 3.13, p<.05). Some of these findings run counter to what would be expected based on the literature (e.g., %FRL and time in collaborative discussion) but given that they only hold for the smaller Complete Sample and do not seem to change dramatically with the addition of unproductive VSMC, I do not attend to them in greater detail.

Based on model (1a) there is some indication that teachers with higher mathematical knowledge for teaching are less likely to pose a level 2 task over a level 3 task (r = -0.79, p<.1), but that does not hold for the Complete Sample and the estimated coefficient decreases even more when the model controls for unproductive VSMC. Also, results from models (1a) and (1b) suggest that teachers whose classrooms have students with greater variation in background knowledge (indicated by SD_Prior) are more likely to pose a level 4 task (model (1a): r = 0.65, p<.05 and model (1b): r = 1.02, p<.1), rather than a level 3 task. With the addition of unproductive VSMC (see model (1c) in Table 3), statistical significance of SD_Prior does not hold.
After estimating the baseline models, models were estimated for each of the moderating hypotheses. While these tests of moderators may have an impact on relationships between factors and task potential, those results are generally not discussed here. Recall that hypotheses 1 and 2 both pertain to interactions between mathematical knowledge for teaching and beliefs about the role of the teacher. But, the models testing these hypotheses attempt to match the causal explanations given for the interrelationships in the literature (e.g., a baseline level of knowledge limits the effect of teacher beliefs, or unsophisticated beliefs limit the impact of teacher knowledge). I report the results from the models individually here and discuss the implications in greater detail in the Discussion section.

For hypothesis 1, suggesting that beliefs about the role of the teacher moderate the relationship between mathematical knowledge for teaching and the task potential, two different models were estimated to test out different critical thresholds for beliefs about the role of the teacher (see Table 2). Generally, results from these two models suggest that beliefs about the role of the teacher do moderate the relationship between mathematical knowledge for teaching and the tasks that teachers pose to students. The results are slightly more consistent for the threshold of RT<=2, so I focus on those results here. First, the addition of the interaction between RT<=2 and MKT made it so that MKT is significantly related to the task potential for both score comparisons (r = -2.23, p<.05 and r = 2.82, p<.1). Further, the coefficients suggest that that relationship is in the expected direction: as mathematical knowledge for teaching increases, teachers pose tasks with higher potential. The negative coefficient for the moderator on the 3 v. 4 model suggests that an RT score less than or equal to 2 combined with increased MKT offsets the generally positive effects of higher mathematical knowledge for teaching, and
teachers with higher MKT and low RT scores are actually more likely to choose a task with level 3 potential over a task with level 4 potential.

With regard to hypothesis 2, the alternate interpretation of the interaction between MKT and beliefs about the role of the teacher-- a basic level of MKT moderating the relationship between beliefs about the role of the teacher and the task potential-- there is some evidence that such a moderating relationship exists. The inclusion of the interaction between the dummy variable representing an MKT score below the national average and teachers’ beliefs about the role of the teacher demonstrates that more sophisticated beliefs about the role of the teacher make it less likely that teachers will pose a level 2 task over a level 3 task (r = -0.81, p<.1), but if teachers have below average MKT scores, then the positive effect of sophisticated beliefs about the role of the teacher are negated (r = 0.97, p<.1). There is no parallel result for a level 4 task over a level 3 task, but this model still gives some evidence of a moderating effect of MKT.

Concerning hypothesis 3, pertaining to beliefs about students’ mathematical capabilities moderating the relationship between MKT and the task potential, results do not verify the hypothesized relationship. In particular, the estimated model closely resembled model (1b) (see Table 3). The coefficients for the interaction term between unproductive VSMC and MKT are not significantly different from zero. It is worth noting that the coefficients for the interaction terms are relatively large in magnitude, but the standard errors are also large in magnitude (r = -2.43 with SE = 3.06 and r = -5.49 and SE = 3.48), hence a larger sample might help with further examination of this relationship.

Lastly, with regard to hypothesis 4, pertaining to alignment between teachers’ beliefs moderating the relationship between beliefs about teaching and learning mathematics and the
task potential, results suggest that alignment between different aspects of teachers’ beliefs do seem to moderate the relationship (see Table 3). Results from the estimated model suggest that if teachers’ beliefs about students’ mathematical capabilities are unproductive but teachers’ beliefs about the role of the teacher are more closely aligned with the NCTM vision (indicated by VSMC<RT) then teachers are more likely to pose a level 2 task over a level 3 task \( (r = 5.82, p<.05) \) and are more likely to pose a level 3 task over a level 4 task \( (r = -11.0, p<.05) \). With regard to the opposite situation, when a teacher has relatively unsophisticated beliefs about the role of the teacher but productive beliefs about students’ mathematical capabilities (indicated by VSMC>RT), then it appears that those teachers are more likely to choose a level 2 task over a level 3 task \( (r = 2.84, p<.05) \). Lastly, it is also important to note that the dummy variable accounting for cases where both types of beliefs are high is not significantly related to the task potential. The implications of this set of findings are discussed more below, but it seems that there is no systematic relationship when beliefs are aligned, and a lack of alignment (in either direction) has a negative impact on task potential.

**Maintenance of the Cognitive Demand of High-Level Tasks**

The second major research question that this study addressed was how these teacher characteristics and contextual factors are related to maintenance of the cognitive demand of high-level tasks. First, since I am just interested in maintenance of the cognitive demand for teachers who initially posed high-level tasks, this sample is even more limited: of the 56 people in the original sample, 41 of them posed high level tasks. So, this set of models is limited to just those 41 teachers who posed high-level tasks. And, of those 41 teachers, only 7 of them maintained the cognitive demand.
In attempting to estimate a baseline model containing all of the factors, the first thing that I discovered was that being new to CMP2 and holding unproductive views of students' mathematical capabilities are both perfectly predictive of a decrease in the cognitive demand. It is important to note that the sample of teachers with scores for VSMC is limited to just 25 of the 41 teachers. Figure 2 shows the distributions of different score combinations and blue areas of the circles indicate that the cognitive demand of a high level task was maintained. The full red circles in four of the cells indicate that for those particular score combinations, none of the teachers maintained the cognitive demand of the high-level task. All 7 of the teachers who maintained the cognitive demand of the high level tasks they posed were not new to CMP2 and had either no score (3 of them) or were deemed to espouse productive VSMC (4 of them). So, while being new to CMP2 and/or having an unproductive VSMC is perfectly predictive of a decrease in cognitive demand, it is important to note that the majority of teachers (10 of 14) who were experienced with CMP2 and had productive VSMC still had lessons with decreases in cognitive demand. Since those two variables are perfectly predictive of a decrease in cognitive demand, they are removed from all of the other models, and hence, I was unable to test many of the hypotheses that involved teachers’ views of students’ mathematical capabilities as they relate to maintenance of the cognitive demand.

First, the baseline model, including all of the teacher characteristics and contextual factors except new to CMP2 and unproductive VSMC is given in the first two columns of Table 4. The primary finding is that when controlling for all of the other factors, beliefs about the role of the teacher is significantly related to maintenance of the cognitive demand of high level tasks ($r = 1.01, p<.05$). The odds ratio of 2.76 suggests that a 1 unit score increase in beliefs about RT,
would make it 2.76 times more likely that that teacher would maintain the cognitive demand of the high-level task he or she posed.

Given the predictive nature of unproductive VSMC, the only hypotheses that I was able to test were hypotheses 1 and 2, both involving different types of interactions between beliefs about the role of the teacher and MKT. Results from models testing those hypotheses are given in Table 4, but the results reveal that the moderating relationships do not seem to hold in the same way for maintenance of the cognitive demand as they did for task potential. Even with the interactions included in the model, beliefs about the role of the teacher are consistently related to maintenance of the cognitive demand. The inclusion of the interaction did increase the strength of the relationship between beliefs about the role of the teacher and maintenance of the cognitive demand (e.g., for MKT<0 on RT, r = 6.33 compared with r = 1.01 in the baseline model), so it is possible that there is some slight moderating relationship that is not statistically significant due to the size of the sample.

Discussion

By drawing on existing literature to form a set of hypothesized factors and moderating relationships, and combining that with an interesting set of measures of the critical constructs, I have identified some critical factors and moderating relationships related to cognitive demand of mathematical tasks in middle school classrooms. In particular, I examined task potential and maintenance of the cognitive demand separately and found different results suggesting that there are different mechanisms at play with regard to task potential and maintenance of the cognitive demand.
Across all of the models investigating task potential, I found that being new to the curriculum is related to the potential of the task that teachers pose to students, and that teachers who are new to the curriculum tend to pose tasks at a level 2 or a level 4, rather than a level 3. One possible explanation is that teachers who are new to the curriculum may either take the task from the curriculum as is and pose it to students (which would often result in a 4) or they may change the nature of the task or even supplement with a task from a different set of materials (which often results in the use of a level 2 task).

In general, results suggest that teachers’ mathematical knowledge for teaching is related to the potential of the task that teachers pose to students. This is consistent with the larger body of literature suggesting that MKT is related to the nature of classroom mathematical activity (e.g., see Escudero & Sánchez, 2007; Hill, et al., 2007; Hill, et al., 2008; Shechtman, et al., 2010) and Charalambous’ (2010) examination of task unfolding. While Charalambous found that MKT did appear to be related to the paths of cognitive demand through the different phases of the lesson (e.g., from high level in the book, to low level as posed to students, to low level as implemented), he did not specifically examine the relationship between MKT and the cognitive demand of tasks posed to students. Hence, this finding adds some specificity to that phase of task unfolding and its relationship with MKT.

Several student-related factors were found to be related to the cognitive demand of tasks posed to students, but there is some evidence many of those findings may be unique to the particular samples. Lastly, the two variables measuring teacher supports were related to the cognitive demand of tasks teachers pose to students for the smaller complete sample, but not for the larger sample. Hence, it is difficult to determine the nature of the relationship between these contextual factors and the cognitive demand of tasks posed to students in general.
Several results suggest that many of the hypothesized moderating relationships are true for the cognitive demand of tasks posed to students. First, there is a significant interaction between mathematical knowledge for teaching and beliefs about the role of the teacher, in their relationship with the cognitive demand of tasks posed to students. The first hypothesis tested the interpretation that beliefs about the role of the teacher moderate the relationships between mathematical knowledge for teaching and the cognitive demand of tasks posed to students. The results from this investigation suggest that, if a teacher has beliefs about the role of the teacher that are not consistent with an inquiry-based approach then they negate the effect of high levels of mathematical knowledge for teaching. So, it is not enough to have high levels of mathematical knowledge for teaching, the teacher must also have beliefs about the role of the teacher that are consistent with an inquiry-based approach in order for the high levels of mathematical knowledge for teaching be positively related to the cognitive demand of the tasks posed to students. This is consistent with the case-study findings of Schoenfeld (2011) and Ball (1991), and extends those findings to a larger sample of teachers.

The second hypothesis suggested another interaction between mathematical knowledge for teaching and beliefs about the role of the teacher. The results from modeling of this hypothesis suggest that mathematical knowledge for teaching moderates the relationship between beliefs about the role of the teacher and the cognitive demand of tasks posed to students. This result held for the choice between a level 2 task and a level 3 task, but not for the choice between a level 3 and a level 4 task. When MKT is above the national average, more sophisticated beliefs about the role of the teacher make it more likely that the teacher will choose a high-level task over a low-level task, but if MKT is below the national average, then that relationship no longer holds. Hence, these results are consistent with what Ball (1991) and Putnam and colleagues
(1992) found generally with regard to high quality mathematics instruction, and go further to specifically validate the moderating relationship for the cognitive demand of tasks posed to students.

While it is clear that there is a complicated interrelationship between mathematical knowledge for teaching and beliefs about the role of the teacher, it is not clear which of these interpretations suggesting some directionality to the relationship is actually the mechanism at work. Given that prior research suggests that both interpretations are likely and viable, it could be that both mechanisms are at work simultaneously. In order to investigate these hypotheses with an eye toward directionality of the moderating relationship, further investigation with longitudinal data is necessary.

The third hypothesis was that teachers’ beliefs about students’ mathematical capabilities moderate the relationship between mathematical knowledge for teaching and the cognitive demand of tasks posed to students. Results for that model were not as definitive. Yet, the results were not at all contradictory with regard to the hypothesized relationship and perhaps would have been consistent if the size of the sample were not so small. Hence, further investigation of this relationship with a larger sample of data is necessary.

Beyond investigating mathematical knowledge for teaching as a key moderator of the relationship between beliefs about teaching and learning mathematics and the cognitive demand of mathematical tasks posed to students, I explored the impact of alignment of different aspects of beliefs about teaching and learning math on the cognitive demand of tasks posed to students. Interestingly, I found that alignment between beliefs about the role of the teacher and beliefs about students’ mathematical capabilities does seem to be related to cognitive demand of
mathematical tasks posed to students. Further, I found that when VSMC is unproductive and unaligned with RT, then teachers are more likely to choose lower cognitive demand tasks; when VSMC is productive and RT is low, then teachers are more likely to choose a 2 over a 3. Given that the main effects of unproductive VSMC and beliefs about the role of the teacher are not statistically significant in the baseline model, this suggests that it is not just about one or the other being dominant, but, instead, that a lack of alignment between beliefs is problematic in itself. This is generally consistent with what Cooney (1985) found which suggested that a lack of alignment between beliefs about the role of the teacher and beliefs about students seemed to negatively influence the teacher’s choice of tasks.

The second set of analyses pertained to maintenance of the cognitive demand of high-level tasks. First, consistent with the work of Stein and colleagues (e.g., see Stein, et al., 1996) I found that it is common for teachers to decrease the cognitive demand of high level tasks. The next step was to figure out which hypothesized teacher characteristics and contextual factors are associated with these decreases in the cognitive demand. In our sample, all of the teachers who were new to the curriculum and posed high-level tasks, decreased the cognitive demand of those tasks. Similarly, all of the teachers whom we characterized as having unproductive beliefs about students’ mathematical capabilities, who posed high-level tasks, decreased the cognitive demand of those tasks. Hence, being new to the curriculum and/or having unproductive beliefs about students’ mathematical capabilities seems to make it challenging to maintain the cognitive demand of high-level tasks. Therefore, we should take this into consideration when we design supports for teachers: teachers need lots of support in their first year of curriculum implementation and we may need to attend to teachers’ views of students’ mathematical capabilities more directly as they seem to have an impact on their instructional decisions.
While mathematical knowledge for teaching and views of students’ mathematical capabilities featured prominently as factors related to the tasks that teachers pose to students, teachers’ beliefs about the role of the teacher were consistently related to maintenance of the cognitive demand of high-level tasks. Since maintenance of the cognitive demand of high-level tasks is not just the mathematical activity that is intended for students but actually how it is enacted in the classroom, it makes sense that beliefs about the role of the teacher might feature more prominently during this phase of instruction. For example, if a teacher believes that the teacher should let students struggle with little teacher intervention (RT=3) then that might result in a different task enactment than if the teacher believes that students should first be shown how to do some problems and then should have opportunities to work on similar problems in groups (RT=2), even if in both situations the same task was initially posed.

Lastly, only two of the hypothesized relationships were tested for maintenance of the cognitive demand of high-level tasks and both involved interactions between mathematical knowledge for teaching and beliefs about the role of the teacher. While the inclusion of the interaction term increased precision and strength of the relationship between beliefs about the role of the teacher and maintenance of the cognitive demand, the interaction term was not statistically significant. Hence, though mathematical knowledge for teaching may slightly moderate the relationship between beliefs about the role of the teacher and maintenance of the cognitive demand, that hypothesis warrants further investigation with a larger sample.

Limitations and Conclusions

Despite the fact that this study has produced some interesting findings, it is important to acknowledge several limitations related to the sample. First, in general, the sample size in this
study is very small and I have had to make decisions about including variables and testing hypotheses that may violate general guidelines followed by the statistically conservative. Also, the use of logistic regression models is generally not recommended for samples of this size. The primary problem is that I may be violating some of the statistical assumptions associated with the models, especially with regard to asymptotic properties. But, given that the logistic regression models fit my research questions more appropriately than linear regression models, I made the decision to move forward with logistic regression models. In the future, it will be important to verify these results with larger samples.

The other limitation is the further reduced sample due to the lack of unproductive VSMC scores for all of the participants. While I have done my best to ensure that the sample with scores is not significantly different from the larger sample, there are no guarantees. It is possible that a certain characteristic in teachers makes it impossible to code their interview transcripts for their views of students’ mathematical capabilities and I have no way of knowing what that is at this stage. I am hopeful that a lack of a VSMC is more of a limitation related to the interviewers which can be explained as external to the situation of the teachers.

Overall, it is clear that a subset of the hypothesized teacher characteristics and contextual factors are systematically related to the cognitive demand of mathematical tasks in the classroom for this sample. Yet, given the differences in findings for the two outcomes and the verification of the moderating relationships, it is clear that the relationships are very complicated. For example, while mathematical knowledge for teaching is related to the tasks that teachers pose, beliefs about students’ mathematical capabilities and the role of the teacher have an impact on that relationship, and limit the effect that mathematical knowledge for teaching can have on the cognitive demand of tasks posed to students. Also, although I have not placed as much emphasis
on the contextual factors included in the analysis, it was their inclusion that increased precision and allowed for the hypothesized interrelationships to be tested. Hence, this study also provides further evidence of the complicated nature of the teaching system (Ball & Forzani, 2007).

Lastly, the findings of this study have implications for professional development for mathematics teachers. Given that mathematical knowledge for teaching, beliefs about the role of the teacher, and beliefs about students’ mathematical capabilities are all related to and mutually necessary for cognitively demanding mathematical activity in the classroom, it seems that professional development cannot focus on just developing one of these teacher characteristics, but, instead, we must find integrated ways to develop teachers on all of these dimensions simultaneously.
References:


